

Propagation of electromagnetic waves in layered anisotropic medium

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In this paper, wave propagation in two-layered anisotropic media when uniaxially anisotropic medium has its optic axis oriented vertically and tilted around z-axis has been investigated. The Fresnel coefficients are derived for horizontally and vertically polarized incident waves with arbitrary incident angles. The contribution of the cross polarized components of the reflected and transmitted waves due to anisotropic- isotropic or isotropic-anisotropic interfaces have been analyzed. The numerical results for the Fresnel coefficients have been presented. The results of this work can be used in calculation of the electric and magnetic fields in radiation and scattering problems.

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1. Introduction

Recent advances in microwave devices increased the significance of the material properties in the design of high frequency devices. The material characteristics are very important in the development of high performance devices at the microwave ranges. Material characteristics can be best understood by the analysis of the wave propagation inside these materials. One of the material types that have been very attractive for high frequency applications is the anisotropic type material. Anisotropic materials have been widely used in radiation, scattering and remote sensing problems [1-11] due to their several advantages. The anisotropic behavior of the material can be used as a design knob to adjust critical parameters since electric field vector is no longer always parallel with the electric flux density for this type of medium.

Anisotropic medium can be classified based on its permittivity or permeability tensor. Uniaxially anisotropic medium with its optic axis oriented vertically, i.e. oriented in z-direction, is defined with the following permittivity tensor

$$\bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_{1z} \end{bmatrix} \quad (1)$$

When the optic axis of the medium is tilted around z-axis by an angle ψ , its permittivity tensor takes the following form

$$\bar{\bar{\epsilon}}^{oz} = \begin{bmatrix} \epsilon_{11}^{oz} & 0 & 0 \\ 0 & \epsilon_{22}^{oz} & \epsilon_{23}^{oz} \\ 0 & \epsilon_{32}^{oz} & \epsilon_{33}^{oz} \end{bmatrix} \quad (2)$$

where

$$\epsilon_{11}^{oz} = \epsilon_1 \quad (3a)$$

$$\epsilon_{22}^{oz} = \epsilon_1 \cos^2 \psi + \epsilon_{1z} \sin^2 \psi \quad (3b)$$

$$\epsilon_{23}^{oz} = \epsilon_{32}^{oz} = (\epsilon_1 - \epsilon_{1z}) \cos \psi \sin \psi \quad (3c)$$

$$\epsilon_{33}^{oz} = \epsilon_1 \sin^2 \psi + \epsilon_{1z} \cos^2 \psi \quad (3d)$$

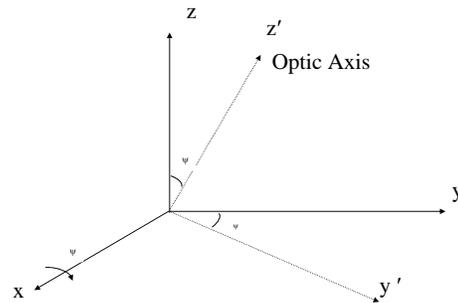


Fig.1. Geometry of the anisotropic medium.

Fig. 2 illustrates the wave propagation through two-layered anisotropic media for vertically or horizontally polarized waves. Horizontally polarized waves are TE waves which are perpendicular to the plane formed by the wave normal and the wave vector in the isotropic medium. Vertically polarized waves propagate parallel to this plane. The transmitted wave in anisotropic region is an *o-ordinary wave* if the incident wave from isotropic region is TE wave, and *e-extraordinary wave* if the incident wave is TM wave. The transmitted or the reflected waves in the isotropic region can be TE wave or TM wave based on the incident wave and can have cross

polarized component if the optic axis of the anisotropic medium is tilted.

In this paper, the analysis of the wave propagation in a two-layered anisotropic media will be given starting from the single layered case when the anisotropic medium has its optic axis oriented vertically and then tilted around one of its axis. The numerical results illustrating the contribution of the cross polarized wave due to tilted optic axis will be presented for isotropic-anisotropic interface. The reflection and transmission coefficients for the layered structure will be derived using half-space reflection coefficients. Practical application example will be presented using our results.

