

Optical solitons in multiple-core couplers

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This paper obtains 1-soliton solutions in multiple-core nonlinear directional couplers. Bright, dark and singular soliton solutions are obtained. The necessary constraint conditions are exhibited. Coupling with nearest neighbors as well as all neighbors are considered. There are five types of nonlinear media that are studied. They are Kerr law, power law, parabolic law, dual-power law and log law.

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1. Introduction

The dynamics of solitons in nonlinear directional couplers (NLDC) has been studied for the past few years in the context of nonlinear optical fibers [1-30]. There are several forms of results that are being produced and they are mostly numerical simulations. However, what is not visible is the analytical aspect of NLDC. Analytical results are truly missing from this literature. Therefore, it is important to address this aspect so that the gap can be filled in.

This paper addresses the analytical aspects of optical solitons in multiple-core optical couplers where coupling is with nearest neighbors as well as with all neighbors. Bright, dark and singular optical soliton solutions will be retrieved along with several necessary constraint conditions on the soliton parameters. There are five forms of nonlinear media that will be studied in this context. They are Kerr law, power law, parabolic law, dual-power law and log law. The results will be extremely helpful in the study of optical routing as well as switching and other studies. The detailed study will now be conducted in the following two sections.

2. Coupling with nearest neighbors

The governing equation for multiple-core couplers is given by [1, 9, 10, 30]

$$i q_t^{(l)} + a q_{xx}^{(l)} + b F(|q^{(l)}|^2) q^{(l)} = K [q^{(l-1)} - 2q^{(l)} + q^{(l+1)}] \quad (1)$$

where $1 \leq l \leq N$. Equation (1) represents the general model for optical couplers where coupling with nearest neighbors is considered. In (1), a is the coefficient of group velocity

dispersion (GVD) and b is the coefficient of nonlinearity. The functional F represents non-Kerr law nonlinear media that will be studied in details in the next five subsections. On the right hand side, K represents coupling coefficient. It must be noted that the general case of optical couplers with the inclusion of spatio-temporal dispersion is already studied earlier [30]. This paper however considers only GVD.

In order to address this model for the five forms of nonlinear media, the initial hypothesis is taken to be

$$q^{(l)}(x, t) = P_l(x, t) e^{i\phi(x, t)} \quad (2)$$

where the amplitude component of soliton is $P_l(x, t)$ while the phase component is defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta \quad (3)$$

Here, κ is the soliton frequency, while ω is the wave number and θ is the phase constant. After substituting hypothesis (2) into (1) while utilizing (3), the resulting expression is split into real and imaginary components. The imaginary part gives the speed of the soliton as

$$v = -2a\kappa \quad (4)$$

The speed of the soliton stays the same for any kind of nonlinearity as well as for all types of nonlinear media and all kinds of solitons. Next, the real part implies

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - b F(P_l^2) P_l - K [P_{l-1} - 2P_l + P_{l+1}] = 0 \quad (5)$$

It is this real part equation that will be further analyzed in the next five subsections based on the types of nonlinearity and types of solitons.

2.1. Kerr law nonlinearity

For Kerr law, governing equation (1) modifies to [9, 27-30]

$$iq_t^{(l)} + aq_{xx}^{(l)} + b|q^{(l)}|^2 q^{(l)} = K[q^{(l-1)} - 2q^{(l)} + q^{(l+1)}] \quad (6)$$

Substituting hypothesis (2) into equation (6) reduces it to

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - bP_l^3 + K[P_{l-1} - 2P_l + P_{l+1}] = 0 \quad (7)$$

For Kerr law nonlinearity, the study will be further split into the following three subsections, depending on the type of soliton. These are bright, dark and singular solitons.

2.1.1 Bright solitons

For bright solitons, starting hypothesis for Kerr law is [9, 27-30]

$$P_l = A_l \operatorname{sech}^{p_l} \tau \quad (8)$$

where

$$\tau = B(x - vt) \quad (9)$$

Here, B represents the inverse width of the soliton. Substituting (8) into (7) leads to

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \operatorname{sech}^p \tau + aA_l B^2 p(p+1) \operatorname{sech}^{p+2} \tau - bA_l^3 \operatorname{sech}^{3p} \tau + K(A_{l-1} - 2A_l + A_{l+1}) \operatorname{sech}^p \tau = 0 \quad (10)$$

after simplification. Balancing principle implies

$$p = 1 \quad (11)$$

Setting the coefficients of linearly independent functions to zero gives

$$\omega = \frac{1}{A_l} [aA_l(B^2 - \kappa^2) - K_l(A_{l-1} - 2A_l + A_{l+1})] \quad (12)$$

$$B = \sqrt{\frac{b}{2a}} A_l \quad (13)$$

which naturally poses the constraint

$$ab > 0 \quad (14)$$

This shows that GVD and nonlinearity must bear the same sign for bright solitons to exist. Hence, the 1-soliton solution to the NLSE in multiple-core couplers with Kerr law nonlinearity is

$$q^{(l)}(x, t) = A_l \operatorname{sech}[B(x - vt)] e^{i(-\kappa x + \omega t + \theta)} \quad (15)$$

where the amplitude-width relation is given by (13), while the wave number is in (12).

