Oscillatory parabolic law optical spatial solitons

S. CRUTCHER[a], A. OSEI[b], AHMET YILDIRIM[c,d], ANJAN BISWAS[e*]
[a]*US Army Research, Development, and Engineering Command, WDID Electronics and Computer Technology Function, Redstone Arsenal, AL 35898-5000 USA*
[b]*Department of Mathematics & Computer Sciences, Oakwood University, Huntsville, AL 35896 USA*
[c]*Department of Mathematics, Ege University, 35100 Bornova, Izmir, Turkey*
[d]*Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620-5700, USA*
[e]*Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA*

We apply equivalence particle principle to a higher order spatial Nonlinear Schrödinger Equation (NLSE) that models the propagation of a beam with higher order nonlinearity ($\chi^{(5)}$). Using this principle, expressions for acceleration, spatial frequency, spatial period and other variables for a spatial soliton can be derived from the solution of a dual power law (or parabolic law) homogenous Nonlinear Schrödinger Equation(NLSE). These results agree well with numerical simulations of the perturbed Nonlinear Schrödinger Equation. We show that if the expression of the acceleration is bounded this means the spatial soliton propagates with a swing effect. Taking one step further in this theoretical study, we investigate the swing effect through the use of numerical simulations.

(Received January 9, 2012; accepted February 20, 2012)

Keywords: Nonlinear optics, Optical spatial soliton, Equivalent particle theory, Nonlinear Schrödinger Equation, Swing effect, Parabolic law

1. Introduction

Optical solitons often occur as self-trapped transverse spatial profiles (or self induced transparency) in non-linear planar waveguides. The most interesting property for such propagating wave packets is their robustness when colliding with each other, while maintaining space-invariance. Spatial optical solitons have been observed in different experiments [13,19], providing the opportunity of the all-optical devices: as arithmetic units, logical operators, switchers and modulators. Recently, NLSE models have been derived in nanotechnology in particular plasmonics or spatial plasmon solitons [1-4,28]. These subwavelength spatial solitons were induced by nonlinear Kerr/metal interface [3,4], tapering nonlinear dielectric/metal waveguides [5,6], and active dielectric/metal/passive dielectric waveguides [7]. So current soliton theoretical methods have been applied to these models, but there is still research to be done at the microscale regime[28-30,32].

Kerr spatial soltions have been shown to oscillate during the propagation when subjected to perturbation representing a refractive index profile change. Previously, the oscillatory behavior of spatial solitons propagating in an inhomogeneous refractive index profiles has been studied. F. Garzia *et. al.* [8] investigated the swing effect of spatial soliton for the Gaussian refractive index profile. A. Suryanto investigated the oscillatory motion in his Ph.D. thesis in 2003[9]. L.W. Dong and H. Wang et. al [10] have studied oscillatory behavior of spatial soliton in a gradient refractive index waveguide with nonlocal nonlinearity. M. Edwards et. al [11,12] has studied the spatial soliton under triangular refractive index profile perturbation. These results have been compared with the swing effect from the Gaussian profile.

The homogeneous Nonlinear Schrödinger Equation (NLSE) is the model that describes spatial and temporal soliton propagation is solved using the inverse transform method [13,14]. This method cannot be used to solve equations with higher order diffraction or nonlinearity, which are more applicable to real optical soliton systems. These equations are not integrable and the integrals of motion are not conserved [15]. The equivalence particle theory which considers the light beam as a particle can be used to describe propagation of a single optical spatial soliton in nonlinear media [8-11,16-19]. Under an index of refraction perturbation, the soliton beam behaves as a particle in a Newtonian gravitational potential well. This theory applies to short optical devices and bulk materials that can produce propagational diffraction and the Kerr Effect. With this property, we can calculate deterministic features of the soliton propagation.

In this paper, we consider the oscillatory behavior of nonparaxial spatial solitons or quasi-solitons in a waveguide reduced to a parabolic law NLSE perturbed by a medium with a Gaussian refractive index profile change. We apply the equivalent particle approach to the perturbed Nonlinear Schrödinger Equation with dual power law or higher order nonlinearity to calculate the transverse acceleration and power. In section 2, we briefly introduce previous results for clarity, the scalar nonlinear nonparaxial evolution equation and discuss some of its features using soliton theory. In section 3, we use the equivalence particle approach to calculate the power and acceleration to show a swing effect under a Gaussian profile perturbation. Lastly, we numerically simulate the higher order NLSE investigating one and two spatial beam propagation.

2. Model

Considering a nonlinear medium with a material polarization expansion in terms of susceptibility tensors that are represented by dielectric tensor considering one diagonal component with an amplitude B, we have

$$\varepsilon = n^2 = n_0^2 + 2n_0 n_2 |B|^2 + 2n_0 n_4 |B|^4 + \ldots\ldots\ldots \quad (1)$$

where ε is the dielectric constant,

$$n_0^2 = 1 + \chi^{(1)} \quad (2)$$

$$n_2 = \frac{3\chi^{(3)}}{8n_0} \quad (3)$$

$$n_4 = \frac{5\chi^{(5)}}{16n_0} \quad (4)$$

are the index of refraction, the cubic nonlinear index, and the quintic nonlinear index respectively. The approximation can be rewritten as

$$n = n_0 + n_2|B|^2 + \left(n_4 - \frac{n_2^2}{2n_0}\right)|B|^4 + \ldots\ldots\ldots \quad (5)$$

The last two terms of equation (5) are the intrinsic and effective quintic nonlinear indices. With only the intrinsic index n_4 considered, this provides stability for the fundamental eigenmode solutions to multi-dimensional Nonlinear Schrödinger Equations (NLSEs). Based on equation (5), we can consider the nonlinear Helmholtz equation considered by Blair[20]

with z being the longitudinal propagation coordinate, x the transversal coordinate, B_y the amplitude of the field, and β the wave vector for the guided mode. This is a (1+1)-D scalar equation in transverse electric (TE) mode. The y direction can be ignored on the basis that the optical field is infinitely extended. Normalizing equation(6) one gets

$$\frac{\partial^2 B_y}{\partial z^2} + 2i\beta\frac{\partial B_y}{\partial z} + \frac{\partial^2 B_y}{\partial x^2} + 2\beta^2\frac{n_2}{n_0}\left[|B_y|^2 + \frac{n_4}{n_2}|B_y|^4\right]B_y = 0 \quad (6)$$

$$i\frac{\partial q}{\partial \vartheta} + \frac{1}{2}\frac{\partial^2 q}{\partial \zeta^2} + 2|q|^2 q + \kappa^2\left[\frac{1}{4}\frac{\partial^2 q}{\partial \vartheta^2} + 2\nu|q|^4\right]q = 0 \quad (7)$$

with

$$q = \beta w_0\left(\frac{n_2}{n_0}\right)^{1/2} B_y \quad (8)$$

$$\vartheta = z\frac{1}{2\beta w_0^2} \quad (9)$$

$$\zeta = \frac{x}{w_0}, \quad \kappa \equiv \frac{1}{\beta w_0} = \lambda\frac{1}{2\pi w_0} \quad \text{and} \quad \nu \equiv \frac{w_0^2 n_4 n_0}{n_2^2} \quad (10)$$

where w_0 is the width of the beam. The last two terms in equation (7) are considered perturbations with κ being the strength of the perturbation called the fractional bandwidth. The variable ν in equation (7) is relative strength of the intrinsic quintic nonlinearity. The equation is a non-slow varying approximation (NSVA) nonparaxial wave equation that contains terms of the order κ^2. The nonparaxial effect in the equation is higher order diffraction and nonlinearity (or $\chi^{(5)}$ effect[21]). Ignoring the nonparaxial effect, we have a type of homogenous NLSE,

$$i\frac{\partial q}{\partial \vartheta} + \frac{\partial^2 q}{\partial \zeta^2} + 2|q|^2 q = 0 \quad (11)$$

