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Optical solutions for linear Diophantine equations

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Determining whether a Diophantine equation has a solution or not is the most important challenge in solving this type of problems. In this paper a special computational device which uses light rays is proposed to answer this question, namely check the existence of nonnegative solutions for linear Diophantine equations. The way of representation for this device is similar to an directed graph, having a number of nodes equal to the number of variables of the equation plus the destination node. The arcs connecting these nodes have assigned a number (length) which corresponds to coefficients of the equation or it is a predefined constant. The light traversing the device follows all possible routes. In each arc it will be delayed by an amount of time indicated by the length of that arc. At the destination node, if a light ray arrives at the moment equal to the free term of the equation plus some constants we may infer that the equation has solution, otherwise it has not.

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1. Introduction

Solving Diophantine equations belong to the class of difficult problems. Hilbert's 10^{th} problem [17] asked if an algorithm exists for determining whether an arbitrary Diophantine equation has a solution or not.

In this paper we use light in a special device for deciding whether a linear Diophantine equation has nonnegative solutions or not. The problem asks to find if the equation $\sum_{i=1}^{n} a_i * x_i = c$ has positive integer solutions, where the coefficients a_i and c are positive integer values.

Since the problem is difficult there is no other way to solve it, but generating all possible combinations of values for the variables. We do this by taking into account the massive parallelism of the light rays.

For the purpose of generating all possible solutions we propose a device which has a graph-like structure. To each arc we assign either a number corresponding to coefficients of the equation, either a predefined constant. The length of an arc is directly proportional to the number assigned to it.

Initially a light ray is sent to the starting node. In each node the incoming light is divided into 2 subrays. One of the subrays is directed to the next node and the other subray is sent back to the current node.

Each arc delays the ray by an amount of time equal to the number assign to it. At the destination node we will check if there is a ray arriving in the destination node at the moment equal to c (plus some constants introduced by the system).

The paper is organized as follows: Related work in the field of optical computing is briefly overviewed in section 2. The definition of a Diophantine equation is given in section 3. The proposed device and the list of components required by this system are presented in section 4. Complexity is discussed in section 5. Physical implementation and difficulties encountered by the proposed device are 6 discussed in section6. Suggestions for improving the device are given in section 6.4. Further work directions are suggested in section 7.

2. Related work

Most of the major computational devices today are using electric power in order to perform useful computations.

Another idea is to use light instead of electrical power. It is hoped that optical computing could advance computer architecture and can improve the speed of data input and output by several orders of magnitude [7]. Optical computation has some advantages, one of them being the fact that it can perform some operations faster than conventional devices.

Many theoretical and practical light-based devices have been proposed for dealing with various problems.

An important practical step was made by Intel researchers who have developed the first continuous wave all-silicon laser using a physical property called the Raman Effect [6, 30, 34, 35]. The device could lead to such practical applications as optical amplifiers, lasers, wavelength converters, and new kinds of lossless optical devices.

Another solution comes from Lenslet [18] which has created a very fast processor for vector-matrix multiplications. This processor can perform up to 8000 Giga Multiple-Accumulate instructions per second. Lenslet technology has already been applied to data analysis using *k*–mean algorithm and video compression.

In [14] was presented a new, principally non-dissipative digital logic architecture which involves a distributed and parallel input scheme where logical functions are evaluated at the speed of light. The system is based on digital logic vectors rather than the Boolean scalars of electronic logic. This new logic paradigm was specially developed with optical implementation in mind. Since each such gate has two input signals and only one output signal, such architectures are fundamentally dissipative in information and energy. Their serial nature also induces latency in the processing time.

A recent paper [36] introduces the idea of sorting by using some properties of light. The method called Rainbow Sort is based on the physical concepts of refraction and dispersion. It is inspired by the observation that light that traverses a prism is sorted by wavelength. For implementing the Rainbow Sort one need to perform the following steps:

- encode multiple wavelengths (representing the numbers to be sorted) into a light ray,
- send the ray through a prism which will split the ray into *n* monochromatic rays that are sorted by wavelength,
- read the output by using a special detector that receives the incoming rays.

