Phase transition in fiber bundle model

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In the present work, we study the phase transition in the composite materials by using the Langevin equation within the framework of the fiber bundle model, subject to the global load-sharing rule in which the load failing elements is shared equally among all surviving elements. We show that fracture in the case of global load sharing can be seen as a second-order phase transition and then we define the order parameter related to this phase transition. Finally we present some properties related to failure phenomenon obtained in the critical point like the failure time and the susceptibility.

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1. Introduction

Fracture of composite materials has received a lot of attention in the last years not only for the very important technological applications but also for the fundamental statistical aspects. Recently the development reached by statistical physics has had a perform impact on this phenomenon. Different methods and technics have been successfully used in studying fracture processes in composite materials.

In the last years, the introduction of models of material failure has led to the evidence that fracture can be viewed as a kind of critical phenomenon [1–6]. However, the question of whether fracture exhibits the properties of a first-order or a second-order phase transition remains under discussion as well as what is the order parameter that determines the type of transition [7].

In this field, it is important to use models that are able to describe the complexity of the failure process, despite they should be simple enough to permit analytical insights. To this class of models belong the well-known fiber bundle models (FBM) which have been the subject of intense research during the last years [7-10]. FBM can be classified in two groups, static and dynamic. In static version of FBM, a set of fibers (elements) is located on a supporting lattice and one assigns to its elements a random strength threshold sampled from a probability distribution. The lattice is loaded and fibers break if their loads exceed their threshold values.

On the other hand, the dynamic FBM simulates failure by creep rupture. Usually a constant load is maintained on the system and the fibers break by fatigue after a period of time.

One can assume different load transfer rules to mimic the range of interactions among the fibers in the set.

The global load sharing (GLS) rule is the simplest theoretical approach one can adopt to make the problem analytically tractable, which implies that the load carried by failed elements is equally distributed among the surviving elements of the system, this model is known as democratic fiber bundle, assumes long-range interaction among the fibers which makes it a mean-field approximation which makes it a mean-field approximation.

In the local load sharing (LLS), the load of failed elements is given equally to all the intact neighbors. This case assumes short-range interaction among the fibers.

The rest of the paper is organized as follows. In the next section, we describe the FBM model under the GLS rule. In section 3, we present and discuss the numerical results of the Langevin dynamic simulation. Finally, section 4 is devoted to conclusions.

2. The model

We simulate the behavior of an heterogeneous material subjected to an imposed load F using a FBM. For this purpose, we model the material as a system of N parallel elastic fibers whose extremities are fixed on a rigid support as shown in Fig. 1.

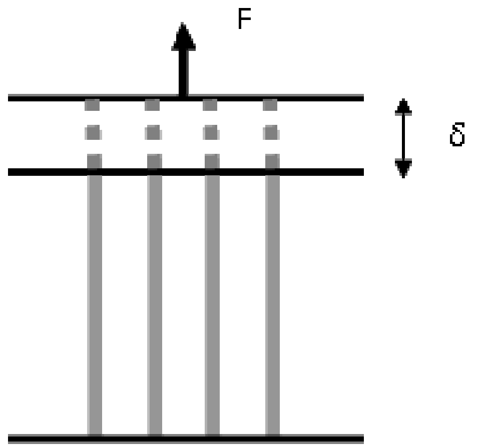

Fig. 1. A schematic illustration of the fiber bundle model.

This model is equivalent to N fibers in parallel subjected to a total force F. Specifically, we have studied the model by using the following rules:

The applied force produces a local force f_i on each fiber. F is democratically distributed in the system

$$F = \sum_{i=1}^{N} f_i .$$

Recently, in order to simulate the dynamics of fibers, we have introduced a model [11] that use the Langevin equation [12,13]. This approach is characterized by the use of a stochastic differential equation which denotes all the ingredients necessary to study the failure in fiber bundle model such as thermal noise and frictional force.

Recently, we have introduced a model that use

The dynamics of the system is completely determined by the Langevin equation:

$$m_i \frac{d^2 z_i}{dt^2} = -m_i \gamma_i \dot{z}_i + f_i(z_i) + \vec{R}_i(t)$$

Where Z_i, m_i and γ_i denote the position, mass and viscous friction coefficient of the fiber, respectively. This equation is a stochastic differential equation in which two force terms have been added to Newton's second law to approximate the effects of neglected degrees of freedom. One term represents a frictional force, $\gamma_i \dot{z}_i$, the other a random force $\vec{R}$, which describes here the thermal noise. It is usually modelled by a Gaussian white noise with zero time average, $\left\langle \vec{R}_i(t) \right\rangle = 0$, and autocorrelation function:

$$\left\langle \vec{R}_i(t) \vec{R}_i(t') \right\rangle = 2 m_i \gamma_i k_B T \delta(t - t')$$

The angular brackets denote here an average, and k_B is Boltzmann's constant.

We impose that the system is large frictional which allows to neglect inertial effects, in this case the system is overdamped.