According to [20], last two terms of equation(7) can be approximated by expansion of the high order diffraction terms

$$\frac{\kappa^2}{4}\frac{\partial^2 q}{\partial \vartheta^2} \approx i\frac{\kappa^2}{4}\left[\frac{\partial^3 q}{\partial \vartheta \zeta^2} + 2\frac{\partial |q|^2 q}{\partial \vartheta}\right]$$

$$\kappa^2\left[-\frac{1}{4}\frac{\partial^4 q}{\partial \zeta^4} - \frac{1}{2}\frac{\partial^2 |q|^2 q}{\partial \zeta^2} + \frac{1}{2}q^2\frac{\partial^2 q^*}{\partial \zeta^2} - |q|^2\frac{\partial^2 q}{\partial \zeta^2}\right] \quad (12)$$

Using the expansion (equation (12)) and equation (7), the new high order equation

$$i\frac{\partial q}{\partial \vartheta} + \frac{1}{2}\frac{\partial^2 q}{\partial \zeta^2} + |q|^2 q + + 2\nu^{total}|q|^4 q\Big) = 0$$

$$\kappa^2\left(-\frac{1}{4}\frac{\partial^4 q}{\partial \zeta^4} - \frac{1}{2}\frac{\partial^2 |q|^2 q}{\partial \zeta^2} + \frac{1}{2}q^2\frac{\partial^2 q^*}{\partial \zeta^2} - |q|^2\frac{\partial^2 q}{\partial \zeta^2}\right. \quad (13)$$

with

$$\nu^{total} = \nu - \frac{1}{2} = \frac{n_0 n_4^{total}}{n_2^2} \quad (14)$$

$$n_4^{total} = n_4 - \frac{n_2^2}{2n_0}. \quad (15)$$

Self focusing can be enhanced or opposed by quintic nonlinearity. The last term in equation (13) will saturate at maximum value and then decrease when n_2 is positive and n_4^{total} is negative. The nonparaxial terms also decrease the longitudinal wavenumber by $-k_x^4/8\beta^3$ where k_x is the transverse wave number with $k_x=\beta\sin\theta$ [21]. Equations of motion for the power, momentum, Hamiltonian, phase, and wavenumber can be derived for equation (13) according to Biswas[15,21-26]. The normalized solution can be obtained by assuming a solution of the form

$$q(\zeta) = \left[\sec h(\zeta) + \kappa^2 g(\zeta)\right]\exp i(\alpha + \kappa^2\gamma)\vartheta \tag{16}$$

and solving the equation with perturbation expansion

$$g(\zeta) = \frac{\nu}{3}\left[\sec h(\zeta) - 2\sec h(\zeta)\right] \tag{17}$$

with $\gamma = -\dfrac{1}{4}$

Transition back to real units the solution now is

$$B_y(x,z) = B_0\left[\operatorname{sec}h(\frac{x}{w_0}) + \frac{\nu}{3\beta^2 w_0^2}\left(\operatorname{sec}h^3(\frac{x}{w_0}) - 2\operatorname{sec}h(\frac{x}{w_0})\right)\right]\exp i\eta z \tag{18}$$

$$B_0 = \left(\frac{n_0}{n_2}\right)^{1/2}\frac{1}{\beta w_0} \tag{19}$$

$$\eta = \frac{1}{2\beta w_0} - \frac{1}{8\beta^3 w_0^4}, \quad \text{for} \tag{20}$$

$$\begin{aligned} &2i\beta\frac{\partial B_y}{\partial z} + \frac{\partial^2 B_y}{\partial x^2} - \frac{1}{4\beta^2}\frac{\partial^4 B_y}{\partial x^4} + 2\beta^2\frac{n_2}{n_0}\left|B_y\right|^2 B_y \\ &-\frac{n_2}{2n_0}\left[\frac{\partial\left|B_y\right|^2 B_y}{\partial x^2} + 2\left|B_y\right|^2\frac{\partial^2 B_y}{\partial x^2} - B_y{}^2\frac{\partial^2 B_y^*}{\partial x^2}\right] + 2\beta^2\frac{n_4^{total}}{n_0}\left|B_y\right|^4 B = 0 \end{aligned} \tag{21}$$

This is the solution for the nonparaxial wave equation (21), which is considered “quasi-soliton”.

Going back to equation (7), one can use a similar transformation like equations (8-10) and the resulting equation is

$$i\frac{\partial q}{\partial\vartheta} + \frac{1}{2}\frac{\partial^2 q}{\partial\zeta^2} + |q|^2 q + \kappa^2\left[\frac{1}{2}\frac{\partial^2 q}{\partial\vartheta^2} + \nu|q|^4\right]q = 0. \tag{22}$$

Here, again there is a higher order of diffraction and nonlinearity, but one can ignore the high order diffraction come up with the equation for the parabolic power law which is

$$i\frac{\partial q}{\partial\vartheta} + \frac{1}{2}\frac{\partial^2 q}{\partial\zeta^2} + \left[|q|^2 + \nu'|q|^4\right]q = 0 \tag{23}$$

and

$$\nu' = \nu\kappa^2$$

removing the prime from ν. According Biswas[21], the stationary solution of equation(23) is

$$q(\vartheta,\zeta) = \frac{A}{\left[a\cosh(C(\zeta - \bar{\zeta}(\vartheta))) + 1\right]^{1/2}}\exp i(-k_{wa}\zeta + \omega\vartheta + \sigma_0) \tag{24}$$

with

$$k_{wa} = v_l = \frac{d\bar{\zeta}}{d\vartheta} \tag{25}$$

$$\omega = \frac{A^2}{4} - \frac{k_{wa}}{2} \tag{26}$$

$$C = \sqrt{2}A \quad \text{and} \tag{27}$$

$$a = \left(1 + \frac{4}{3}\nu A^2\right)^{1/2}. \tag{28}$$

Also, for this homogenous equation, the conserved integrals of motion are

$$E = \int_{-\infty}^{\infty}|q|^2 d\zeta = \frac{A^2}{aB}F(1,1,\frac{3}{2};\frac{a-1}{2a})B(1,\frac{1}{2}) \tag{29}$$

$$M = \frac{i}{2}\int_{-\infty}^{\infty}(q^* q_\zeta - q q_\zeta^*)d\zeta = k_{wa}\frac{A^2}{aB}F(1,1,\frac{3}{2};\frac{a-1}{2a})B(1,\frac{1}{2}) \tag{30}$$