2.1.2 Dark solitons

For dark solitons, the starting hypothesis is [27-30]

$$P_l = A_l \tanh^p \tau \quad (16)$$

with unknown exponent p . Here A_l and B are free parameters. Substituting this hypothesis into (7) leads to

$$A_l(\omega + a\kappa^2 + 2ap^2 B^2) \tanh^p \tau - aA_l B^2 p(p-1) \tanh^{p-2} \tau - aA_l B^2 p(p+1) \tanh^{p+2} \tau - bA_l^3 \tanh^{3p} \tau + K(A_{l-1} - 2A_l + A_{l+1}) \tanh^p \tau = 0 \quad (17)$$

By balancing principle, same value of p as in (11) is revealed. Also, this same value of p is obtained from coefficient of the standalone linearly independent function $\tanh^{p-2} \tau$. From other linearly independent functions,

$$\omega = \frac{1}{A_l} [-aA_l(2B^2 + \kappa^2) - K(A_{l-1} - 2A_l + A_{l+1})] \quad (18)$$

and

$$B = \sqrt{-\frac{b}{2a}} A_l \quad (19)$$

which implies the natural constraint

$$ab < 0 \quad (20)$$

Thus for dark solitons to exist, GVD and nonlinearity must carry opposite signs. Therefore, dark 1-soliton solution to the NLSE in multiple-core couplers with Kerr law nonlinearity is

$$q^{(l)}(x, t) = A_l \tanh[B(x - vt)] e^{i(-\kappa x + \omega t + \theta)} \quad (21)$$

with the respective parameters as defined.

2.1.3 Singular solitons

Here, one starts with the hypothesis [27-30]

$$P_l = A_l \operatorname{csch}^p \tau \quad (22)$$

For singular solitons, A_l and B are also referred to as free parameters. Upon substituting this hypothesis into (7),

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \operatorname{csch}^p \tau - aA_l B^2 p(p+1) \operatorname{csch}^{p+2} \tau - bA_l^3 \operatorname{csch}^{3p} \tau + K(A_{l-1} - 2A_l + A_{l+1}) \operatorname{csch}^p \tau = 0 \quad (23)$$

Balancing principle again leads to the same value of p as given by (11). The linearly independent functions yield the wave number as in (12) while the connection between

the free parameters is the same as (19) along with its constraint (20). Finally, singular 1-soliton solution to NLSE in NLDC with Kerr law is

$$q^{(l)}(x, t) = A_l \operatorname{csch}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{24}$$

2.2 Power law nonlinearity

For power law, the coupled NLSE modifies to [27-30]

$$i q_t^{(l)} + a q_{xx}^{(l)} + b |q^{(l)}|^{2n} q^{(l)} = K [q^{(l-1)} - 2q^{(l)} + q^{(l+1)}] \tag{25}$$

Here n is the power law nonlinearity parameters. For solitons to exist, it is necessary to have $0 < n < 2$. In particular $n \neq 2$, in order to avoid self-focusing singularity. With hypothesis given by (2), equation (25) reduces to

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - b P_l^{2n+1} + K [P_{l-1} - 2P_l + P_{l+1}] = 0 \tag{26}$$

The study, for power law nonlinearity, will now be split into the following three subsections based on the same three types of solitons as in Kerr law.

2.2.1. Bright solitons

For bright solitons, starting hypothesis with power law nonlinearity is same as given by (8). Substituting (8) into (26) leads to [1, 9, 27-30]

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \operatorname{sech}^p \tau + aA_l B^2 p(p+1) \operatorname{sech}^{p+2} \tau - bA_l^{2n+1} \operatorname{sech}^{(2n+1)p} \tau + K(A_{l-1} - 2A_l + A_{l+1}) \operatorname{sech}^p \tau = 0 \tag{27}$$