A stable version of the Rainbow Sort is proposed in [24].

Naughton (et al.) proposed and investigated [25, 41] a model called the continuous space machine which operates in discrete time-steps over a number of two-dimensional complex-valued images of constant size and arbitrary spatial resolution. The (constant time) operations on images include Fourier transformation, multiplication, addition, thresholding, copying and scaling.

2.1 Related optical devices for NP-complete problems

Our purpose is to solve NP-complete problems. This is why we also review optical devices specially designed for attacking NP-complete problems. Currently, acording to our knowledge, 5 problems have been solved using optical priciples. They are: Hamiltonian Path problem [26, 27], Traveling Salesman Problem [3, 5, 37, 11, 12, 13], standard subset sum problem [28, 42], Exact Cover [29] and unbounded subset sum [23].

A system which solves the Hamiltonian path problem (HPP) [9] by using light and its properties has been proposed in [26, 27]. The device has the same structure as the graph where the solution is to be found 1. The light is delayed within nodes, whereas the delays introduced by arcs are constants. Because the problem asks that each node has to be visited exactly once, a special delaying system was designed. At the destination node we will search for a ray which has visited each node exactly once. This is very easy due to the special properties of the delaying system.

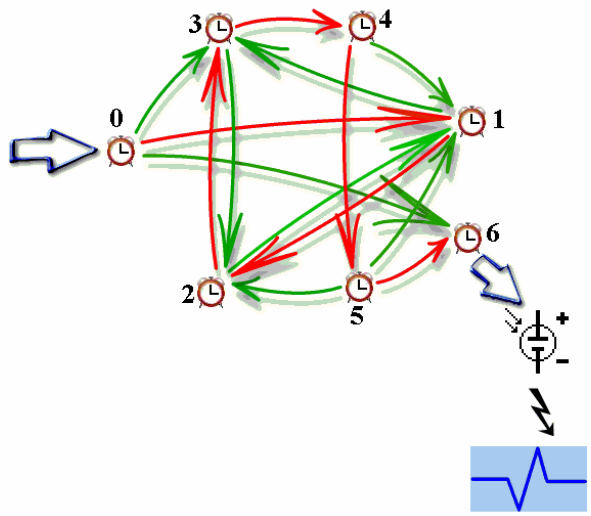

Fig. 1. A schematic representation of the device used for solving an instance of HPP with 6 nodes. In each node the light is delayed by a certain amount of time. The device generates all possible paths in the graph and finally decides if there is a Hamiltonian path or not.

An optical device for the standard subset sum problem (each element can appear no more than once in the solution) was proposed in [28]. The basic idea was to generate all possible subsets and then to select the good one (whose sum of numbers is equal to the target sum). Note that the generation of all posibile subsets is done in $O(B)$ time, but with an exponential consumption of energy. A possible design for this device is give in Figure 2.

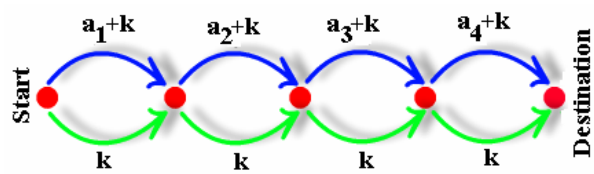

Fig. 2. A schematic representation of the device used for solving an instance of standard subset sum with 4 numbers. On each arc we have depicted its length. There are n cables of length k and n cables of length a_i+k *(*$1 \le i \le n$*,* a_i *belongs to set A and k is a constant). This device does generate all possible subsets of A.*

Another idea for solving the subset sum problem is to use discrete convolution. It is widely known that the *n*-point discrete Fourier transform computation can be performed, optically, in only unit time [10, 33]. Based on that, a solution to the subset sum problem can be obtained by discrete convolution [42]. The idea is that the convolution of 2 functions is the same as the product of their frequencies representation.