$$\begin{aligned} H = &\int_{-\infty}^{\infty}\{\frac{1}{2}\left|q_\zeta\right|^2 - \frac{1}{2}|q|^4 - \frac{\nu}{3}|q|^6\}d\zeta = \\ &\frac{\sqrt{2}A}{12a^2}[3a^2A^2F(1,1,\frac{5}{2};\frac{a-1}{2a})B(1,\frac{3}{2}) \\ &+6k_{wa}a^2F(1,1,\frac{3}{2};\frac{a-1}{2a})B(1,\frac{1}{2}) - 3aF(2,2,\frac{5}{2};\frac{a-1}{2a})B(2,\frac{1}{2}) \\ &-\nu A^4F(3,3,\frac{7}{2};\frac{a-1}{2a})B(3,\frac{1}{2})] \end{aligned} \tag{31}$$

where equations (29-31), (Biswas [21]) $F(\alpha,\beta_h;\gamma_h,t)$ is the Gauss hypergeometric function defined as

$$F(\alpha,\beta_h,\gamma_h;t) = \frac{\Gamma(\gamma_h)}{\Gamma(\alpha)\Gamma(\beta_h)}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha+n)\Gamma(\beta_h+n)t^n}{\Gamma(\gamma_h+n)n!} \tag{32}$$

and *B(l,m)* is the beta function that is defined as

$$B(l,m)=\int_0^1 \zeta^{l-1}(1-\zeta)^{m-1}d\zeta. \tag{33}$$

For temporal solitons, the temporal type of equation (22) represents nonlinear interaction between the high frequency Langmuir waves and the ion-acoustic wave by pondermotive forces [14,15,21]. Also, with this optical higher nonlinearity $\chi^{(5)}$, this model could be used to investigate CdS_xSe_{1-x} doped glasses [14,15,21]. The $\chi^{(5)}$ nonlinearity becomes significant in transparent glass with intense femtosecond pulses at a wavelength of 620nm[29,31,32].

The nonlinear scalar Helmholtz spatial equation has been reduced to a nonlinear scalar spatial parabolic law equation. Lastly, the term $\nu\kappa^2$ must be sufficiently small for the perturbed solution equation (22) to be valid. This is considered in the next section using the equivalence particle theory to examine spatial beam propagation.

3. Equilvalent-particle theory with the Parabolic Law Nonlinearity

Suppose one sets a perturbation on the index refraction in equation(34) below, which can be expressed as

$$\frac{\partial^2 B_y}{\partial z^2}+2i\beta\frac{\partial B_y}{\partial z}+\frac{\partial^2 B_y}{\partial x^2}+ 2\beta^2\frac{n_2}{n_0(1+\Delta n_0(x))}\left[\left|B_y\right|^2+\frac{n_4}{n_2}\left|B_y\right|^4\right]B_y=0 \tag{34}$$

provided $\Delta n(x)<<1$. So we assume higher order diffraction for this case ,

$$\frac{\partial^2 B_y}{\partial z^2}=0 \tag{35}$$

and the normalize equation (34)

$$2i\frac{\partial q}{\partial \vartheta}+\frac{\partial^2 q}{\partial \varsigma^2}+2\left(\left|q\right|^2+\nu_{pert}\left|q\right|^4\right)q=Vq \tag{36}$$

with transformations

$$B_y=\left(n_0\frac{1}{n_2}\right)^{1/3}\frac{1}{\beta^{2/3}}q \tag{37}$$

$$z=\left(n_0\frac{\beta}{n_2}\right)^{1/3}\vartheta \tag{38}$$

$$x=\left(n_0\frac{1}{n_2}\right)^{1/6}\frac{1}{\beta^{1/3}}\varsigma,\ \text{and}\quad \nu_{pert}=n_4\frac{\beta^{-4/3}}{n_2}\left(\frac{n_0}{n_2}\right)^{2/3}. \tag{39}$$

V is the perturbation potential responsible for transversal trajectory of the soliton beam. It depends on the index profile, intensity profile of the beam, and the higher order nonlinearity. Here, similar to Garzia[8], Suryanto[9] and Crutcher and Edwards[11,12], we assume the electromagnetic field moves as a particle qq* and is a function of $\varsigma-\bar{\varsigma}$. This is considered the equivalent particle theory description [9]. We can calculate several parameters based on this theory, such as normalized power (similar to equation(29)):

$$E=\int_{-\infty}^{\infty}qq^*d\varsigma, \tag{40}$$

the average position,

$$\bar{\varsigma}=(E)^{-1}\int_{-\infty}^{\infty}\varsigma qq^*d\varsigma, \tag{41}$$

the average velocity,

$$v_s=i(E)^{-1}\int_{-\infty}^{\infty}\left(q\frac{\partial q^*}{\partial\varsigma}-q^*\frac{\partial q}{\partial\varsigma}\right)d\varsigma, \tag{42}$$

for the normalized field q. So, the acceleration that includes the perturbation potential is

$$a=\frac{dv_s}{d\varsigma}=-2(E)^{-1}\int_{-\infty}^{\infty}\frac{\partial V}{\partial\varsigma}qq^*d\varsigma \tag{43}$$

with

$$\frac{dp}{d\vartheta}=0 \tag{44}$$

$$\frac{d\bar{\varsigma}}{d\vartheta}=v_s. \tag{45}$$

The approach here is to show if there is swing effect with the soliton beam by calculating the power and acceleration with higher order nonlinearity [20]. We will assume a perturbation Gaussian profile similar to Garzia [8] which is

$$V=-2\frac{\Delta n_0\exp(-b\varsigma^2)}{1+\Delta n_0\exp(-b\varsigma^2)}\left(\left|q\right|^2+\nu_{pert}\left|q\right|^4\right) \tag{46}$$

where q is a solution given by equation (24)

$$q(\vartheta,\varsigma)=\frac{A}{\left[a\cosh(C(\varsigma-\bar{\varsigma}(\vartheta)))+1\right]^{1/2}}\exp i(-k_{wa}\varsigma+\omega\vartheta+\sigma_0) \quad (47)$$

Equation (42) can be approximated so

$$V=-2\frac{\Delta n_0\exp(-b\varsigma^2)}{1+\Delta n_0\exp(-b\varsigma^2)}\left(\frac{A^2}{\left[a\cosh(C(\varsigma-\bar{\varsigma}(\vartheta)))+1\right]}+v_{pert}\frac{A^4}{\left[a\cosh(C(\varsigma-\bar{\varsigma}(\vartheta)))+1\right]^2}\right). \quad (48)$$

We assume change of index of refraction to be smaller than one and the variable b << C,A. Now the perturbation potential V can be expanded in a power series about $\varsigma=\bar{\varsigma}$. So, the expansion to first order is

$$V(\varsigma)\cong\left[-2\Delta n_0\exp(-b\bar{\varsigma}^2)+4b\Delta n_0\bar{\varsigma}\exp(-b\bar{\varsigma}^2)(\varsigma-\bar{\varsigma})\right]\cdot\left(\frac{A^2}{\left[a\cosh(C(\varsigma-\bar{\varsigma}(\vartheta)))+1\right]}+v_{pert}\frac{A^4}{\left[a\cosh(C(\varsigma-\bar{\varsigma}(\vartheta)))+1\right]^2}\right). \quad (49)$$

Using equation (40) the beam power and taking the limit to infinity is

$$E=\int_{-\infty}^{\infty}qq^*d\varsigma=\frac{A4Tanh^{-1}\left(\left(\frac{a-1}{a+1}\right)^{1/2}\right)}{\sqrt{2}(a^2-1)^{1/2}} \quad (50)$$