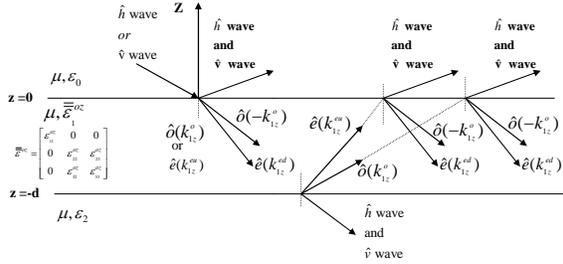


Fig 2. Wave propagation in two-layered anisotropic media

2. Wave propagation in a single layered anisotropic media – vertically uniaxial case

In this section, the analysis of the wave propagation for a single layer anisotropic media shown in Fig. 3 is described. The anisotropic layer is considered to be uniaxially anisotropic medium with its optic axis oriented in z-direction (vertically uniaxial case). The wave is incident from isotropic region designated by Region 0 to uniaxially anisotropic region designated by Region 1. The incident wave in isotropic region can be either a horizontally polarized wave (TE wave) or vertically polarized wave (TM wave). The transmitted wave in anisotropic region is an *o*-ordinary wave if the incident wave is TE wave and *e*-extraordinary wave if the incident wave is TM wave. The reflected wave in isotropic region is a horizontally polarized wave when incident wave is TE wave and vertically polarized wave when incident wave is TM wave. Two vectors, $\hat{h}_0(\pm k_{0z})$ and $\hat{v}(\pm k_{0z})$, represent the unit vectors for horizontally polarized (TE) and vertically polarized waves (TM) in isotropic region, respectively. Unit vectors $\hat{o}(-k_{1z}^o)$ and $\hat{e}(k_{1z}^{ed})$ represent the downward propagating ordinary and extraordinary waves that exist in a vertically uniaxial anisotropic medium. The unit vectors are defined as

$$\hat{h}_0(\pm k_{0z}) = \frac{\hat{z} \times \bar{k}_0}{k_\rho} = \frac{(-\hat{x}k_y + \hat{y}k_x)}{k_\rho} \quad (4)$$

$$\hat{v}_0(k_{0z}) = \frac{\hat{h}_0(k_{0z}) \times \bar{k}_0}{k_0} = \frac{1}{k_0} \left[\frac{k_{oz}(\hat{x}k_x + \hat{y}k_y)}{k_\rho} - \hat{z}k_\rho \right] \quad (5)$$

$$\hat{v}_0(-k_{0z}) = \frac{\hat{h}_0(-k_{0z}) \times \bar{k}_0}{k_0} = \frac{1}{k_0} \left[\frac{k_{oz}(\hat{x}k_x + \hat{y}k_y)}{k_\rho} - \hat{z}k_\rho \right] \quad (6)$$

$$\hat{o}(-k_{1z}^o) = \frac{\hat{z} \times \bar{\kappa}_1^o}{|\hat{z} \times \bar{\kappa}_1^o|} = \frac{(-\hat{x}k_y + \hat{y}k_x)}{k_\rho} = \hat{h}_1(-k_{1z}) \quad (7)$$

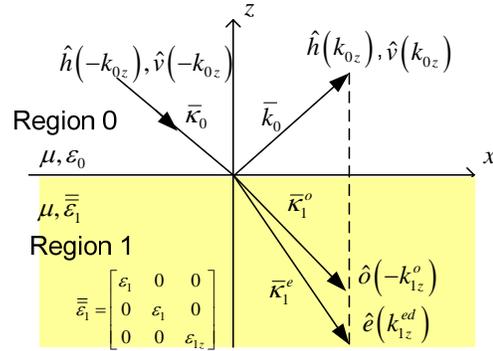


Fig. 3 Reflection and transmission of waves in single-layered anisotropic slab.

$$\hat{e}(k_{1z}^{ed}) = \frac{\hat{h}_1(-k_{1z}) \times \bar{\kappa}_{1u}}{k_{1u}} = \frac{1}{k_{1u}} \left[\frac{-\bar{k}_\rho k_{1z}^{ed} \epsilon_{1z}}{\epsilon_1 k_\rho} - \hat{z}k_\rho \right] \quad (8)$$

The wave vectors showing the direction of wave propagation are defined as

$$\bar{k}_0 = \bar{k}_\rho + \hat{z}k_0 \quad (9)$$

$$\bar{\kappa}_0 = \bar{k}_\rho - \hat{z}k_0 \quad (10)$$

$$\bar{k}_\rho = \hat{x}k_x + \hat{y}k_y \quad (11)$$

$$\bar{k}_1^o = \bar{k}_\rho + \hat{z}k_{1z}^o \quad (12)$$

$$\bar{\kappa}_1^o = \bar{k}_\rho - k_{1z}^o \hat{z} \quad (13)$$

$$\bar{k}_1^e = \bar{k}_\rho + \hat{z}k_{1z}^{eu} \quad (14)$$

$$\bar{\kappa}_1^e = \bar{k}_\rho + k_{1z}^{ed} \hat{z} \quad (15)$$

$$\bar{k}_{1u} = \bar{\epsilon} \cdot \bar{k}_1^e \quad (16)$$

$$\bar{\kappa}_{1u} = \bar{\epsilon} \cdot \bar{\kappa}_1^e \quad (17)$$

The wave numbers are found from the dispersion relations as

$$k_{0z}^o = \left[k_0^2 - k_\rho^2 \right]^{1/2} \quad (18)$$

$$k_{1z}^o = [k_1^2 - k_\rho^2]^{1/2} \quad (19)$$

$$k_{0,1} = \omega \sqrt{\mu \varepsilon_{0,1}} \quad (20)$$

$$\begin{bmatrix} k_{1z}^{eu} \\ k_{1z}^{ed} \end{bmatrix} = \pm \frac{1}{\varepsilon_{1z}} [k_1^2 \varepsilon_{1z}^2 - \varepsilon_1 \varepsilon_{1z} k_\rho^2]^{1/2} \quad (21)$$