Balancing principle yields

$$p = \frac{1}{n} \tag{28}$$

Next, setting the coefficients of linearly independent functions to zero leads to

$$\omega = \frac{1}{n^2 A_l} [aA_l(B^2 - n^2 \kappa^2) - n^2 K(A_{l-1} - 2A_l + A_{l+1})] \tag{29}$$

and

$$B = n \sqrt{\frac{b}{(n+1)a}} A_l \tag{30}$$

which naturally poses the constraint given by (14). Hence, finally, bright 1-soliton solution to the NLSE in multiple-core couplers with power law nonlinearity is

$$q^{(l)}(x, t) = A_l \operatorname{sech}^{\frac{1}{n}} [B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{31}$$

2.2.2. Dark solitons

For dark solitons, starting hypothesis stays the same as (16), so that the real part equation (26), simplifies to

$$A_l(\omega + a\kappa^2 + 2ap^2 B^2) \tanh^p \tau - aA_l B^2 p(p-1) \tanh^{p-2} \tau - aA_l B^2 p(p+1) \tanh^{p+2} \tau - bA_l^{2n+1} \tanh^{(2n+1)p} \tau + K(A_{l-1} - 2A_l + A_{l+1}) \tanh^p \tau = 0 \tag{32}$$

By balancing principle (28) is revealed. Also, this same value of p as given by (11) is obtained from coefficient of the standalone linearly independent function $\tanh^{p-2} \tau$. Thus, (11) and (28) together imply that for dark soliton solutions to exist, power law nonlinearity must collapse to Kerr law nonlinearity. This implies that all results of dark solitons for Kerr law nonlinearity, from previous subsection, will remain valid for power law nonlinearity.

2.2.3. Singular solitons

Here, the hypothesis given by (22) will work. Therefore real part equation, based on (22), leads to

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \operatorname{csch}^p \tau - aA_l B^2 p(p+1) \operatorname{csch}^{p+2} \tau - bA_l^{2n+1} \operatorname{csch}^{(2n+1)p} \tau + K(A_{l-1} - 2A_l + A_{l+1}) \operatorname{csch}^p \tau = 0 \tag{33}$$

Balancing principle leads to the same value of p as given by (28). Also, the wave number is the same as in bright solitons as given by (29), while the relation between the free parameters is

$$B = n \sqrt{\frac{b}{(n+1)a}} A_l \tag{34}$$

that naturally implies (20). Hence, finally singular 1-soliton solution to the NLSE in multiple-core couplers with Kerr law nonlinearity is

$$q^{(l)}(x, t) = A_l \operatorname{csch}^{\frac{1}{n}} [B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{35}$$

2.3 Parabolic law nonlinearity

In this case, the governing equation reduces to [1, 9, 27-30]

$$i q_t^{(l)} + a q_{xx}^{(l)} + (b_1 |q^{(l)}|^2 + b_2 |q^{(l)}|^4) q^{(l)} = K [q^{(l-1)} - 2q^{(l)} + q^{(l+1)}] \tag{36}$$

where $1 \leq l \leq N$, while b_j for $j = 1, 2$ are constants. The real part equation therefore is

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - b_1 P_l^3 - b_2 P_l^5 + K [P_{l-1} - 2P_l + P_{l+1}] = 0 \tag{37}$$

Equation (37) will now be analyzed for bright and singular solitons only. Dark solitons for multiple-core couplers with parabolic law nonlinearity cannot be retrieved by this ansatz method.

2.3.1. Bright solitons

For parabolic law nonlinearity, the hypothesis for the waveform is [9]

$$P_l(x, t) = \frac{A_l}{(D + \cosh\tau)^{p_l}} \tag{38}$$

for some unknown exponent p and D is a newly introduced parameter. Here A_l and B represent, as usual, the soliton amplitude and inverse width. Substituting (38) into (37), the real part equation modifies to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2B^2)A_l}{(D + \cosh\tau)^p} - \frac{aDB^2p(2p+1)A_l}{(D + \cosh\tau)^{p+1}} - \\ & - \frac{aA_lB^2p(p+1)(D^2+1)}{(D + \cosh\tau)^{p+2}} - \frac{b_1A_l^3}{(D + \cosh\tau)^{3p}} - \\ & - \frac{b_2A_l^5}{(D + \cosh\tau)^{5p}} + \frac{K_l(A_{l-1} - 2A_l + A_{l+1})}{(D + \cosh\tau)^p} = 0 \end{aligned} \tag{39}$$

Balancing principle gives

$$p = \frac{1}{2} \tag{40}$$

From the linearly independent functions,

$$\omega = \frac{1}{4A_l} [aA_l(B^2 - 4\kappa^2) - 4K_l(A_{l-1} - 2A_l + A_{l+1})] \tag{41}$$

$$D = \frac{-2b_2A_l^2 + \sqrt{4b_2^2A_l^4 + 9b_1^2}}{3b_1} \tag{42}$$

and

$$B = n\sqrt{\frac{b_1}{aD}}A_l \tag{43}$$

which shows that

$$ab_1 > 0 \tag{44}$$

Therefore, for parabolic law nonlinearity, bright 1-soliton solution for optical couplers is given by

$$q^{(l)}(x, t) = \frac{A_l}{\sqrt{D + \cosh\tau}} e^{i(-\kappa x + \omega t + \theta)} \tag{45}$$