Substituting equation (49 and 50) into equation(43) one gets

$$a(\bar{\varsigma})=-2(E)^{-1}\int_{-\infty}^{\infty}\frac{\partial V}{\partial\varsigma}qq^*d\varsigma=$$
$$-2(E)^{-1}\frac{A\Delta n\exp(-b\bar{\varsigma}^2)}{\sqrt{2}C}\{(4b+2a^2+4A^2+bv_{pert})\bar{\varsigma}\bullet$$
$$\frac{4(2+a^2)Tanh^{-1}\left(\left(\frac{a-1}{a+1}\right)^{1/2}\right)}{(a^2-1)^{5/2}}+\frac{3a^2}{(a^2-1)^2}+4ab(2+A^2v_{pert})\bullet$$
$$\frac{\bar{\varsigma}}{2C}\left(-12aTanh^{-1}\left(\left(\frac{a-1}{a+1}\right)^{1/2}\right)\right)+\frac{a+2a^3}{(a^2-1)^2}$$
$$+\frac{ab\bar{\varsigma}}{C}\left[\frac{6a^3Tanh^{-1}\left(\left(\frac{a-1}{a+1}\right)^{1/2}\right)}{(a^2-1)^{5/2}}-\right.$$
$$\frac{a(5a^2-2)}{(a^2-1)^2}-(4abC+$$
$$8aA^2bCv_{pert})\bar{\varsigma}\frac{1}{4a^2C}F(2,2,\frac{5}{2};\frac{(a-1)}{2a})B(2,\frac{1}{2})-$$
$$\left.\frac{b\bar{\varsigma}}{2a}F(2,2,\frac{5}{2};\frac{(a-1)}{2a})B(2,\frac{1}{2})\right]\} \quad (51)$$

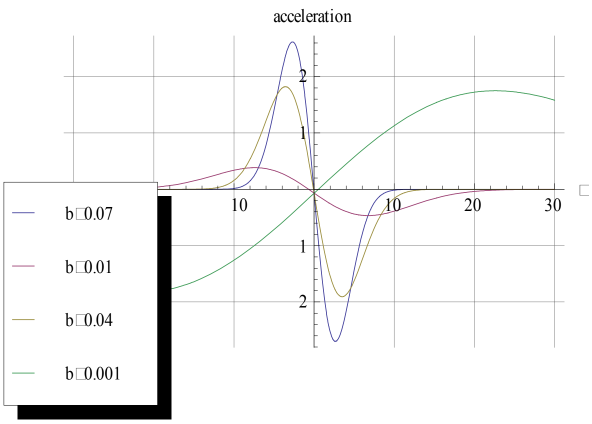
being the acceleration . Again, the hypergeometric and Beta function (*B*) is defined as

$$F(\alpha,\beta_h,\gamma_h;t)=\frac{\Gamma(\gamma_h)}{\Gamma(\alpha)\Gamma(\beta_h)}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha+n)\Gamma(\beta_h+n)t^n}{\Gamma(\gamma_h+n)n!} \quad (52)$$

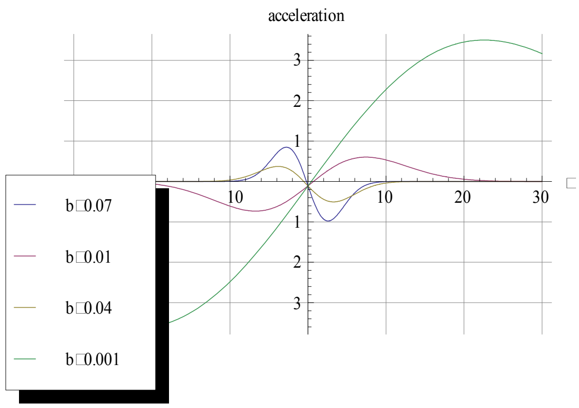
and

$$B(l,m)=\int_0^1\varsigma^{l-1}(1-\varsigma)^{m-1}d\varsigma. \quad (53)$$

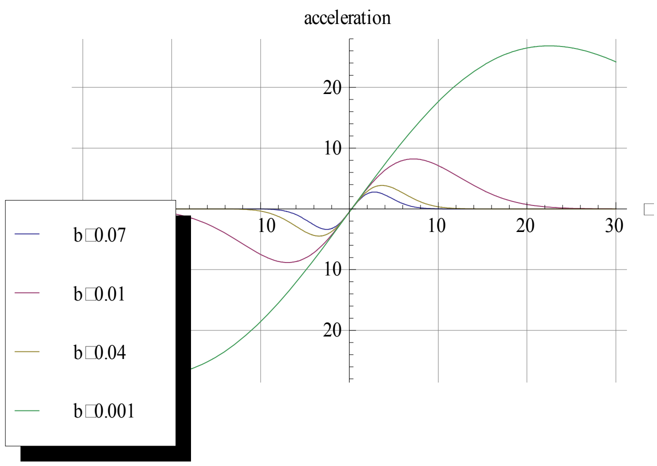
In Fig. 1 (a-c), the acceleration profile tends to zero in both directions depending on the average position ς. The initial profile is shifted with respect to the center, and the change in refractive index Δn with proper sign can keep the beam moving toward the center. Here, maximum velocity is reached and there is a force that inverts it sign when the beam passes through it. So the same force acts on the beam in the opposite direction. An oscillatory behavior is created on the beam. This swing effect has the same behavior on a perturbed homogenous NLSE without higher order nonlinearity [8,9,11,12]. The results of Fig. 1, show an swing effect with parabolic law spatial solitons. This effect has also been shown for power law spatial solitons with a triangular perturbed potential [26,27].



(a) A=1.104 a=3.36 b=0.01,0.07,0.04,0.001
Δn=0.05



(b)A=1.104 a=2.51 b=0.01,0.07,0.04,0.001
Δn=0.05



(c) A=1.104 a=1.46 b=0.01,0.07,0.04,0.001
Δn=0.05

Fig. 1. (a) and (b) are acceleration(equation) versus the mean ς showing a swing effect of the spatial soliton with higher order nonlinearity. In (c), for different value of a the swing effect can start negative.

Thus, the parabolic law soliton transverse motion follows from the principles of classical mechanics. More specifically, once the equation (43) is determined the wave trajectory is unequivocally provided. Lastly, various intensity profiles in the same index profile are subject to different forces and therefore move along different trajectories.