Initially each fiber has a length z_0. When we apply a load $f_0 = k \min \delta_c(i)$, which $\delta_c(i)$ is a different random elongation, taken from an uniform distribution, the fiber dilates and his length becomes: $z = z_0 + \delta$, here δ is the fiber elongation.

If δ exceeds the elongation threshold value δ_c, the fiber is removed and its load is transferred equally to all the intact ones. We consider only the case of global load sharing (GLS) for the redistribution of load following fiber failure.

We use the periodic boundary conditions which is an important technique in a molecular dynamics simulation. We make it in simulation in order to remove the effect of surface and it consists from a system with a few hundred fibers behave like an infinte one.

In numerical simulation, the cycle of complete breakdown of the material is performed many times in order to average out the effect of fluctuation.

3. Numerical results

The idea that fracture can be seen as a phase transition has a long history. For example, the Griffith theory of fracture is very similar in spirit to the classical theory of nucleation in first order phase transitions. In bubble nucleation, a critical droplet will form when the loss in free energy due to the bulk forces exceeds the increase in the interfacial energy. Similarly fracture occurs if the external stress prevail over the resistance at surface of the crack. Further analogies come from the scaling behavior observed in fracture experiments, such as for the crack roughness and the acoustic emission distributions.

Phase transitions are characterized by changes in the internal symmetries of a material as external control parameters are varied. Familiar examples are the melting of a crystal, or the ferromagnetic transition in a magnet [14]. In the first example, we have an abrupt first-order phase transition, with latent heat, coexistance and no precursors, while the latter is a continuous second-order transitions.

It is clear that at the critical stress value σ_c, GLS fiber bundles show phase transition from partially broken state to completely broken state. What is the order of this phase transition Zapperi et al. [4,5] considered the fraction of unbroken fibers as the order parameter and as it has a discontinuity at the critical stress value, they suggested, after a mean-field analysis, that it can be seen as a first-order phase transition similar to spinodal instability [15]. The additional reason for identifying the transition at $\sigma = \sigma_c$ as a first order spinodal point had been [16] that in the presence of short-range interactions, the transition becomes discontinuous and first-order like. It is indeed hard to identify continuously changing order parameter there. We, however, believe that the transition in GLS to be second-order. Chronologically, a little later, a new parameter was identified [17]: the branching ratio (ζ), which is defined as the probability of triggering further breaking given an individual failure. The branching ratio continuously approaches the value 1 at the critical stress (σ_c) starting from 0 value (for very small σ). Also it shows a power law variation: $1-\zeta \propto (\sigma_c-\sigma)^{\beta}$, with β=1/2. Therefore $1-\zeta$ acts as the order parameter showing a continuous transition at the critical point, signaling a second-order phase transition.

Let Nt be the number of fibers that survive after step t, where t indicates the number of stress redistribution steps.

Now we introduce σ = F/N, the applied stress and $\rho(t)$ = Nt/N, the surviving fraction of total fibers.

In Fig. 3 we plot the behavior of the fraction of unbroken fibers. We see that the fraction of surviving fibers decrease as the time increase tell the total failure.

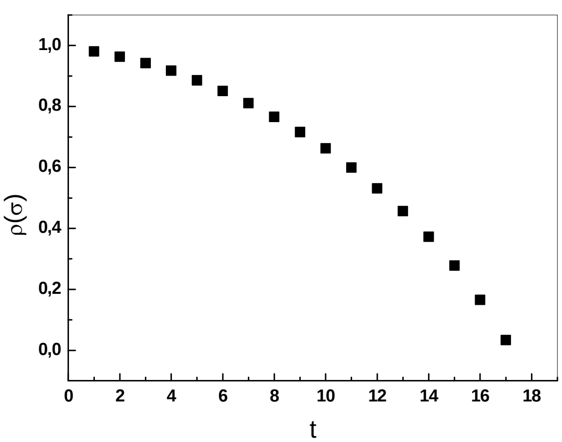

Fig. 2. Behavior of the fraction of surviving fibers for system size L=512 and T=315 K.

Now we explore in Fig. 3 quantity Φ which represents the difference of the fraction of surviving fibers $\rho(\sigma)-\rho(\sigma_c)$ versus $\sigma_c-\sigma$.

It may be noted that the quantity $\rho(\sigma)-\rho(\sigma_c)$ behaves like an order parameter that determines a transition from a state of partial failure ($\sigma \le \sigma_c$) to a state of total failure
($\sigma > \sigma_c$):

$$\Phi = \rho(\sigma)-\rho(\sigma_c) \sim (\sigma_c-\sigma)^{\beta} \qquad \text{with } \beta=1/2$$

Here σ_c is the critical value of initial applied stress beyond which the bundle fails completely.