2.3.2. Singular solitons

For singular solitons, the waveform is assumed to be of the form [27, 28]

$$P_l(x, t) = \frac{A_l}{(D + \sinh\tau)^p} \tag{46}$$

Here A_l and B once again represent free parameters. Substituting into (37), the real part equation simplifies to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2B^2)A_l}{(D + \sinh\tau)^p} + \frac{aDB^2p(2p+1)A_l}{(D + \sinh\tau)^{p+1}} - \\ & - \frac{aA_lB^2p(p+1)(D^2+1)}{(D + \sinh\tau)^{p+2}} - \frac{b_1A_l^3}{(D + \sinh\tau)^{3p}} - \\ & - \frac{b_2A_l^5}{(D + \sinh\tau)^{5p}} + \frac{K_l(A_{l-1} - 2A_l + A_{l+1})}{(D + \sinh\tau)^p} = 0 \end{aligned} \tag{47}$$

Balancing principle yields (40). From linearly independent functions, wave number and the free parameter B are given as in (41), (43) along with (44). However,

$$D = \frac{-2b_2A_l^2 + \sqrt{4b_2^2A_l^4 - 9b_1^2}}{3b_1} \tag{48}$$

which will make sense for

$$2|b_2|A_l^2 > 3|b_1| \tag{49}$$

Therefore, for parabolic law nonlinearity, singular 1-soliton solution for optical couplers is given by

$$q^{(l)}(x, t) = \frac{A_l}{\sqrt{D + \sinh\tau}} e^{i(-\kappa x + \omega t + \theta)} \tag{50}$$

2.4 Dual-Power law nonlinearity

In this case, the governing equation reduces to [9]

$$\begin{aligned} & iq_t^{(l)} + aq_{xx}^{(l)} + \left(b_1|q^{(l)}|^{2n} + b_2|q^{(l)}|^{4n} \right) q^{(l)} = \\ & = K[q^{(l-1)} - 2q^{(l)} + q^{(l+1)}] \end{aligned} \tag{51}$$

where $1 \leq l \leq N$ and the exponent n represents the dual power law nonlinearity parameter. The real part equation therefore is

$$\begin{aligned} & P_l(\omega + a\kappa^2) - a\frac{\partial^2 P_l}{\partial x^2} - b_1P_l^{2n+1} - b_2P_l^{4n+1} + \\ & + K[P_{l-1} - 2P_l + P_{l+1}] = 0 \end{aligned} \tag{52}$$

Equation (52) will now be analyzed for bright and singular solitons. It needs to be noted that, just as in parabolic law nonlinearity, dark solitons for multiple-core couplers with parabolic law nonlinearity cannot be retrieved by the aid of this integration scheme.

2.4.1. Bright solitons

For parabolic law nonlinearity, the hypothesis for the waveform is given by (38). Substituting into (52), the real part equation is

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2B^2)A_l}{(D + \cosh\tau)^p} + \frac{aDB^2p(2p+1)A_l}{(D + \cosh\tau)^{p+1}} - \\ & - \frac{aA_lB^2p(p+1)(D^2-1)}{(D + \cosh\tau)^{p+2}} - \frac{b_1A_l^{2n+1}}{(D + \cosh\tau)^{(2n+1)p}} - \\ & - \frac{b_2A_l^{4n+1}}{(D + \cosh\tau)^{(4n+1)p}} + \frac{K_l(A_{l-1} - 2A_l + A_{l+1})}{(D + \cosh\tau)^p} = 0 \end{aligned} \tag{53}$$

Balancing principle yields

$$p = \frac{1}{2n} \tag{54}$$

The special case, when $n = 1$, the result collapses to parabolic law nonlinearity. From the linearly independent functions,

$$\begin{aligned} \omega &= \frac{1}{4n^2A_l} [aA_l(B^2 - 4n^2\kappa^2) - \\ & - 4n^2K_l(A_{l-1} - 2A_l + A_{l+1})] \end{aligned} \tag{55}$$

$$D = \frac{-(n+1)b_2A_l^2 + \sqrt{(n+1)^2b_2^2A_l^4 + (2n+1)^2b_1^2}}{(2n+1)b_1} \tag{56}$$

and

$$B = n \sqrt{\frac{2b_1}{(n+1)aD}} A_l^n \tag{57}$$

which will exist when (44) holds. Therefore, for dual-power law nonlinearity, bright 1-soliton solution for optical couplers is given by

$$q^{(l)}(x,t) = \frac{A_l}{(D + \cosh\tau)^{\frac{1}{2n}}} e^{i(-\kappa x + \omega t + \theta)} \tag{58}$$