To verify certain transversal motion of the spatial soliton , we simulate some of the Nonlinear Schrödinger Equations with and without perturbation. First, we simulate

$$i\frac{\partial q}{\partial \vartheta}+\frac{1}{2}\frac{\partial^2 q}{\partial \varsigma^2}+|q|^2 q=0 \qquad (54)$$

with an initial profile hyperbolic secant

$$q(\zeta,\vartheta=0)=\operatorname{sec}h(\zeta) \qquad (55)$$

In Fig. 2(a),with N=10 grid points, the beam profile is constant with no deviation from its path which is what is predicted using soliton theory[13,20]. But in the next figure 2(b), we consider a perturbed Nonlinear Schrödinger Equation

$$i\frac{\partial q}{\partial \vartheta}+\frac{1}{2}\frac{\partial^2 q}{\partial \varsigma^2}+|q|^2 q+\frac{\Delta n_0 \exp(-b\zeta^2)}{1+\Delta n_0 \exp(-b\zeta^2)}|q|^2 q=0 \qquad (56)$$

with a Gaussian type profile perturbation potential with initial profile

$$q(\varsigma,\vartheta=0)=\operatorname{sec}h(\varsigma)\exp(i\frac{\varsigma}{2}). \qquad (57)$$

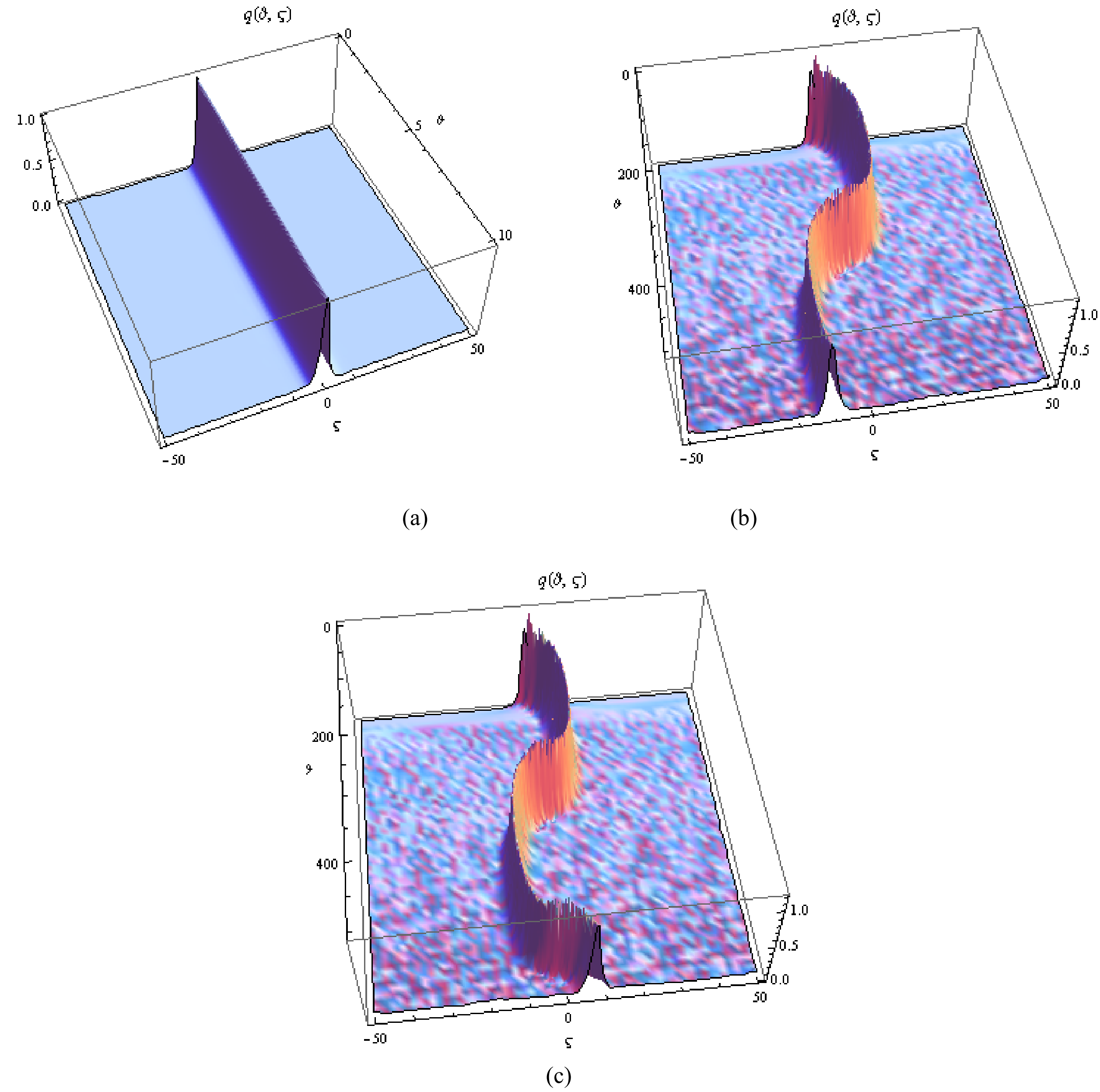


Fig. 2. (a) and (b) are acceleration(equation) versus the mean ς showing a swing effect of the spatial soliton with higher order nonlinearity. In (c), for different value of a the swing effect can start negative.

This was done with N=500 steps with b=0.07 and Δn=0.5. The beam profile mostly remains constant but swings periodically as it propagates along ϑ. Using the equation for acceleration one can calculate the period of oscillation. To do this first calculate the maximum point of the acceleration and then calculate the mean acceleration using

$$a_{mean} = \frac{1}{w_3}\int_0^{W_3} a(\bar{\varsigma})d\bar{\varsigma} \tag{58}$$

where w_3 is the initial position of the beam with respect to the center of the index profile. Then using the classical mechanics equation of motion

$$\bar{\varsigma} = \frac{1}{2}a_{mean}\,\vartheta^2 \tag{59}$$

and

$$\frac{M_{\vartheta}}{2} = \left(\frac{2w_3}{a_{mean}}\right)^{1/2}, \tag{60}$$

with $\varsigma=w_3$, M is the distance needed to propagate a swing to transverse distance $2w_3$. F. Garzia [8] using equations (40, 43,and 54) came up with a mean acceleration

$$a_{mean} = \frac{4}{3}\sqrt{2b}\Delta n C_0 (1-\exp(\frac{-1}{2})) \tag{61}$$

with initial beam profile

$$q(\varsigma, \vartheta = 0) = C_0 \sec h(C_0(\varsigma - \bar{\varsigma})) \tag{62}$$

the resulting period of oscillation is

$$T(C_0) = \left(\frac{12}{\Delta n b(1-\exp(\frac{-1}{2}))} \right)^{1/2} C_0^{-1}. \tag{63}$$

Edwards and Crutcher [11,12] calculated a mean acceleration and period of oscillation with a triangular index profile perturbation

$$\begin{aligned} a_M = \frac{\Delta n_0 C}{6Sb_T}\{[\frac{1}{2}[2\tanh^2(CS) + 2\tanh^2(Cb_T) \\ - \tanh^2(CS - Cb_T) - \tanh^2(CS + Cb_T)] \\ + 2[2\ln\cosh(CS) + 2\ln\cosh(Cb_T) \\ - \ln\cosh(CS - Cb_T) - \ln\cosh(CS + Cb_T)]\} \end{aligned} \tag{64}$$

and

$$\begin{aligned} \tau = \{-\frac{192 b_T S^2}{\Delta n_0 C}\}^{1/2}\{\frac{1}{2}[2\tanh^2(CS) + 2\tanh^2(Cb_T) \\ - \tanh^2(CS - Cb_T) - \tanh^2(CS + Cb_T)] \\ + 2[2\ln\cosh(CS) + 2\ln\cosh(Cb_T) \\ - \ln\cosh(CS - Cb_T) - \ln\cosh(CS + Cb_T)\}^{-1/2}, \end{aligned} \tag{65}$$

where b_T controls the width of the profile, C is the amplitude, and S is the initial position of the beam with respect to the center of the index profile. Since the focus in this study is whether there is a swing effect with higher order nonlinearity, calculation of the mean acceleration and period of oscillation for higher order nonlinearity will be a topic considered for future investigation.