2.1 Case 1 – TE Wave Incidence

When the incident wave from Region 0 is TE wave, the electric field vector can be represented as

$$\bar{E}_{i,0} = \hat{h}_0(-k_{0z}) e^{i(\bar{k}_o \cdot \bar{r})} e^{-i\omega t} \quad (22)$$

$$\bar{E}_{r,0} = [\hat{h}_0(k_{0z}) R_{01HH} e^{i(\bar{k}_o \cdot \bar{r})}] e^{-i\omega t} \quad (23)$$

$$\bar{E}_{t,1} = [\hat{o}(-k_{1z}^o) T_{Ho} e^{i(\bar{k}_1^o \cdot \bar{r})}] e^{-i\omega t} \quad (24)$$

The first subscript shows the wave type (incident, reflected, or transmitted) whereas the second subscript shows the region. R_{01HH} is the reflection coefficient when incident and reflected waves are both horizontally polarized. T_{Ho} is the transmission coefficient when incident wave is horizontally polarized and transmitted wave is an ordinary wave. The reflection and transmission coefficients, Fresnel coefficients, are found using boundary conditions at $z=0$ as described in the previous section. Application of boundary conditions at $z=0$ gives following equations

$$\hat{z} \times \bar{E}_0 = \hat{z} \times \bar{E}_1 \quad (25)$$

$$\hat{z} \times \nabla \times \bar{E}_0 = \hat{z} \times \nabla \times \bar{E}_1 \quad (26)$$

where

$$\bar{E}_0 = \bar{E}_{i,0} + \bar{E}_{r,0} \quad (27a)$$

$$\bar{E}_1 = \bar{E}_{t,1} \quad (27b)$$

Substituting (22),(23),(24), and (27) into (25) and (26) gives

$$R_{01HH} = \frac{k_{oz} - k_{1z}^o}{k_{oz} + k_{1z}^o} \quad (28)$$

$$T_{Ho} = \frac{2k_{oz}}{(k_{oz} + k_{1z}^o)} \quad (29)$$

2.2 Case II – TM Wave Incidence

Now, assume that the incident wave from Region 0 is TM wave. Then, the field vectors in each region can be expressed as

$$\bar{E}_{i,0} = \hat{v}_0(-k_{0z}) e^{i(\bar{k}_o \cdot \bar{r})} e^{-i\omega t} \quad (30)$$

$$\bar{E}_{r,0} = [\hat{v}_0(k_{0z}) R_{01VV} e^{i(\bar{k}_o \cdot \bar{r})}] e^{-i\omega t} \quad (31)$$

$$\bar{E}_{t,1} = [\hat{e}(k_{1z}^{ed}) T_{Ve} e^{i(\bar{k}_1^e \cdot \bar{r})}] e^{-i\omega t} \quad (32)$$

R_{01VV} is the reflection coefficient when incident and reflected waves are both vertically polarized. T_{Ve} is the transmission coefficient when incident wave is vertically polarized and transmitted wave is an extraordinary wave. Repeating the same procedure outlined in Section II.A, we can find the reflection and transmission coefficients for TM wave incidence as

$$R_{01VV} = \frac{\varepsilon_1 k_{oz} - \varepsilon_0 k_{1z}^e}{\varepsilon_1 k_{oz} + \varepsilon_0 k_{1z}^e} \quad (33)$$

$$T_{Ve} = \frac{2\varepsilon_1 k_{oz}}{(\varepsilon_1 k_{oz} + \varepsilon_0 k_{1z}^e)} \quad (34)$$

2.3 Numerical Example

The Fresnel coefficients derived for the single layer uniaxially anisotropic medium with a vertically oriented optic axis are numerically calculated for Taconic TLY-5A material and illustrated in Fig. 4. TLY-5A is a negatively uniaxial anisotropic medium with a dissipation factor of 0.0014 in (x, y)-direction and 0.00066 in z-direction at 10GHz. The observation angle is taken to be $\phi = 45^\circ$. The Brewster angle is numerically calculated to be 56.96° . The theoretical value of the Brewster angle is found from

$$\theta_B = a \sin \left(\sqrt{\frac{\varepsilon_{1z}(\varepsilon_1 - \varepsilon_0)}{\varepsilon_1 \varepsilon_{1z} - \varepsilon_0^2}} \right) \quad (35)$$

This value matches with the angle shown in Fig. 4. As it is illustrated in the figure, there is no cross polarized term. This is due to the vertically oriented optic axis.

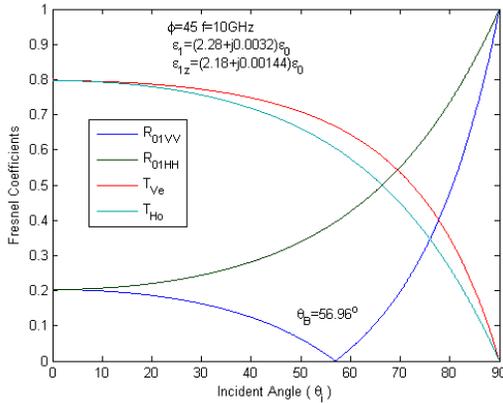


Fig. 4 Fresnel coefficients for single layer uniaxially anisotropic medium with a vertically oriented optic axis.

