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Optical solitons with nonlinear dispersion in polynomial law medium

Q. ZHOU[a], X. XIONG[a], Q. ZHU[a], Y. LIU[a], H. YU[a], P. YAO[a], A. BISWAS[b,c,*], M. BELIC[d]
[a]*School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan, 430212, P.R. China*
[b]*Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA*
[c]*Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah-21589, Saudi Arabia*
[d]*Science Program, Texas A & M University at Qatar, Doha, Qatar*

This paper studies optical solitons with polynomial law nonlinearity and nonlinear dispersion. The P^6 model expansion method is employed to solve the nonlinear Schrödinger equation. As a result, singular, dark and bright soliton solutions are constructed.

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1. Introduction

Optical solitons, the outcome of perfect balance between the dispersion (or diffraction) and nonlinearity in the nonlinear media, are widely applied in the area of telecommunications and electromagnetics [1-6]. The study of non-Kerr optical solitons have attracted a lot of interest in recent years [1.2.7-14], current researches focus mostly on the optical solitons with parabolic law nonlinearity, while there are very few studies to the optical solitons in the polynomial law media [1,11,12]. The innovative point of this paper is to study the optical solitons in the polynomial law medium with nonlinear dispersion.

The polynomial law nonlinearity, also known as the cubic–quintic–septic (CQS) nonlinearity, is closely related to the seventh-order susceptibility of the material [1,11]. Generally speaking, the higher-order susceptibility can be neglected, but recent studies show that CdSxSe1−x-doped glasses has considerable $\chi^{(7)}$ [15]. So the study on the optical solitons in the higher-order nonlinear media plays a very important role in the field of nonlinear optics.

2. Governing Equation

The model equation for the propagation of optical solitons through optical fibers with polynomial law nonlinearity and nonlinear dispersion is given by the following NNLSE [11]

$$\begin{aligned} &iq_t + aq_{xx} + b|q|^2 q + c|q|^4 q \\ &+ d|q|^6 q + e(|q|^2)_{xx} q = 0 \end{aligned} \tag{1}$$

where the dependent variable $q(x,t)$ represents the slowly varying wave packet envelope, while the independent variables x and t represent the spatial and temporal respectively. The first term in Eq. (1) is the linear temporal evolution term, while the second term represents the group velocity dispersion (GVD), and the third, fourth and fifth terms account for the polynomial law nonlinearity, and the sixth term is due to the nonlinear dispersion.

In order to construct analytical soliton solutions to Eq. (1), the starting hypothesis is given by [13,16]

$$q(x,t) = AP[\eta(x,t)]\exp[i\phi(x,t)] \tag{2}$$

where $\eta = B(x - vt)$ and $\phi(x,t) = -\kappa x + \omega t + \theta$.

In Eq. (2), $P(\eta)$ is the soliton amplitude component that represents the pulse shape, while $\phi(x,t)$ is the soliton phase. The soliton amplitude A, soliton width B, soliton velocity v, soliton frequency κ, wave number ω and phase constant θ are all the real constants.

Substituting Eq. (2) into Eq. (1) and then decomposing into real and imaginary parts, one obtains

$$\begin{aligned} &-\omega P + aB^2P'' - a\kappa^2 P + bA^2P^3 + cA^4P^5 \\ &+ dA^6P^7 + 2eA^2B^2(PP'^2 + P^2P'') = 0 \end{aligned} \tag{3}$$

$$-vBP' - 2a\kappa BP' = 0 \tag{4}$$

From the imaginary part (4), the soliton velocity gives

$$v = -2a\kappa \tag{5}$$

3. Soliton solutions

This section will focus on the integration to Eq. (3). The integration tools that is the $P^6(\eta)$ model expansion method [17] will be shown in details.

Assume that $P(\eta)$ satisfies the $P^6(\eta)$ model in the form

$$\begin{cases} P'^2(\eta) = h_0 + h_2P^2(\eta) + h_4P^4(\eta) + h_6P^6(\eta) \\ P''(\eta) = h_2P(\eta) + 2h_4P^3(\eta) + 3h_6P^5(\eta) \end{cases} \tag{6}$$

with h_i for $i = 0, 2, 4, 6$ being real constants.