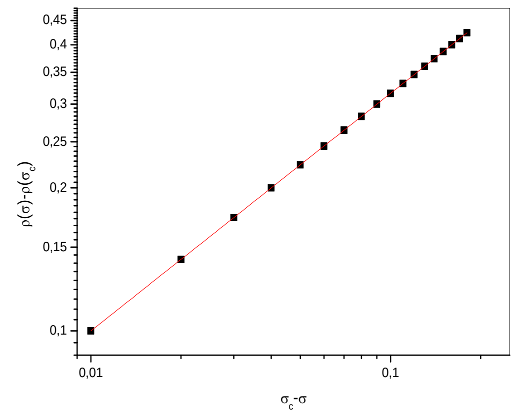

Fig. 3. Behavior of the difference of the fraction of surviving fibers for system size L=512 and T=315 K, the solid line represents the exponent β = ½.

Static critical behaviour of the fiber bundle is observed in the susceptibility of the surviving fraction of fibers. One may define a breakdown susceptibility χ by the change of $\rho(\sigma)$ due to an infinitesimal increment of the applied stress σ. We use in the simulation $d\sigma = 0.01$
we see that the susceptibility diverges in the form of a power law as the initial applied stress approaches its critical value from below:

$$\chi = \left|\frac{d\rho}{d\sigma}\right| \sim (\sigma_c-\sigma)^{-\gamma} \quad \text{with } \gamma=1/2$$

the susceptibility diverges as the applied stress σ approaches the critical value σ_c
Such a divergence in χ had already been reported in several studies [4 ,5].

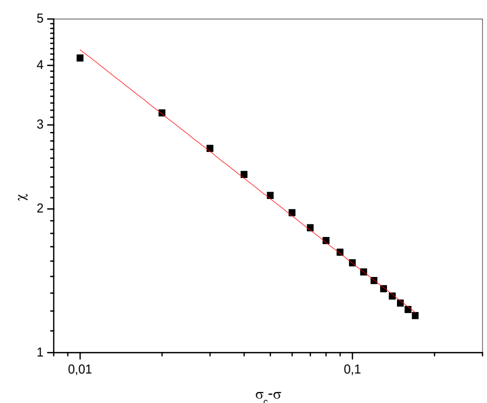

Fig. 4. The susceptibility of the material for system size L=512 and T=315 K, the dashed line represents the exponent γ = ½.

Near the critical point we explore in Fig. 5 the behavior of the relaxation time τ, Our numerical study shows that τ has a power law divergence at σ_c with a universal exponent $\alpha = 1/2$. This power law can be written as $\tau \sim (\sigma_c - \sigma)^{-\alpha}$ and it's verified previously [18,19]

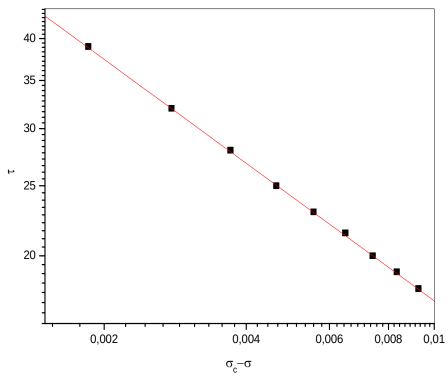

Fig. 5. The relaxation time τ for system size L=512 and T=315 K, the solid line represents the exponent α=1/2.

At the critical point ($\sigma = \sigma_c$), a dynamic critical behavior has been observed in the relaxation of the failure process and we can write

$$\rho(\sigma) - \rho(\sigma_c) \sim t^{-\delta} \quad \text{with } \delta=1$$

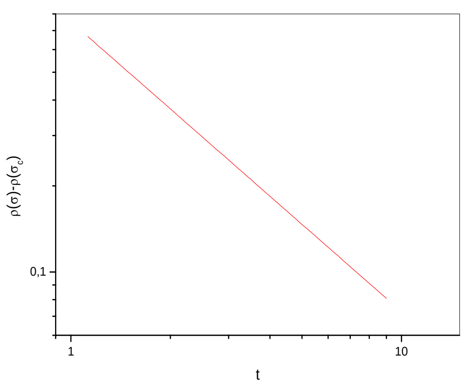

Fig. 6. The quantity $\rho(\sigma) - \rho(\sigma_c)$ versus time for system size L=512 and T=315 K, the solid line represents the exponent δ=1.

As mentioned earlier we considered the difference between the fraction of unbroken fibers at any σ and at σ_c, as the order parameter (Φ): it shows a similar continuous variation with the applied stress: $\Phi \sim (\sigma_c - \sigma)^{\beta}$, with $\beta = 1/2$. Apart from this, the susceptibility and relaxation time diverge at the critical point following power laws having universal exponent values [18,19]. One may therefore conclude that at the critical point the GLS fiber bundles show a second order phase transition.

4. Conclusion

In summary we have studied the phase transition in the composite materials by using the Langevin equation, then we have shown the critical properties of failure in a class of fiber bundle models under an applied stress. The model is simple dynamical system that show an irreversible phase transition. We have determined the dynamic critical properties associated with the phase transition. We have defined an order parameter which shows that the transition is of second-order. It is supported by facts which are characteristic of second-order transitions: the susceptibility diverges at the critical point and the decay of surviving fraction of fibers with time at the critical point follows a power-law.

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