An optical solution for solving the traveling salesman problem (TSP) was proposed in [37]. The power of optics in this method was done by using a fast matrix-vector multiplication between a binary matrix, representing all feasible TSP tours, and a gray-scale vector, representing the weights among the TSP cities. The multiplication was performed optically by using an optical correlator. To synthesize the initial binary matrix representing all feasible tours, an efficient algorithm was provided. However, since the number of all tours is exponential the method is difficult to be implemented even for small instances.

TSP was also solved in [11, 12] by using a similar idea as in [26, 27].

An optical system which finds solutions to the 6-city TSP using a Kohonen-type network was proposed in [3]. The system shows robustness with regard to the light intensity fluctuations and weight discretization which have been simulated. The authors suggested a relatively large number of TSP cities can be handled by using this method.

In the case of Exact Cover the original problem was decomposed in 2 subproblems: generating all subsets of the given set and then selecting the correct one [29]. For the first step we have designed a light-based device which has a graph-like structure. Each arc can represent either an element of *C* or can be a skipping arc. An arc will actually delay the signal (light) which passes through by a certain amount of time. The nodes are connected by arcs in such way all possible subsets of *C* are generated. For the second part we have assigned, to each item from the set to be covered, a special positive number such that sum of all numbers assigned to that set is not equal to any other combination of numbers assigned to items from the set. These numbers have the same property as in the case of optical solution for the Hamiltonian Path problem [26].

3. Linear Diophantine equations

Diophantine equations are indeterminate polynomial equations in which only integer solutions are allowed. So, if we are given a polynomial equation $f(x_1,\dots x_n) = 0$, with integer coefficients, we are asked to find any integer solution.

In 1900, in recognition of their depth, Hilbert proposed the solvability of all Diophantine problems as the tenth of his celebrated problems [17]. In 1970, a novel result in mathematical logic known as Matiyasevich's theorem settled the problem negatively: in general Diophantine problems are unsolvable [20].

This is why solving Diophantine equations is not an easy task. Even the decision whether an equation has solutions or not can be a real challenge. In [21] it was proved that the problem of deciding if there are positive integer solutions for the equation $a*x^2+b*y=c$ where a,b and c are positive integers, is NP-complete [9]. Some specific cases of Diophantine equations and their computational complexities were studied [39].

A linear Diophantine equation is an equation of the general form $\sum_{i=1}^{n} a_i * x_i = c$, where a_i ($1 \le i \le n$) and c are integer values.

For the case of linear Diophantine equation in two variables $a*x+b*y=c$ with integer solutions the existence of solutions is a very simple problem concerning divisibility of numbers. If c is a multiple of the least common divisor of a and b then the equation has solution. In all other situations the equation has no solution.

However, in this paper we are interested in finding if a Diophantine equation with positive coefficients has nonnegative solutions. This case is more difficult than the previous one since more restrictions are imposed here [2]. Moreover, we work with equations that have more than 2 variables, so the problem gets even harder.

4. The proposed idea

This section deeply describes the proposed system. Section 4.1 describes the properties of light which are useful for our device. Section 4.2 introduces the operations performed by the components of our device. Basic ideas behind our concept are given in section 4.3.

4.1 Light and its properties

In our device we are considering the light because it has two properties which are useful for our purposes:

- The speed of light has a limit. The value of the limit is not very important at this stage of explanation. The speed will become important when we will try to measure the moment when rays arrive at the destination node (see section 6.1). What is important now is the fact that we can delay the ray by forcing it to pass through an optical fiber cable of a certain length.
- The ray can be easily divided into multiple rays of smaller intensity/power. Beam-splitters are used for this operation.

4.2 Operations performed within our device

The proposed device has a graph like structure. Generally speaking one operation is performed when a ray passes through a node and one operation is performed when a ray passes through an arc.

- When passing through an arc the light ray is delayed by the amount of time assigned to that arc.
- When the ray is passing through a node it is divided into a number of rays equal to the external degree of that node. Each obtained ray is directed toward one of the nodes connected to the current node.

At the destination node we will check if there is a ray which has arrived in the destination node at moment c (plus a constant value).