2.4.2. Singular solitons

For singular solitons, the waveform is given by (46). Substituting into (52), the real part equation transforms to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2B^2)A_l}{(D + \sinh\tau)^p} + \frac{ap(2p+1)A_lDB^2}{(D + \sinh\tau)^{p+1}} - \\ & - \frac{ap(p+1)A_lB^2(D^2+1)}{(D + \sinh\tau)^{p+2}} - \frac{b_1A_l^{2n+1}}{(D + \sinh\tau)^{(2n+1)p}} - \\ & - \frac{b_2A_l^{4n+1}}{(D + \sinh\tau)^{(4n+1)p}} + \frac{K_l(A_{l-1} - 2A_l + A_{l+1})}{(D + \sinh\tau)^p} = 0 \end{aligned} \tag{59}$$

Balancing principle yields (54). From the linearly independent functions, wave number and the free parameter B are given as in (55) and (57) along with (44). However,

$$D = \frac{-(n+1)b_2A_l^2 + \sqrt{(n+1)^2b_2^2A_l^4 - (2n+1)^2b_1^2}}{(2n+1)b_1} \tag{60}$$

which will make sense for

$$(n+1)|b_2|A_l^2 > (2n+1)|b_1| \tag{61}$$

Therefore, for dual-power law nonlinearity, singular 1-soliton solution in NLDC is given by

$$q^{(l)}(x,t) = \frac{A_l}{(D + \sinh\tau)^{\frac{1}{2n}}} e^{i(-\kappa x + \omega t + \theta)} \tag{62}$$

2.5 Log-law nonlinearity

For log law nonlinear medium, NLSE for multiple-core couplers is given by [1, 10, 27-30]

$$iq_t^{(l)} + aq_{xx}^{(l)} + bq^{(l)} \ln|q^{(l)}|^2 = K[q^{(l-1)} - 2q^{(l)} + q^{(l+1)}] \tag{63}$$

It is only Gaussons that can be retrieved for this model [10, 30]. In this case, the real part equation (5) changes to

$$\begin{aligned} & P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - 2bP_l \ln|P_l| + \\ & + K[P_{l-1} - 2P_l + P_{l+1}] = 0 \end{aligned} \tag{64}$$

The starting hypothesis for Gaussons is [10, 30]

$$P_l(x,t) = A_l e^{-\tau^2} \tag{65}$$

Substituting this hypothesis into (64) simplifies it to

$$\begin{aligned} & A_l\omega + 2aA_lB^2 - 2bA_l \ln A_l + aA_l\kappa^2 + \\ & + (2bA_l - 4aA_lB^2)\tau^2 + K(A_{l-1} - 2A_l + A_{l+1}) = 0 \end{aligned} \tag{66}$$

Setting the coefficients of the linearly independent functions τ^j , for $j = 0, 2$ leads to the width of Gaussons being

$$B = \sqrt{\frac{b}{2a}} \tag{67}$$

which indicates the same constraint condition as in (14), for Gaussons to exist. Also, the wave number of Gaussons from linearly independent functions falls out to

$$\begin{aligned} \omega &= -\frac{1}{A_l} [aA_l(2B^2 + \kappa^2) - 2bA_l \ln A_l - \\ & - K_l(A_{l-1} - 2A_l + A_{l+1})] \end{aligned} \tag{68}$$

Hence, finally optical Gausson solution to multiple-core couplers with log-law nonlinearity is

$$q^{(l)}(x,t) = A_l e^{-B^2(x-v)^2} e^{i(-\kappa x + \omega t + \theta)} \quad (69)$$

3. Coupling with all neighbors

The governing equation for multiple-core couplers is given by [1, 30]

$$iq_t^{(l)} + aq_{xx}^{(l)} + bF(|q^{(l)}|^2)q^{(l)} = \sum_{m=1, m \neq l}^N \lambda_{lm} P_m \quad (70)$$

where $1 \leq l \leq N$. Here λ_{lm} represents the coupling coefficients with all neighbors, except with itself. The starting hypothesis for the solution is the same as in (2) with the definition of the phase component as given by (3). Substituting (2) into (70) leads to the speed of the soliton given by (4), from the imaginary part. The real part, however, reduces to:

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - bF(P_l^2)P_l + \sum_{m=1, m \neq l}^N \lambda_{lm} P_m = 0 \quad (71)$$

It is this real part equation that will be further analyzed in the next five subsections based on the types of nonlinearity and types of solitons.