Now taking into account simulating equation (34) for the higher order nonlinearity for spatial which is

$$\begin{aligned} i\frac{\partial q}{\partial \vartheta} + \frac{1}{2}\frac{\partial^2 q}{\partial \varsigma^2} + |q|^2 q + \nu_{pert}|q|^4 q \\ + \frac{\Delta n_0 \exp(-b\varsigma^2)}{1+\Delta n_0 \exp(-b\varsigma^2)}\left(|q|^2 q + \nu_{pert}|q|^4 q\right) = 0 \end{aligned} \tag{66}$$

In Fig. 3(a) below, the NLSE without perturbation was simulated using Biswas[21] solution as initial profile

$$a = (1+\nu_{pert}\frac{4}{3}A^2)^{1/2} \tag{67}$$

$$q(\varsigma, \vartheta = 0) = A(1 + a\cosh(\sqrt{A}(\varsigma))^{-1/2} \tag{68}$$

with A=0.75, ν_{pert} =0.001 and 50 x 10 grid points. The beam is a spatial soliton that propagates with no digression from its straight line path and unvarying amplitude with higher order nonlinearity. But in figure 3(b), we include the perturbation potential from equation (66) which there is an oscillation with approximately one period on a 50 x 500 grid points.

This was done with an initial profile of equation (67). By means of equation(66), in figure 3(c) and 3(d) there was a swing effect with 50x350 grid points with A=1 amplitude, ν_{pert} =0.000001, and a complex factor exp(iς/2) multiplied to the initial profile. Also, in this case we chose to use a=0.5 instead of using equation (68). In Fig. 3(e) and 3(f),we simulated a higher frequency of the spatial soliton beam with 50x350 grid points with A=1.75, ν_{pert} =0.000001,and a complex factor exp(iς/2). In figure 4(a) and 4(b) below, we increased the number of transverse points to 12000. Here, there was no swing effect via equation (67) as the initial profile, but there was a movement away from a straight line path.

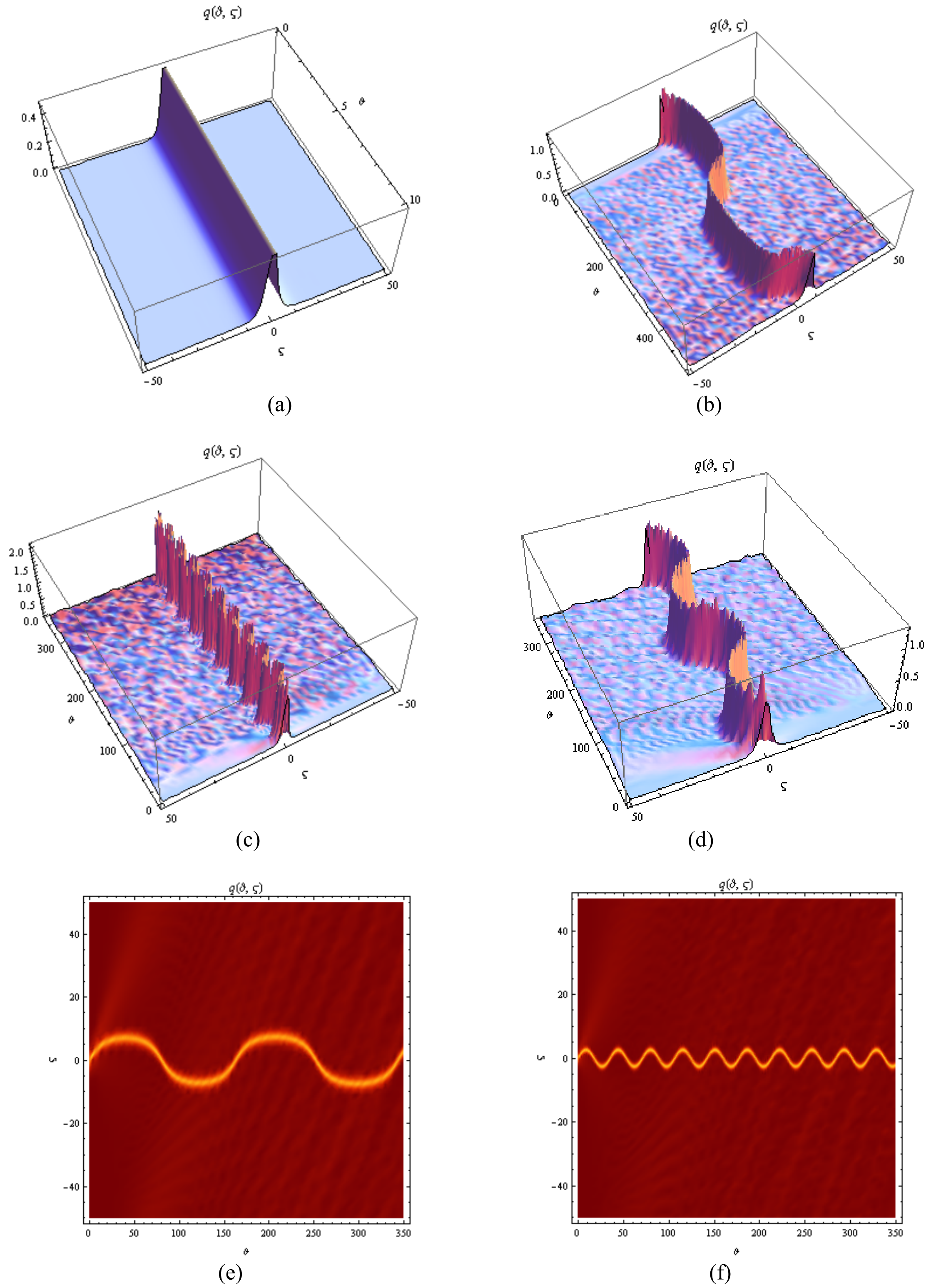


Fig. 3.Simulation of the higher order nonlinearity NLSE (a)Δn=0 , A=0.75, v_{pert} =0.001 a=1.000037 (b) Δn=0.5 , A=1, v_{pert} =0.000001 a=0.5,b=0.07, (c) and(d) Δn=0.5 , A=1, v_{pert} =0.000001 a=0.5,b=0.07 with initial profile equation (67) (e) and (f), and (d) Δn = 0.5, A = 1.75, v_{pert} = 0.000001 a = 0.5, b = 0.07 with initial profile equation (67).

$$q(\varsigma, \vartheta = 0) = A1(1 + a\cosh(\sqrt{A}1(\varsigma))^{-1/2} + A2(1 + a\cosh(\sqrt{A}2(\varsigma))^{-1/2} \tag{69}$$

In Fig. 4(c) and 4(d) above, using equation (66), a two beam profile was used to start the NLSE for simulation of the higher order nonlinearity. There was interaction of the two spatial beams one fourth of a period from the initial position. But one of the beams continues to swing at a certain period while the other beam approaches the negative longitudinal boundary. The oscillatory beam has a higher amplitude (A1=1) compared to the beam with an amplitude (A2=0.75) that approaches the negative boundary. This could lead to the possibility that a spatial beam with a swing effect can control another beam differently than the normal spatial beam interaction of repulsion and interaction by just passing through each other. In figure 5(a) and 5(b), repulsion is demonstrated up to a grid step 150. Both beams had incomplete swing effects with smaller amplitude beam (A1=.75) approaching the negative boundary. Both beams shared the similar paths from 0 to approximately 140 grid points. In figure 5(c) and 5(d), mixing of two beams with swing effect is demonstrated up to 200 grid points before the smaller amplitude beam (A1=1) approaches the positive boundary. In figure 5(b), the smaller amplitude beam has a distorted swing before diverges to the boundary. Here, theoretically we have shown that light beams with a swing effect can possibly control other straight line spatial beams by putting them into an oscillatory period. Oscillatory spatial beams can also stop other oscillatory beams from propagating in a certain direction and shifting them back to straight path spatial beams. Further study would be needed to see if two interacting swing spatial beams can oscillate together at longer distances. Lastly, one could also investigate the conservation of energy and required conditions to set off these beam interactions which is a possible focus of future investigation.