3. Wave propagation in a single layered anisotropic media – uniaxial case with tilted optic axis

The cross polarization effect due to isotropic-anisotropic interface can be obtained if the optic axis of the uniaxially anisotropic medium is tilted around one of its optic axis. Cross polarization effect is very important in radiation and wave scattering problem and can be used as an design knob. The geometry of the uniaxial anisotropic medium when its optic axis is tilted around z-axis by angle ψ is shown in Fig. 1. The permittivity tensor of the anisotropic medium with tilted optic axis is given by equation (2).

3.1 Case I – TE Wave Incidence

The geometry of the problem is given in Fig. 5. Fig. 5 shows the wave vectors for the single layer structure when the incident wave is horizontally polarized. Upon reflection from the isotropic-anisotropic interface the reflected wave in Region 0 will have the cross polarized component R_{01HV} .

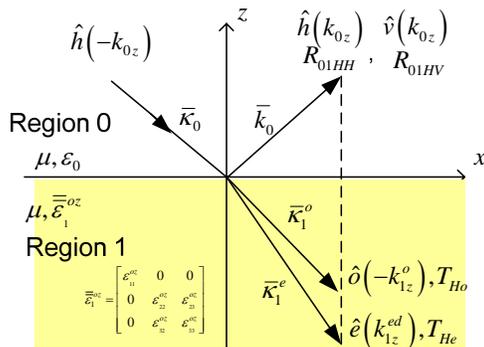


Fig. 5 Reflection and transmission of TE waves in single-layered anisotropic slab with a tilted optic axis in one

direction.

The first subscript of the cross polarized component represents the polarization of the incident wave whereas the second subscript represents the polarization of the reflected wave. The transmitted wave also has the cross polarized component due to tilted optic axis. T_{He} is the transmitted cross polarized component and represents the transmitted extraordinary wave when the incident wave is TE wave. Hence, the field vectors in each region can be expressed as

$$\vec{E}_{i,0} = \hat{h}_0(-k_{0z})e^{i(\vec{k}_o \cdot \vec{r})}e^{-i\omega t} \quad (36)$$

$$\vec{E}_{r,0} = \left[\hat{h}_0(k_{0z})R_{01HH}e^{i(\vec{k}_o \cdot \vec{r})} + \hat{v}_0(k_{0z})R_{01HV}e^{i(\vec{k}_o \cdot \vec{r})} \right] e^{-i\omega t} \quad (37)$$

$$\vec{E}_{t,1} = \left[\hat{o}(-k_{1z}^o)T_{Ho}e^{i(\vec{k}_1^o \cdot \vec{r})} + \hat{e}(k_{1z}^{ed})T_{He}e^{i(\vec{k}_1^o \cdot \vec{r})} \right] e^{-i\omega t} \quad (38)$$

Application of the boundary conditions given by (25)-(26) gives the reflection and transmission coefficients as

$$R_{01HH} = -1 + \frac{T_{Ho}}{\alpha_d} \frac{1}{k_\rho} (k_\rho^2 \cos \psi + k_y k_{1z}^o \sin \psi) - \frac{k_x^2}{k_\rho} k_x \sin \psi \frac{T_{He}}{\beta_d} \quad (39)$$

$$R_{01HV} = -\frac{T_{Ho}}{\alpha_d} \frac{k_o k_x}{k_\rho k_{oc}} \sin \psi + \frac{T_{He}}{\beta_d} \frac{k_o}{k_\rho k_{oc}} (k_x^2 k_{1z}^{ed} \cos \psi - k_y (k_{1z}^o)^2 \sin \psi) \quad (40)$$

$$T_{He} = -\frac{\beta_d}{\lambda_e} \frac{2k_{oc}}{k_\rho} k_x (k_\rho^2 + k_{oc} k_{1z}^o) \sin \psi \quad (41)$$

$$T_{Ho} = -\frac{\alpha_d}{\lambda_e} \frac{2}{(k_{oc} + k_{1z}^o)} \frac{k_{oc}}{k_\rho} \left[k_\rho^2 (k_{1z}^o k_{oc} - k_o^2 k_{1z}^{ed}) \cos \psi + k_y (k_o^2 (k_{1z}^o)^2 - k_{oc}^2 k_{1z}^{ed}) \sin \psi \right] \quad (42)$$

$$\lambda_e = \cos^2 \psi k_\rho^2 (k_{1z}^o k_{oc} - k_o^2 k_{1z}^{ed}) + \sin^2 \psi \left[k_{1z}^o (k_{oc} - k_{1z}^{ed}) (k_x^2 + k_{oc} k_{1z}^o) + k_y^2 k_{1z}^o (k_{1z}^o - k_o^2) \right] + \cos \psi \sin \psi k_y (k_{1z}^o + k_{oc}) (k_{1z}^o - k_{1z}^{ed}) (k_\rho^2 + k_{1z}^o k_{oc}) \quad (43)$$