3.1 Kerr law nonlinearity

For Kerr law, the coupled NLSE modifies to

$$iq_t^{(l)} + aq_{xx}^{(l)} + b|q^{(l)}|^2 q^{(l)} = \sum_{m=1, m \neq l}^N \lambda_{lm} q^{(m)} \quad (72)$$

For hypothesis given by (8), equation (71) reduces to

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - bP_l^3 + \sum_{m=1, m \neq l}^N \lambda_{lm} P_m = 0 \quad (73)$$

Equation (73) will now be analyzed in the following three subsections for the types of solitons.

3.1.1. Bright solitons

For bright solitons, starting hypothesis for Kerr law is given by (8). Substituting this hypothesis into (73) gives

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \text{sech}^p \tau - bA_l^3 \text{sech}^{3p} \tau + aA_l B^2 p(p+1) \text{sech}^{p+2} \tau + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \text{sech}^p \tau = 0 \quad (74)$$

Balancing principle yields (11). Next, setting the coefficients of linearly independent functions to zero leads to the wave number of the solitons as

$$\omega = \frac{1}{A_l} \left[aA_l(B^2 - \kappa^2) - \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \right] \quad (75)$$

while the amplitude-width relation is given by (13) along with the constraint (14). Hence, finally the 1-soliton solution to the NLSE in multiple-core couplers with Kerr law nonlinearity is given as in (15).

3.1.2 Dark solitons

For dark solitons, the starting hypothesis is (16). Substituting into the real part equation (73) leads to

$$A_l(\omega + a\kappa^2 + 2ap^2 B^2) \tanh^p \tau - aA_l B^2 p(p-1) \tanh^{p-2} \tau - aA_l B^2 p(p+1) \tanh^{p+2} \tau - bA_l^3 \tanh^{3p} \tau + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \tanh^p \tau = 0 \quad (76)$$

By balancing principle (11) is revealed. Also, this same value of p is recovered from coefficient of the standalone linearly independent function $\tanh^{p-2} \tau$. From other linearly independent functions,

$$\omega = \frac{1}{A_l} \left[-aA_l(2B^2 + \kappa^2) - \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \right] \quad (77)$$

The remaining relations as given by (19)-(21) are also valid in this case.

3.1.3. Singular solitons

Once again, for singular solitons, the starting hypothesis stays the same as (22). Therefore the real part equation simplifies to

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \text{csch}^p \tau - bA_l^3 \text{csch}^{3p} \tau - aA_l B^2 p(p+1) \text{csch}^{p+2} \tau + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \text{csch}^p \tau = 0 \quad (78)$$

Balancing principle again leads to the same value of p as in (11). Then, other linearly independent functions give (75), (19) and (20). Finally, singular 1-soliton solution is (24).

3.2. Power law nonlinearity

For power law, the coupled NLSE modifies to

$$iq_t^{(l)} + aq_{xx}^{(l)} + b|q^{(l)}|^{2n} q^{(l)} = \sum_{m=1, m \neq l}^N \lambda_{lm} q^{(m)} \quad (79)$$

Here n is the power law nonlinearity parameters and all limitations on the parameter n , as discussed earlier, remain valid here as well. With hypothesis (2), equation (79) simplifies to

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - bP_l^{2n+1} + \sum_{m=1, m \neq l}^N \lambda_{lm} P_m = 0 \quad (80)$$

For power law nonlinearity, the study will be further split into the following three subsections, depending on the type of soliton that is being considered.

3.2.1. Bright solitons

For bright solitons, starting hypothesis for power law is again given by (8). Therefore the real part equation (80) simplifies to

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \operatorname{sech}^p \tau - bA_l^{2n+1} \operatorname{sech}^{(2n+1)p} \tau + aA_l B^2 p(p+1) \operatorname{sech}^{p+2} \tau + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \operatorname{sech}^p \tau = 0 \quad (81)$$

Balancing principle yields the same value of the unknown exponent p as in (28). The linearly independent coefficients give

$$\omega = \frac{1}{n^2 A_l} \left[aA_l(B^2 - n^2 \kappa^2) - \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \right] \quad (82)$$

The amplitude-width relation is still given by (30) with its corresponding constraint.