In Ref. [17], we have constructed exact solutions to Eq. (6) by using the Jacobian elliptic equation expansion method [18-20]. The results show that Eq. (6) admits the explicit solutions

$$P(\eta) = \frac{U(\eta)}{\sqrt{fU^2(\eta) + g}} \tag{7}$$

where $U(\eta)$ is the solutions of the Jacobian elliptic equation, f and g are given by

$$f = \frac{h_4(l_2 - h_2)}{(l_2 - h_2)^2 + 3l_0l_4 - 2l_2(l_2 - h_2)} \tag{8}$$

$$g = \frac{3l_0h_4}{(l_2 - h_2)^2 + 3l_0l_4 - 2l_2(l_2 - h_2)} \tag{9}$$

and the constraint condition is given by

$$\begin{aligned} &2h_4^2[\frac{9l_0l_4}{l_2 - h_2} - (2l_2 + h_2)] \\ &+ 3h_6[\frac{3l_0l_4}{l_2 - h_2} - l_2 - h_2]^2 = 0 \end{aligned} \tag{10}$$

Substituting Eq. (6) into Eq. (3) gives

$$\begin{aligned} &-(\omega + a\kappa^2)P + aB^2(h_2P + 2h_4P^3 + 3h_6P^5) \\ &+ bA^2P^3 + cA^4P^5 + dA^6P^7 \\ &+ 2eA^2B^2P(h_0 + h_2P^2 + h_4P^4 + h_6P^6) \\ &+ 2eA^2B^2P^2(h_2P + 2h_4P^3 + 3h_6P^5) = 0 \end{aligned} \tag{11}$$

Then using the homogeneous balance principle, from Eq. (11), setting the coefficients of each power of $P(\eta)$ to zero leads to

$$\omega = ah_2B^2 + 2eh_0A^2B^2 - a\kappa^2 \tag{12}$$

$$bA^2 + 2ah_4B^2 + 4eh_2A^2B^2 = 0 \tag{13}$$

$$dA^4 + 8eh_6B^2 = 0 \tag{14}$$

$$cA^4 + 3ah_6B^2 + 6eh_4A^2B^2 = 0 \tag{15}$$

From Eqs. (13) and (14), we have

$$A = \sqrt{\frac{\sqrt{-a^2h_4^2K - 32beh_2} - ah_4K^2}{4eh_2K^2}} \tag{16}$$

$$B = \frac{\sqrt{-a^2h_4^2K - 32beh_2} - ah_4\sqrt{-K}}{32eh_2} \tag{17}$$

which introduce the constraint conditions

$$K = d/eh_6 < 0 \tag{18}$$

$$a^2h_4^2K + 32beh_2 \le 0 \tag{19}$$

$$eh_2(\sqrt{-a^2h_4^2K - 32beh_2} - ah_4K^2) > 0 \tag{20}$$

Additionally, Eq (15) poses other constraint condition that is given by

$$\begin{aligned} &3dh_4\sqrt{-a^2h_4^2K + 32beh_2} \\ &= (16ceh_2h_6 + 3adh_4^2 - 6adh_2h_6)K^2 \end{aligned} \tag{21}$$

Thus, finally in this section, the exact Jacobian elliptic periodic traveling wave solutions (see Table 1) to Eq. (1) are constructed. It's noted that when the modulus $m = 0$ and $m = 1$, the Jacobian elliptic periodic traveling wave solutions become to trigonometric periodic solutions (see Table 2), unbounded solutions (see Table 3), singular solutions (see Table 4), singular, bright and dark soliton solutions (see Table 5).

The soliton amplitude and width are given by Eqs. (16) and (17) respectively, while the velocity of the solitons is given by Eq. (5) and finally the wave number is given by Eq. (12). The constraint conditions for analytical solutions to exist are given by Eqs. (40) and (42)-(44). Additionally, Eqs. (18)-(21) give the constraint conditions for these exact solutions to exist.