$$\beta_d = \left[\frac{\epsilon_1}{\epsilon_1 - \epsilon_{1z}} (k_x^2 + k_y^2 + (k_{1z}^{ed})^2 - \omega^2 \mu \epsilon_1) (k_x^2 + k_y^2 + (k_{1z}^o)^2 - \omega^2 \mu (\epsilon_1 + \epsilon_{1z})) \right]^{1/2} \quad (44)$$

$$\alpha_d = \left[k_x^2 + (k_y \cos \psi + k_{1z}^o \sin \psi)^2 \right]^{1/2} \quad (45)$$

3.2 Case II – TM Wave Incidence

Fig. 6 illustrates the single layer structure when the incident wave is vertically polarized. The incident electric field vector can be written as

$$\vec{E}_{i,0} = \hat{v}_0(-k_{0z})e^{i(\vec{k}_o \cdot \vec{r})}e^{-i\omega t} \quad (46)$$

The reflected wave with the cross polarized term is R_{01VH} is

$$\vec{E}_{r,0} = \left[\hat{v}_0(k_{0z})R_{01VV}e^{i(\vec{k}_o \cdot \vec{r})} + \hat{h}_0(k_{0z})R_{01VH}e^{i(\vec{k}_o \cdot \vec{r})} \right] e^{-i\omega t} \quad (47)$$

R_{01VH} indicates that the incident wave is vertically polarized or TM wave, and the reflected wave is

horizontally polarized or TE wave. The transmitted wave in Region 1 can be expressed as

$$\vec{E}_{t,1} = \left[\hat{o}(-k_{1z}^o) T_{Vo} e^{i(\vec{k}_1^o \cdot \vec{r})} + \hat{e}(k_{1z}^{ed}) T_{Ve} e^{i(\vec{k}_1^o \cdot \vec{r})} \right] e^{-i\omega t} \quad (48)$$

T_{Vo} and T_{Ve} are the transmission coefficients when the incident wave is TM wave and transmitted wave ordinary wave or extraordinary wave, respectively. The reflection and transmission coefficients are found using boundary conditions as described before and given as

$$R_{01VV} = 1 - \frac{T_{Vo} \frac{k_o k_x}{\alpha_d} k_{1z}^o \sin \psi + \frac{T_{Ve} \frac{k_o}{\beta_d} k_{1z}^{ed} \cos \psi - k_y (k_{1z}^o)^2 \sin \psi}{\alpha_d k_p k_{oz}} \quad (49)$$

$$R_{01VH} = \frac{T_{Vo}}{\alpha_d} \frac{1}{k_p} (k_\rho^2 \cos \psi + k_y k_{1z}^o \sin \psi) - \frac{T_{Ve} k_1^2}{\beta_d k_p} k_x \sin \psi \quad (50)$$

where

$$T_{Ve} = \frac{\beta_d}{\lambda_e} \frac{2k_o}{k_p} (k_\rho^2 k_{oz} \cos \psi + k_y k_{oz} k_{1z}^o \sin \psi) \quad (51)$$

$$T_{Vo} = \frac{\alpha_d}{\lambda_e} \frac{2}{(k_{oz} + k_{1z}^o)} \frac{k_{oz}}{k_p} \left[k_1^2 k_x k_o (k_{oz} - k_{1z}^{ed}) \sin \psi \right] \quad (52)$$

The coefficients α_d , β_d , and λ_d are given by equations (43)-(45).

3.3 Numerical Example

The Fresnel coefficients derived for the single layer vertically uniaxial anisotropic medium with a tilted optic axis are numerically calculated for Taconic TLY-5A material and illustrated in Fig. 6. Material properties of Taconic TLY-5A have been given in Section IIC. As illustrated, the cross polarization components exist because of the isotropic-anisotropic interface with the existence of the tilted optic axis. The magnitude of the cross polarized components varies with the tilt angle and observation angle. The cross polarized terms are very important due to their contribution on the electric and magnetic field intensities in radiation and scattering problems.

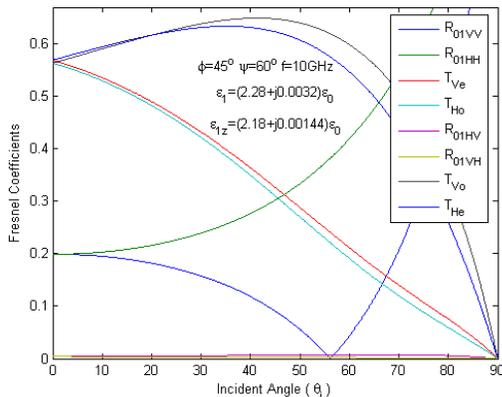


Fig. 6 Fresnel coefficients for single layer uniaxially anisotropic medium with a tilted optic axis

4. Wave propagation in two-layered anisotropic media – vertically uniaxial case

In this section, wave propagation for two-layered anisotropic media when the incident wave is TE or TM polarized is analyzed.