4.3 The device

The aim of our device is to generate all possible values for variables.

The device is represented similar to a directed graph having a number of nodes equal to the number of variables of the Diophantine equation plus the destination node. The coefficients of the equation are assigned to the arcs of the graph and represent the delays induced to the signals (light) that passes through this device. In practice, the delays are induced by forcing a signal to pass through a cable of a given length.

In each node (but the destination one) we place a beam-splitter which will split a ray into 2 subrays of smaller intensity, one subray going back to the same node and the other one to the next node. The subray going back in the same node will actually mean an increase by one unit of the corresponding variable. So, if a sub-ray has visited the arc of length a_2 five times it means that x_2=5. The value of a variable can increase to high values, because the light can be divided multiple times inside a node.

Each arc connecting 2 consecutive nodes was supposed to have length 0 because we are not interested in the delays between nodes. However, in practice, we cannot have cables of length 0. This is why we assigned them a constant value *k*.

An example of representation for an equation with 3 variables is depicted in Figure 3.

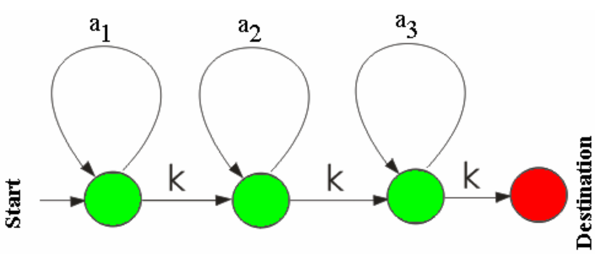

Fig. 3. A schematic representation of the device used for solving an instance with 3 variables. On each arc we have depicted its length. There are n cables of constant length k. The other n cables have the lengths a_i *(1≤i≤n).*

The ray of light skipping all arcs labeled by numbers a_i is actually the ray encoding the value 0 for each variable (x_i=0, 1≤*i*≤*n*).

The ray which has passed once through the arc labeled with a_1 and has skipped all other arcs a_2, a_3, ..., a_n encodes the values (x_1=1, x_2=0, ..., x_n=0) for variables.

We can see that each path from *Start* to *Destination* contains exactly *n* times value *k*. Thus, at the destination we will not wait anymore at moment *c*. Instead we will wait for a solution at moment *c*+*n***k* since all results will have the constant *n***k* added.

The total delay of a particular ray is $\sum_{i=1}^{n} a_i * x_i$. We have two cases here:

- If there is a ray arriving at moment *c*+*n***k* means that the equation has nonnegative solutions.
- If there is no ray arriving at moment *c*+*n***k* means that the equation does not have nonnegative solutions.

If there are 2 rays arriving at the same moment in the *Destination* it simply means that there are multiple solutions of the equation. This is not a problem for us because we want to answer the YES/NO decision problem (see section 3). We are not interested at this moment which are the values of the variables representing the solution.

Because we are working with continuous signal we cannot expect to have discrete output at the destination node. This means that rays arrival is notified by fluctuations in the intensity of the light. These fluctuations will be transformed, by a photodiode, in fluctuations of the electric power which will be easily read by an oscilloscope.

Now it has become clearer why the have added the restriction that all constants and variables in our system should be nonnegative. These values represent either the length of some cables (the coefficients a_i) or tell us how many times a particular arc has been traversed by a given ray (the value of variables x_i).

4.4 How the system works

In the graph depicted in Figure 3 the light will enter in the device from the *Start* direction (left part of the picture). When it enters in a node it will be divided into 2 subrays of smaller intensity.

In the destination node we will have light rays arriving at different moments. This is because some rays have visited the arcs labeled with a_1, a_2, ..., a_n more time than the others.

The ray arriving at moment *n***k* means that $x_i = 0$, $1 \le i \le n$. The ray which has visited the arc of length a_1 three times and the arc labeled with a_2 5 times and has skipped all other arcs (a_3, ..., a_n) means (x_1=3, x_2 = 5, x_3 = 0, ..., x_n=0).