3.2.2. Dark solitons

For dark solitons, starting hypothesis is the same as given by (16). Substituting into (80) leads to

$$A_l(\omega + a\kappa^2 + 2ap^2 B^2) \tanh^p \tau - aA_l B^2 p(p-1) \tanh^{p-2} \tau - aA_l B^2 p(p+1) \tanh^{p+2} \tau - bA_l^{2n+1} \tanh^{(2n+1)p} \tau + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \tanh^p \tau = 0 \quad (83)$$

By balancing principle (28) is revealed. Also, this same value of p as given by (11) is obtained from coefficient of the standalone linearly independent function $\tanh^{p-2} \tau$. Thus, (11) and (28) together imply that for dark soliton solutions to exist, power law nonlinearity must collapse to Kerr law nonlinearity. This implies that all results of dark solitons for Kerr law nonlinearity will remain valid for power law nonlinearity.

3.2.3. Singular solitons

Here, the starting hypothesis is given by (22) so that the real part equation reduces to

$$A_l(\omega + a\kappa^2 - ap^2 B^2) \operatorname{csch}^p \tau - aA_l B^2 p(p+1) \operatorname{csch}^{p+2} \tau - bA_l^{2n+1} \operatorname{csch}^{(2n+1)p} \tau + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m \operatorname{csch}^p \tau = 0 \quad (84)$$

Balancing principle leads to the same value of p as given by (28). Also the wave number is the same as in bright solitons as given by (82), while the relation between the free parameters is (19) along with (20). Finally, the singular 1-soliton solution is given by (24).

3.3. Parabolic law nonlinearity

In this case, the governing equation is

$$iq_t^{(l)} + aq_{xx}^{(l)} + (b_1 |q^{(l)}|^2 + b_2 |q^{(l)}|^4) q^{(l)} = \sum_{m=1, m \neq l}^N \lambda_{lm} q^{(m)} \quad (85)$$

where $1 \leq l \leq N$. The real part equation therefore is

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - b_1 P_l^3 - b_2 P_l^5 - \sum_{m=1, m \neq l}^N \lambda_{lm} P_m = 0 \quad (86)$$

Equation (86) will now be analyzed for bright and singular solitons. Once again, dark solitons for multiple-core couplers with parabolic law nonlinearity cannot be retrieved by this ansatz method.

3.3.1. Bright solitons

For parabolic law nonlinearity, the hypothesis for the waveform is the same as (38). The real part equation, now, simplifies to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2 B^2) A_l}{(D + \cosh \tau)^p} + \frac{aDB^2 p(2p+1) A_l}{(D + \cosh \tau)^{p+1}} - \\ & - \frac{aA_l B^2 p(p+1)(D^2 - 1)}{(D + \cosh \tau)^{p+2}} - \frac{b_1 A_l^3}{(D + \cosh \tau)^{3p}} - \\ & - \frac{b_2 A_l^5}{(D + \cosh \tau)^{5p}} + \frac{\sum_{m=1, m \neq l}^N \lambda_{lm} A_m}{(D + \cosh \tau)^p} = 0 \end{aligned} \quad (87)$$

Balancing principle yields (40). From linearly independent functions,

$$\omega = \frac{1}{4A_l} [aA_l(B^2 - 4\kappa^2) - 4 \sum_{m=1, m \neq l}^N \lambda_{lm} A_m] \quad (88)$$

Subsequently, relations (42)-(45) holds.

3.3.2. Singular solitons

For singular solitons, the waveform is given by (46). Thus, the real part equation transforms to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2 B^2) A_l}{(D + \sinh \tau)^p} + \frac{ap(2p+1) A_l DB^2}{(D + \sinh \tau)^{p+1}} - \\ & - \frac{ap(p+1) A_l B^2 (D^2 + 1)}{(D + \sinh \tau)^{p+2}} - \frac{b_1 A_l^3}{(D + \sinh \tau)^{3p}} - \\ & - \frac{b_2 A_l^5}{(D + \sinh \tau)^{5p}} + \frac{\sum_{m=1, m \neq l}^N \lambda_{lm} A_m}{(D + \sinh \tau)^p} = 0 \end{aligned} \quad (89)$$

The wave number is given by (88). After this, rest stays the same as in the case of singular soliton in multiple-core couplers, where coupling is with nearest neighbors.

3.4. Dual-Power law nonlinearity

In this case, the governing equation is

$$\begin{aligned} & iq_t^{(l)} + aq_{xx}^{(l)} + \left(b_1 |q^{(l)}|^{2n} + b_2 |q^{(l)}|^{4n} \right) q^{(l)} = \\ & = \sum_{m=1, m \neq l}^N \lambda_{lm} q^{(m)} \end{aligned} \quad (90)$$

where $1 \leq l \leq N$. The real part equation therefore is

$$\begin{aligned} & P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - b_1 P_l^{2n+1} - b_2 P_l^{4n+1} + \\ & + \sum_{m=1, m \neq l}^N \lambda_{lm} P_m = 0 \end{aligned} \quad (91)$$

Equation (91) will now be analyzed for bright and singular solitons. To repeat, dark solitons for multiple-core couplers with dual-power law nonlinearity cannot be retrieved by this ansatz method.