4.1 Case 1 – TE Wave Incidence

The geometry of the problem is shown in Fig. 7. The uniaxially anisotropic medium is assumed to have its optic axis oriented in the z-direction.

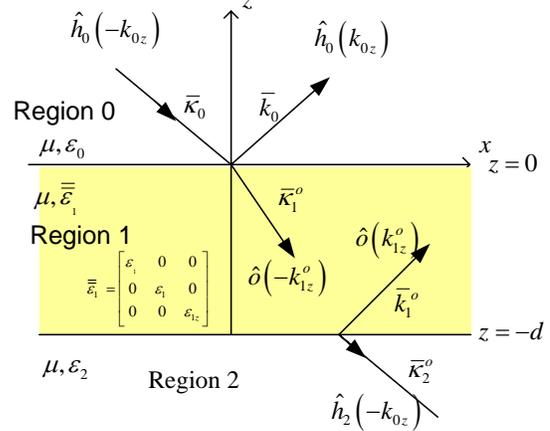


Fig. 7. Reflection and transmission of TE waves in two-layered anisotropic structure.

The electric field vector in Region 0 is expressed as

$$\vec{E}_0 = \left[\hat{h}_0(-k_{0z}) e^{i(\vec{k}_0 \cdot \vec{r})} + \hat{h}_0(k_{0z}) R_H e^{i(\vec{k}_0 \cdot \vec{r})} \right] e^{-i\omega t} \quad (53)$$

The reflected wave in Region 0 is also a horizontally polarized wave. R_H is the reflection coefficient which has the reflected wave components from isotropic-anisotropic interface and anisotropic-isotropic interface when incident wave is horizontally polarized. The transmitted wave in Region 1 is expressed as

$$\vec{E}_1 = \left[\hat{o}(-k_{1z}^o) T_{Ho1} e^{i(\vec{k}_1^o \cdot \vec{r})} + \hat{o}(k_{1z}^o) T_{Ho2} e^{i(\vec{k}_1^o \cdot \vec{r})} \right] e^{-i\omega t} \quad (54)$$

T_{Ho1} is the transmission coefficient for the downward propagating ordinary wave and T_{Ho2} is transmission coefficient for upward propagating ordinary wave. In Region 2, there is only downward propagating wave which is horizontally polarized. The field vector in Region 2 can be expressed as

$$\vec{E}_2 = \left[\hat{h}_2(-k_{2z}^o) T_{H2} e^{i(\vec{k}_2^o \cdot \vec{r})} \right] e^{-i\omega t} \quad (55)$$

T_{H2} is the transmission coefficient in Region 2 when the incident wave in Region 0 is a horizontally polarized wave . The Fresnel coefficients R_H, T_{H01}, T_{H02} and T_{H2} are found by using the following boundary conditions at $z = 0, -d$

$$\hat{z} \times \bar{E}_0 = \hat{z} \times \bar{E}_1 \tag{56}$$

$$\hat{z} \times \nabla \times \bar{E}_0 = \hat{z} \times \nabla \times \bar{E}_1 \tag{57}$$

$$\hat{z} \times \bar{E}_1 = \hat{z} \times \bar{E}_2 \tag{58}$$

$$\hat{z} \times \nabla \times \bar{E}_1 = \hat{z} \times \nabla \times \bar{E}_2 \tag{59}$$

The Fresnel coefficients after application of the boundary conditions (56)-(59) to field vectors are found as

$$R_H = \frac{(R_{H01} + R_{H12}e^{i2k_{1z}d})}{\alpha_{H2}} \tag{60}$$

$$T_{H01} = \frac{(1 + R_{H01})}{\alpha_{H2}} \tag{61}$$

$$T_{H02} = \frac{(1 + R_{H01})R_{H12}e^{i2k_{1z}d}}{\alpha_{H2}} \tag{62}$$

$$T_{H2} = \frac{(1 + R_{H01})(1 + R_{H12})e^{i(k_{1z}-k_{2z})d}}{\alpha_{H2}} \tag{63}$$

where

$$R_{H01} = \frac{k_{0z} - k_{1z}}{k_{0z} + k_{1z}} \tag{64}$$

$$R_{H12} = \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}} \tag{65}$$

$$\alpha_{H2} = 1 + R_{H01}R_{H12}e^{i2k_{1z}d} \tag{66}$$

The unit vectors, in each region are defined as

$$\hat{h}_i(\pm k_{iz}) = \frac{\hat{z} \times \bar{k}_i}{k_\rho} = \frac{(-\hat{x}k_y + \hat{y}k_x)}{k_\rho}, i=0,2 \tag{67}$$

$$\hat{o}(k_{1z}^o) = \frac{\hat{z} \times \bar{k}_1^o}{|\hat{z} \times \bar{k}_1^o|} \tag{68}$$