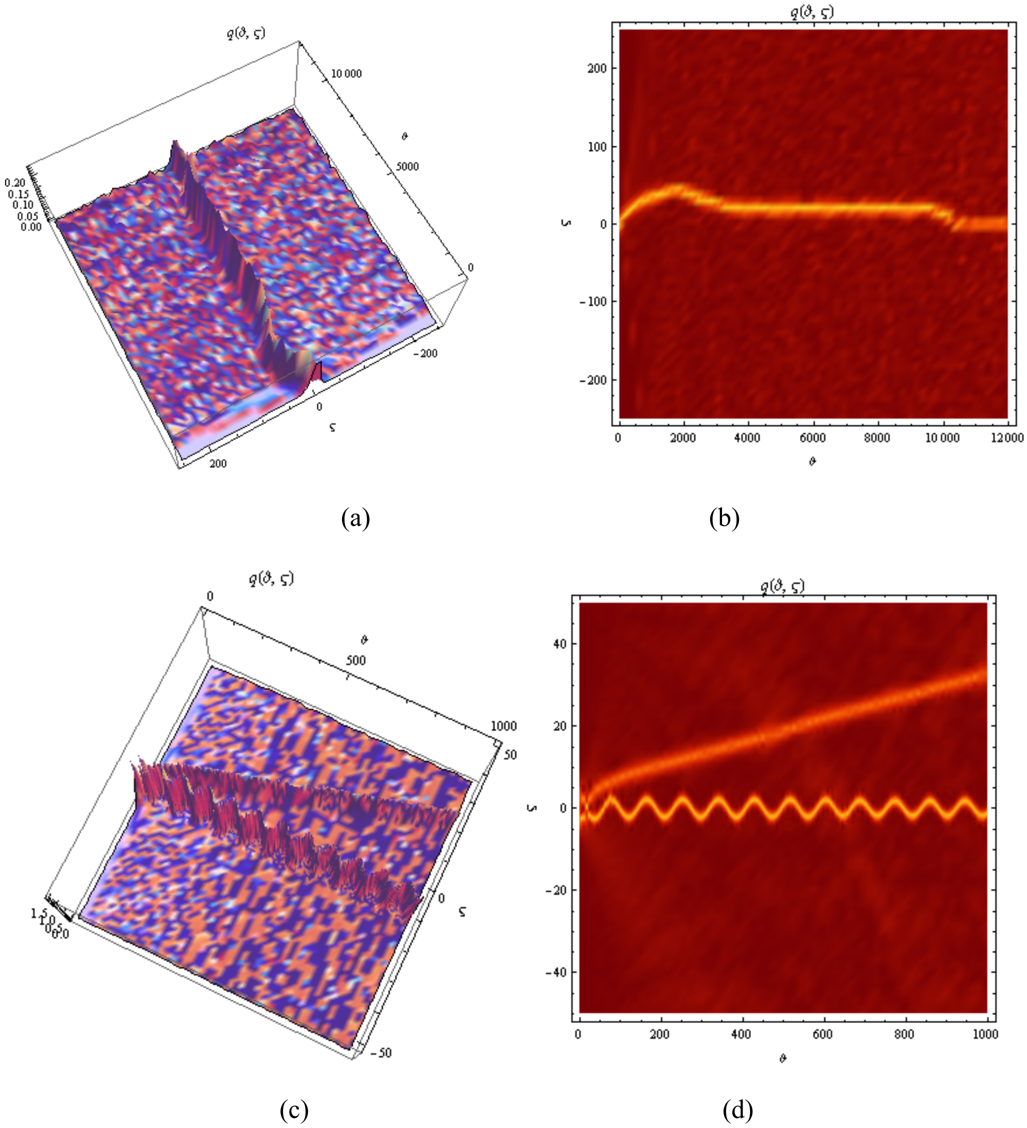


Fig. 4.Simulation of the higher order nonlinearity NLSE with no swing but a deviation from straight path in (a)and (b) with Δn=0.5 , A=0.25, v_{pert} =0.000001 ,b=0.07, on a 250x12000 grid . Two beam interaction (c) and(d) with Δn=0.09, A1=1,A2=0.75, v_{pert} = 0.000001 a = 0.5, b = 0.07 with initial profile equation (69) .