3.4.1. Bright solitons

For parabolic law nonlinearity, the hypothesis for the waveform is given by (38). Substituting into (90), the real part equation is

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2 B^2) A_l}{(D + \cosh \tau)^p} + \frac{aDB^2 p(2p+1) A_l}{(D + \cosh \tau)^{p+1}} - \\ & - \frac{aA_l B^2 p(p+1)(D^2 - 1)}{(D + \cosh \tau)^{p+2}} - \frac{b_1 A_l^{2n+1}}{(D + \cosh \tau)^{(2n+1)p}} - \\ & - \frac{b_2 A_l^{4n+1}}{(D + \cosh \tau)^{(4n+1)p}} + \frac{\sum_{m=1, m \neq l}^N \lambda_{lm} A_m}{(D + \cosh \tau)^p} = 0 \end{aligned} \quad (92)$$

Balancing principle yields the same value of the exponent p given by (54). Next, linearly independent functions yield

$$\omega = \frac{1}{4n^2 A_l} [aA_l (B^2 - 4n^2 \kappa^2) - 4n^2 \sum_{m=1, m \neq l}^N \lambda_{lm} A_m] \quad (93)$$

as well as relations (56)-(58) along with (44).

3.4.2. Singular solitons

For singular solitons, the waveform is given by (46). Substituting into (91), the real part equation transforms to

$$\begin{aligned} & \frac{(\omega + a\kappa^2 - ap^2 B^2) A_l}{(D + \sinh \tau)^p} + \frac{ap(2p+1) A_l DB^2}{(D + \sinh \tau)^{p+1}} - \\ & - \frac{ap(p+1) A_l B^2 (D^2 + 1)}{(D + \sinh \tau)^{p+2}} - \frac{b_1 A_l^{2n+1}}{(D + \sinh \tau)^{(2n+1)p}} - \\ & - \frac{b_2 A_l^{4n+1}}{(D + \sinh \tau)^{(4n+1)p}} + \frac{\sum_{m=1, m \neq l}^N \lambda_{lm} A_m}{(D + \sinh \tau)^p} = 0 \end{aligned} \quad (94)$$

The wave number is given by (93), while the remaining relations are same as in singular solitons for coupling with nearest neighbors.

3.5 Log-law nonlinearity

For log law nonlinear media, NLSE for NLDC in multiple core couplers is [30]

$$iq_t^{(l)} + aq_{xx}^{(l)} + bq^{(l)} \ln |q^{(l)}|^2 = \sum_{m=1, m \neq l}^N \lambda_{lm} q^{(m)} \quad (95)$$

It is only Gaussons that can be retrieved for this model. In this case, the real part equation (71) simplifies to

$$P_l(\omega + a\kappa^2) - a \frac{\partial^2 P_l}{\partial x^2} - 2bP_l \ln |P_l| + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m = 0 \quad (96)$$

The starting hypothesis for Gaussons is still given by (65). Substituting this hypothesis into (96) simplifies it to

$$\begin{aligned} & A_l \omega + 2aA_l B^2 - 2bA_l \ln A_l + aA_l \kappa^2 + \\ & + (2bA_l - 4aA_l B^2) \tau^2 + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m = 0 \end{aligned} \quad (97)$$

Setting the coefficients of the linearly independent functions τ^j , for $j = 0, 2$ leads to wave number of Gaussons

$$\omega = -\frac{1}{A_l} [aA_l (2B^2 + \kappa^2) - 2bA_l \ln A_l + \sum_{m=1, m \neq l}^N \lambda_{lm} A_m] \quad (98)$$

The width of Gausson stays the same as in (67) along with the constraint (14). Finally, the Gaussons are given by (69).

4. Conclusions

This paper extracted exact 1-soliton solution to the governing equation for optical couplers. The coupling was considered with nearest neighbors as well as all neighbors. There are three types of soliton solutions obtained. They are bright, dark and singular solitons. The constraint conditions, for the existence of these solitons, naturally emerged from the structure of soliton solutions of the governing equations. There are five types of nonlinear media that are considered. They are Kerr law, power law, parabolic law, dual-power law and log law. All results, for

each nonlinearity, are exhibited in this paper with appropriate technical details.

These results will be immensely helpful in optical switching and networking in order to address all-optical switching. In future, there is a lot of scope to extend these results. The perturbation terms will be added and the exact soliton solutions, in optical couplers, with perturbation terms will be derived later and will be reported elsewhere. Additionally, conservation laws will be derived for such equations and they will also be available.

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