4. Conclusion

The dynamics for the propagation of optical solitons through optical fibers with polynomial law nonlinearity and nonlinear dispersion, modeled by the NLSE (1), has been studied analytically in this paper. We use the P^6 model expansion method to carry out the integration. We obtain the analytical Jacobian elliptic periodic traveling wave solutions, singular, dark and bright soliton solutions.

Table 1. Jacobian elliptic periodic traveling wave solutions to Eq. (1).

l_0	l_2	l_4	$q(x,t)$
1	$-(1+m^2)$	m^2	$\frac{A\,sn[B(x-vt)]}{\sqrt{f\ sn^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
$1-m^2$	$2m^2-1$	$-m^2$	$\frac{A\,cn[B(x-vt)]}{\sqrt{f\ cn^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
m^2-1	$2-m^2$	-1	$\frac{A\,dn[B(x-vt)]}{\sqrt{f\ dn^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
m^2	$-(1+m^2)$	1	$\frac{A\,ns[B(x-vt)]}{\sqrt{f\ ns^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
$-m^2$	$2m^2-1$	$1-m^2$	$\frac{A\,nc[B(x-vt)]}{\sqrt{f\ nc^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
-1	$2-m^2$	m^2-1	$\frac{A\,nd[B(x-vt)]}{\sqrt{f\ nd^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
1	$2-m^2$	$1-m^2$	$\frac{A\,sc[B(x-vt)]}{\sqrt{f\ sc^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
1	$2m^2-1$	$m^2(m^2-1)$	$\frac{A\,sd[B(x-vt)]}{\sqrt{f\ sd^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
$1-m^2$	$2-m^2$	1	$\frac{A\,cs[B(x-vt)]}{\sqrt{f\ cs^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
1	$-(1+m^2)$	m^2	$\frac{A\,cd[B(x-vt)]}{\sqrt{f\ cd^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
$m^2(m^2-1)$	$2m^2-1$	1	$\frac{A\,ds[B(x-vt)]}{\sqrt{f\ ds^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
m^2	$-(1+m^2)$	1	$\frac{A\,dc[B(x-vt)]}{\sqrt{f\ dc^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$

Table 2. Trigonometric periodic solutions to Eq. (1).

l_0	l_2	l_4	$q(x,t)$
1	-1	0	$\frac{A\sin[B(x-vt)]}{\sqrt{f\ \sin^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$ $\frac{A\cos[B(x-vt)]}{\sqrt{f\ \cos^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$

Table 3. Unbounded solutions to Eq. (1).

l_0	l_2	l_4	$q(x,t)$
-1	1	0	$\frac{A\cosh[B(x-vt)]}{\sqrt{f\ \cosh^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
1	1	0	$\frac{A\sinh[B(x-vt)]}{\sqrt{f\ \sinh^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$

Table 4. Singular solutions to Eq. (1).

l_0	l_2	l_4	$q(x,t)$
			$\frac{A\csc[B(x-vt)]}{\sqrt{f\ \csc^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
0	-1	1	$\frac{A\sec[B(x-vt)]}{\sqrt{f\ \sec^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
			$\frac{A\tan[B(x-vt)]}{\sqrt{f\ \tan^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
1	2	1	$\frac{A\cot[B(x-vt)]}{\sqrt{f\ \cot^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$

Table 5. Singular, dark and bright soliton solutions to Eq. (1).

l_0	l_2	l_4	$q(x,t)$
1	-2	1	$\frac{A\coth[B(x-vt)]}{\sqrt{f\ \coth^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
0	1	1	$\frac{A\,\mathrm{csch}[B(x-vt)]}{\sqrt{f\ \mathrm{csch}^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
1	-2	1	$\frac{A\tanh[B(x-vt)]}{\sqrt{f\ \tanh^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$
0	1	-1	$\frac{A\,\mathrm{sech}[B(x-vt)]}{\sqrt{f\ \mathrm{sech}^2[B(x-vt)]+g}}\exp[i(-\kappa x+\omega t+\theta)]$

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*Corresponding author: biswas.anjan@gmail.com