$$\hat{o}(-k_{1z}^o) = \frac{\hat{z} \times \bar{k}_1^o}{|\hat{z} \times \bar{k}_1^o|} \tag{69}$$

where the wave vectors and dispersion relations are given as

$$\bar{k}_0 = \bar{k}_\rho + \hat{z}k_0 \tag{70}$$

$$\bar{\kappa}_0 = \bar{k}_\rho - \hat{z}k_0 \tag{71}$$

$$\bar{k}_\rho = \hat{x}k_x + \hat{y}k_y \tag{72}$$

$$\bar{k}_1^o = \bar{k}_\rho + \hat{z}k_{1z}^o \tag{73}$$

$$\bar{\kappa}_1^o = \bar{k}_\rho - k_{1z}^o \hat{z} \tag{74}$$

$$\bar{\kappa}_2^o = \bar{k}_\rho - k_{2z}^o \hat{z} \tag{75}$$

$$k_i = \omega \sqrt{\mu \epsilon_i} \quad i=0,2 \tag{76}$$

4.2 Case II – TM Wave Incidence

Now, assume that the incident wave in Region 0 is TM wave for the layered structure shown in Fig. 8. The field vectors in Region 0, 1, and 2 can be expressed as

$$\bar{E}_0 = [\hat{v}_0(-k_{0z})e^{i(\bar{\kappa}_0 \cdot \bar{r})} + \hat{v}_0(k_{0z})R_V e^{i(\bar{k}_0 \cdot \bar{r})}]e^{-i\omega t} \tag{77}$$

$$\bar{E}_1 = [\hat{e}(k_{1z}^{ed})T_{Ve1}e^{i(\bar{\kappa}_1^o \cdot \bar{r})} + \hat{e}(k_{1z}^{eu})T_{Ve2}e^{i(\bar{k}_1^e \cdot \bar{r})}]e^{-i\omega t} \tag{78}$$

$$\bar{E}_2 = [\hat{v}_2(-k_{2z}^o)T_{V2}e^{i(\bar{\kappa}_2^o \cdot \bar{r})}]e^{-i\omega t} \tag{79}$$

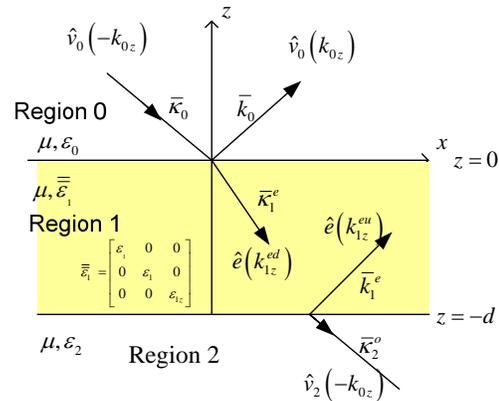


Fig. 8 Reflection and transmission of TM waves in two-layered anisotropic structure.

The reflection and transmission coefficients R_V, T_{Ve1}, T_{Ve2} and T_{V2} are found using the boundary conditions given by (51)-(54) for the electric field vectors given by (69)-(71). The Fresnel coefficients are obtained as

$$R_V = \frac{(R_{V01} + R_{V12}e^{i2k_{1z}d})}{\alpha_{V2}} \tag{80}$$

$$T_{Ve1} = \frac{(1+R_{V01})k_0k_{1u}\varepsilon_1}{\alpha_{V2}k_1^2\varepsilon_{1z}} \quad (81)$$

$$T_{Ve2} = \frac{(1+R_{V01})k_0k_{1u}\varepsilon_1}{\alpha_{V2}k_1^2\varepsilon_{1z}}(1+R_{V12})e^{i2k_{1z}^e d} \quad (82)$$

$$T_{V2} = \frac{k_0}{k_2} \frac{(1+R_{V01})}{\alpha_{V2}} (1+R_{V12}) e^{i(k_{1z}^e - k_{2z})d} \quad (83)$$

where

$$R_{V01} = \frac{\varepsilon_1 k_{0z} - \varepsilon_0 k_{1z}^e}{\varepsilon_1 k_{0z} + \varepsilon_0 k_{1z}^e} \quad (84)$$

$$R_{V12} = \frac{\varepsilon_2 k_{1z}^e - \varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z}^e + \varepsilon_1 k_{2z}} \quad (85)$$

$$\alpha_{V2} = 1 + R_{V01} R_{V12} e^{i2k_{1z}^e d} \quad (86)$$

4.3 Application Example

Application of the electromagnetic wave propagation problem through composite structure can be best illustrated with approximating the second region in Fig. 8 to a perfect conductor. When the permittivity of the second layer is taken to be infinitely large, $\varepsilon_2 \rightarrow \infty$, the second region can be approximated as a perfect conductor. This structure can now be treated as a microstrip. The final configuration of the problem is shown in Fig. 9.