Assuming that we have an equation with 3 variables (x_1, x_2 and x_3). The graph for that problem was depicted in Figure 3. The moments when the rays will arrive in the destination node are:

{
3*k*,
a_1+3*k*, a_2+3*k*, a_3+3*k*
a_1+a_1+3*k*, a_1+a_2+3*k*, a_1+a_3+3*k*, a_2+a_1+3*k*, a_2+a_2+3*k*, a_2+a_3+3*k*, a_3+a_1+3*k*, a_3+a_2+3*k*, a_3+a_3+3*k*,
...
}

The moments are represented as a set because they might not be distinct.

5. Complexity

The time required to build the device has $O(n*c)$ complexity. We assume that all coefficients are shorter than c, otherwise they cannot participate to the final solution.

Because the ray encoding the solution takes $O(c)$ time to reach the destination node we may say that the complexity is $O(c)$.

The intensity of the signal decreases exponentially with the number of nodes. This is why the required power is exponential with the number variables and the value of each variable.

We are also interested to find the maximal value for the coefficients. We need this value, because the coefficients are actually cables in our system. We cannot have any value for the coefficients because we have available a limited length for each cable. Let us suppose that we have 3 kilometers for each cable.

Each coefficient is less or equal to c. So, let us see how large c can be. We know that the shortest delay possible is 0.0003 meters. Having a cable of 3 kilometers we may encode coefficients less than 10^7.

Longer cables may also be available. Take for instance the optical cables linking the cities in a given country. We may easily find cables having 300 km. In this case we may work with coefficients smaller than 10^9. This is a little bit smaller than the largest integer value represented over 32 bits.

6. Tips for physical implementation & difficulties

For implementing the proposed device we need the following components:

- a source of light (laser),
- Several beam-splitters for dividing light rays into 2 subrays. A standard beam-splitter is designed using a half-silvered mirror (see Figure 4),
- A high speed photodiode for converting light rays into electrical power. The photodiode is placed in the destination node,
- A tool for detecting fluctuations in the intensity of electric power generated by the photodiode (oscilloscope),
- A set of optical fiber cables having lengths equals to the coefficients of the Diophantine equation and another set of n cables having fixed length k. These cables are used for connecting nodes.

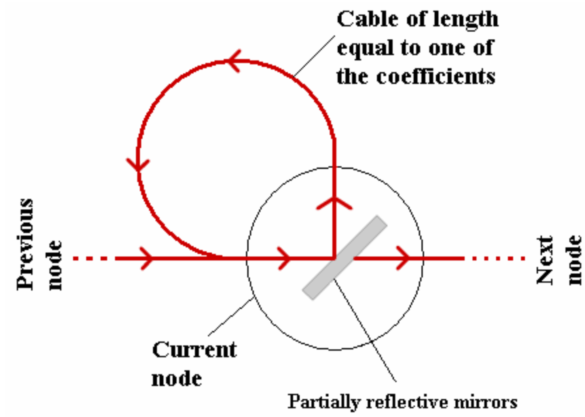

Fig. 4. The mechanism inside a node of the graph from Figure 3. It show the way in which a ray can be split into 2 sub-rays by using a beam-splitter. One subray is sent to the next node. The other subray will loop back in the current node. This means that the value for that variable is increased by 1.

6.1 Precision

One important problem is that we cannot measure the moment $c+n*k$ exactly. We can do this measurement only with a given precision which depends on the tools involved in the experiments. Actually it will depend on the response time of the photodiode and the rise time of the oscilloscope.

The rise-time of the best oscilloscope available on the market is in the range of picoseconds (10^{-12} seconds). This means that we should not have two signals that arrive at 2 consecutive moments at a difference smaller than 10^{-12} seconds. We cannot distinguish them if they arrive in a smaller than 10^{-12}s interval. In our case it simply means that if a signal arrives in the destination in the interval $[c+n*k-10^{12}, c+n*k+10^{12}]$ we cannot be perfectly sure that we have a correct subset or another one which does not have the wanted property.