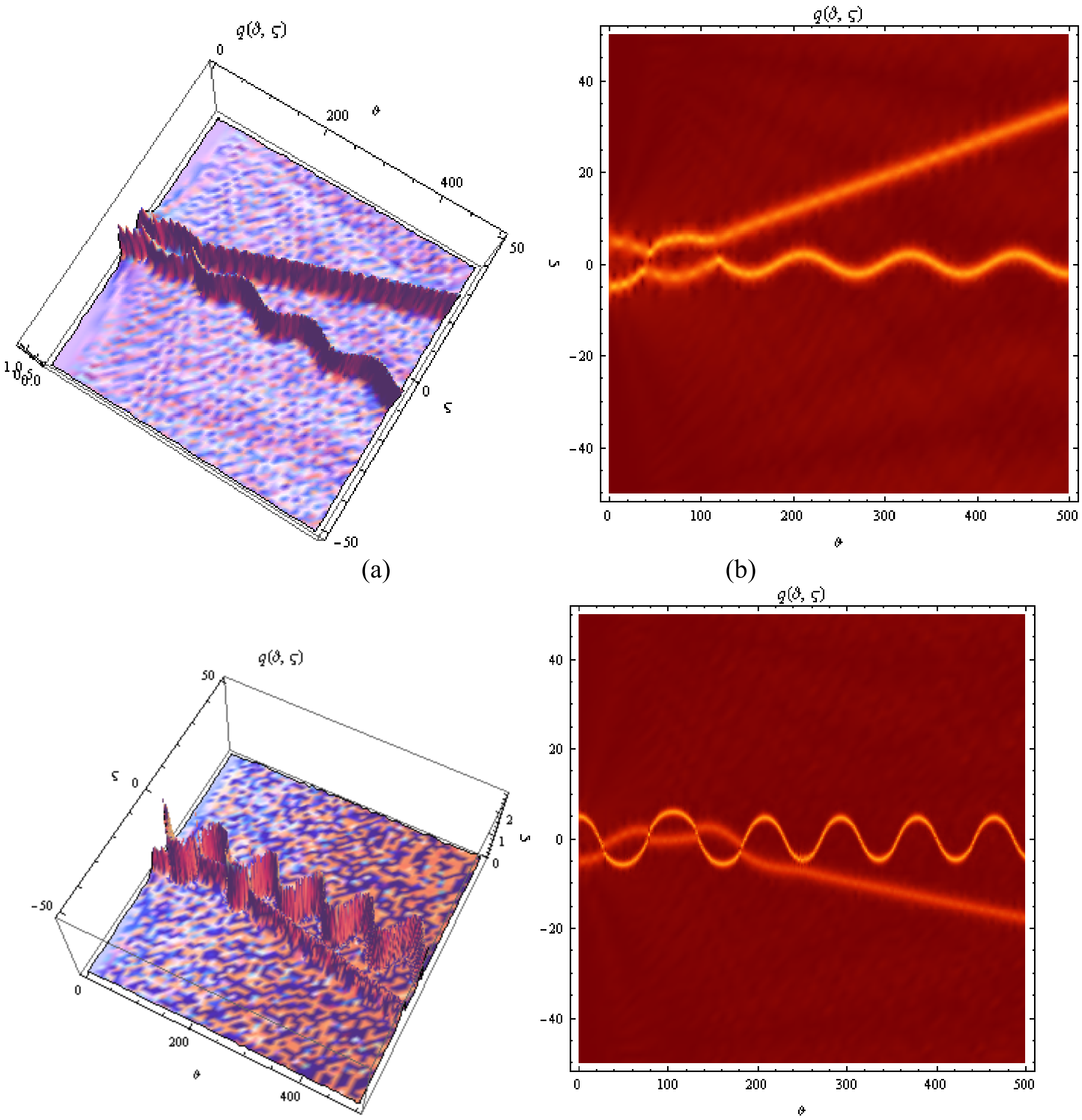


Fig. 5.Simulation of the higher order nonlinearity NLSE. In (a) and (b) Two beam interaction shows repulsion with Δn=0.09, A1=1,A2=0.75, v_{pert} = 0.000001 a=0.5,b=0.07 with initial profile equation (69). In (c) and(d), Two beam interaction shows mixing with each beam having swing up to around N=200 steps with Δn=0.5 , A1=2,A2=1,, v_{pert} =0.000001 a=0.5,b=0.07 with initial profile equation(69) .

4. Conclusions

In order to understand features of the parabolic law Nonlinear Schrödinger Equation, we started with the expansion of dielectric constant (or index of refraction) which included constant and tensor terms with electric field amplitudes. From this, one was able to derive the nonlinear Helmholtz equation which is the NLSE with higher order diffraction and nonlinearity. We normalized the equation and approximated the higher order terms into nonlinear equation that can be studied by methods used by Biswas[21]. Using perturbation theory, a solution to equation presented that was dependent on the coefficient of the higher order nonlinearity ($\chi^{(5)}$). Blair[20] pointed out that these solutions are nonparaxial spatial solitons or quasi-solitons due to the higher order diffraction. We applied the equivalence particle theory to higher order normalized NLSE assuming no higher order diffraction which reduced to a parabolic law spatial NLSE. Even with higher order nonlinearity the spatial solitons have swing effect. We also show the swing by simulation of the perturbed NLSE. Taking one further step, the two beam interaction features were presented simulating the perturbed equation. It was shown that the two beams oscillate together, repulse the oscillation, and change an oscillating beam to a straight line away from the other oscillating beam. By means of further study, this may be practical for optoelectronics or short distance nonlinear optical devices.

References

[1] B. Gisin, B. Malomed, Journal of Optical Society of America **18**, 1356 (2001).
[2] B. Gisin, B.Malomed, Journal of Optics A: Pure and Applied Optics **3**, 284 (2001).
[3] S. Crutcher, A. Osei, Progress In Electromagnetics Research B **29**, 83-103 (2011).
[4] A. Davoyan, I. Shadrivov,Y. Kivshar, Optics Express **17**, 21732 (2009).
[5] S. Crutcher, A.Osei, A. Biswas, Optics & Laser Technology **in press** (2011).
[6] A. Davoyan, I. Shadrivov, A. Zharov, D. Gramotnev, Y. Kivshar, Physics Review Letters **105**, 116804 (2010).
[7] A. Marini, D. Skryabin, B. Malomed, Optics Express **19**, 6616 (2011).
[8] F. Garzia, C. Sibilia, M. Bertolotti, Optics Communications **139**, 193 (1997).
[9] A. Suryanto, Ph.D. Dissertation, University of Twente, AE Enschede, The Netherlands (2003).
[10] L. Dong, H. Wang, Applied Physics B: Lasers and Optics, **84**, 465 (2006).
[11] M. Edwards, S. Crutcher, M. Aggarwal,A. Batra, H. Jaenisch,A. Osei, S. Bairavarasu, SPIE Proceedings **5525,** 163 (2004).
[12] S.Crutcher, M. Edwards, A. Biswas, M. Aggarwal, SPIE Proceedings **5876,** 58760E (2005).
[13] Y. Kivshar, G. Agrawal, Optical Solitons From Fibers to Photonic Crystals, pub. Academic Press San Diego, CA 31-60 (2003).
[14] A. Hasegawa, Y. Kodama,Solitons in Optical Communications, pub. Oxford University Press,USA, (1995).
[15] A. Biswas A, S. Konar, Introduction to Non-Kerr Law Optical Solitons, pub. Chapman & Hall, Florida, (2007).
[16] A. Aceves, J. Moloney, A. Newell, Physical Review **39**, 225 (1980).
[17] A. Suryanto, E. Groesen, Optical and Quantum Electronics **34**, 597 (2002).
[18] A. Suryanto, E. Groesen, Optical Communications **258**, 264 (2006).
[19] N. Akhmediev,A. Ankiewicz, Solitons Nonlinear Pulses and Beams, pub. Chapman and Hall, UK, (1999).
[20] S. Blair, Non paraxial one-dimensional spatial solitons, Chaos **10**, 570 (2000).
[21] K. Russell,A. Biswas, D. Milovic, E. Zerrad, Optics & Laser Technology **40**, 647 (2008).
[22] A. Biswas, J Phys. A: Math. Gen, **36**, 4581 (2003).
[23] B. Sturdevant, D. Lott, A. Biswas, Progress In Electromagnetic Research Letters **10**, 69 (2009).
[24] A. Biswas, Physics Letters A **372**, 5941 (2008).
[25] A. Biswas, Applied Mathematics and Computation **208**, 299 (2009).
[26] A. Biswas, Communication in Nonlinear Science and Numerical Simulations **14**, 1830 (2009).
[27] S. Crutcher, M. Edwards, A.Osei, Proceedings of SPIE, **7056**, 70560Q-1 (2008).
[28] D. Mihalache, J. Optoelectron. Adv. Mater. **13**, 1055 (2011).
[29] D. Mihalache, Proceedings of the Romanian Academy, Series A **11**,142 (2010).
[30] I. V. Mel'nikov, D. Mihalache, N.-C.Panoiu, Optics Communications **181**, 345 (2000).
[31] H. Leblond, D. Mihalache, Romanian Reports in Physics **63**, 1254 (2011).
[32] H. Leblond, H. Triki, F. Sanchez, D.Mihalache, Physical Review A **83**, 063802 (5 pages) (2011).

*Corresponding author: biswas.anjan@gmail.com