Knowing that the speed of light is $3{\cdot}10^8 m/s$ we can easily compute the minimal cable length that should be traversed by the ray in order to be delayed with 10^{-12} seconds. This is obviously 0.0003 meters.

This value is the minimal delay that should be introduced by an arc in order to ensure that the difference between the moments when two consecutive signals arrive at the destination node is greater or equal to the measurable unit of 10^{-12} seconds. This will also ensure that we will be able to correctly identify whether the signal has arrived in the destination node at a moment equal to the sum of delays introduced by each arc.

Once we have the length for that minimal delay is quite easy to compute the length of the other cables that are used in order to induce a certain delay.

Note that the maximal number of nodes can be increased when the precision of our measurement instruments (oscilloscope and photodiode) is increased.

6.2 Power decrease

The intensity of the signal decreases exponentially with the number of nodes that are traversed. When the ray is passing through a node it is divided using beam splitters into 2 subrays. If divided uniformly we could have a 2 times decrease in the intensity for each subray. If a ray is passing through 10 nodes we can have a decrease of 2^{10} times from the initial power.

This means that, at the destination node, we have to be able to detect very small fluctuations in the intensity of the signal. For this purpose we will use a photomultiplier [8] which is an extremely sensitive detector of light in the ultraviolet, visible and near infrared range. This detector multiplies the signal produced by incident light by as much as 10^8, from which even single photons can be detected.

6.3 Technical challenges

There are many technical challenges that must be solved when implementing the proposed device. Some of them are:

- Cutting the optic fibers to an exact length with high precision. Failing to accomplish this task can lead to errors in detecting if there was a fluctuation in the intensity at moment $c+n*k$,
- Finding a high precision oscilloscope. This is an essential step for measuring the moment $c+n*k$ exactly (see section 6.1).

6.4 Improving the device

The speed of the light in optic fibers is an important parameter in our device. The problem is that the light is too fast for our measurement tools. We have either to increase the precision of our measurement tools or to decrease the speed of light.

It is known that the speed of light traversing a cable is significantly smaller than the speed of light in the void space. Commercially available cables have limit the speed of the ray wave up to 60% from the original speed of light. This means that we can obtain the same delay by using a shorter cable.

However, this method for reducing the speed of light is not enough for our purpose. The order of magnitude is still the same. This is why we have the search for other methods for reducing that speed. A very interesting solution was proposed in [16] which is able to reduce the speed of light by 7 orders of magnitude and even to stop it [1, 19]. In [1] they succeeded in completely halting light by directing it into a mass of hot rubidium gas, the atoms of which, behaved like tiny mirrors, due to an interference pattern in two control beams.

This could help our mechanism significantly. However, how to use this idea for our device is still an open question because of the complex equipment involved in those experiments [16, 19].

By reducing the speed of light by 7 orders of magnitude we can reduce the size of the involved cables by a similar order (assuming that the precision of the measurement tools is still the same). This will help us to solve larger instances of the problem.

7. Conclusions and further work

The way in which light can be used for performing useful computations has been suggested in this paper. The techniques are based on the massive parallelism of the light ray.

It has been shown the way in which a light-based device can be used for solving Diophantine equations. Using the today technology we can build a light-based device which can solve small and medium size instances in several seconds.

Further work directions will be focused on:

- Implementing the proposed device,
- Cutting new cables each time when a new instance has to be solved is extremely inefficient. This is why finding a simple way to reuse the previously utilized cables is a priority for our system,
- Automate the entire process,
- Our device cannot find the set of numbers representing the solution. It can only say if there is a solution or not. If there are multiple solutions we cannot distinguish them. However, finding if the equation has solution or not if more than 2 variables are involved is still a difficult problem. We are currently investigating a way to store the order of nodes so that we can easily reconstruct the path,
- Using the device for other types of Diophantine equations,
- Finding other ways to introduce delays in the system. The current solution requires cables that are too long and too expensive,
- Using other type of signals instead of light. Possible candidates are electric power and sound.

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