For the microstrip layered structure involving uniaxially anisotropic medium shown in Fig. 9, the Fresnel coefficients for TE and TM waves are obtained with the substitution of $\varepsilon_2 \rightarrow \infty$ into equations (55)-(61) and (72)-(77). The modified Fresnel coefficients for TE waves are

$$R_H \rightarrow \frac{(R_{H01} - e^{i2k_{1z}^e d})}{\alpha_{H2}} \quad (87)$$

$$T_{H01} = \frac{(1 + R_{H01})}{\alpha_{H2}} \quad (88)$$

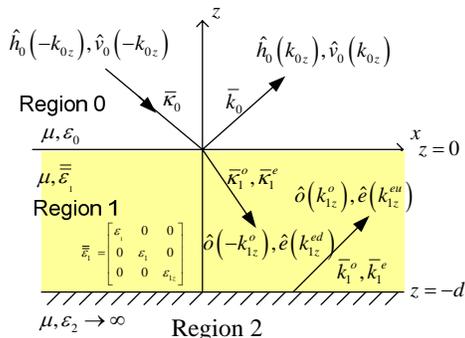


Fig. 9 Microstrip configuration with a vertically uniaxial anisotropic medium.

$$T_{H02} \rightarrow -\frac{(1+R_{H01})e^{i2k_{1z}^e d}}{\alpha_{H2}} \quad (89)$$

$$T_{H2} \rightarrow 0 \quad (90)$$

where

$$R_{H01} = \frac{k_{0z} - k_{1z}}{k_{0z} + k_{1z}} \quad (91)$$

$$R_{H12} \rightarrow -1 \quad (92)$$

$$\alpha_{H2} \rightarrow 1 - R_{H01} e^{i2k_{1z}^e d} \quad (93)$$

Similarly, the modified Fresnel coefficients for TM waves are found as

$$R_V \rightarrow \frac{(R_{V01} + e^{i2k_{1z}^e d})}{\alpha_{V2}} \quad (94)$$

$$T_{Ve1} = \frac{(1+R_{V01})k_0k_{1u}\varepsilon_1}{\alpha_{V2}k_1^2\varepsilon_{1z}} \quad (95)$$

$$T_{Ve2} \rightarrow \frac{(1+R_{V01})k_0k_{1u}\varepsilon_1}{\alpha_{V2}k_1^2\varepsilon_{1z}} 2e^{i2k_{1z}^e d} \quad (96)$$

$$T_{V2} \rightarrow 0 \quad (97)$$

where

$$R_{V01} = \frac{\varepsilon_1 k_{0z} - \varepsilon_0 k_{1z}^e}{\varepsilon_1 k_{0z} + \varepsilon_0 k_{1z}^e} \quad (98)$$

$$R_{V12} \rightarrow 1 \quad (99)$$

$$\alpha_{V2} \rightarrow 1 + R_{V01} e^{i2k_{1z}^e d} \quad (100)$$

5. Wave propagation in two-layered anisotropic media – uniaxial case with tilted optic axis

The geometry of uniaxially anisotropic medium with its optic axis tilted around z-direction by an angle ψ is illustrated in Fig. 1. The permittivity tensor and its elements are given by equations (2)-(3). When the optic axis of the anisotropic medium is tilted, it allows existence of the cross polarization components across the isotropic-anisotropic interface. The numerical calculation of the Fresnel coefficients showing the contribution of the cross polarized waves is plotted in Fig. 6 for isotropic-anisotropic interface for Taconic TLY-5A material.

Case 1 – TE Wave Incidence

The geometry of the problem is given in Fig. 10. When the incident wave is TE wave or horizontally polarized wave, $\hat{h}_o(-k_{oz})$, the electric field vector in Region 0 will have both horizontally polarized and vertically polarized components due to isotropic-anisotropic interface. The cross polarized term in Region 0 is accompanied with reflection coefficient R_{01HV} and unit vector \hat{v} . The field vector in Region 0 can be expressed as

$$\bar{E}_0 = \left[\begin{matrix} \hat{h}_o(-k_{oz})e^{i(\bar{k}_o \cdot \bar{r})} + \hat{h}_o(k_{oz})R_{01HH}e^{i(\bar{k}_o \cdot \bar{r})} \\ + \hat{v}_o(k_{oz})R_{01HV}e^{i(\bar{k}_o \cdot \bar{r})} \end{matrix} \right] e^{-i\omega t} \quad (101)$$

Upon transmission from Region 0 to Region 1, the transmitted horizontally polarized wave is decomposed into downward propagating ordinary and extraordinary wave components, $\hat{o}(-k_{1z}^o)$ and $\hat{e}(k_{1z}^{ed})$, with transmission coefficients T_{Ho} and T_{He} . Extraordinary waves in anisotropic region exist for TE wave incidence due to its tilted optic axis. The ordinary and extraordinary downward propagating wave components will be reflected back from the anisotropic-isotropic interface as illustrated in Fig. 10. The wave components reflected from anisotropic-isotropic interface will be upward propagating and are designated by unit vectors $\hat{o}(k_{1z}^o)$ and $\hat{e}(k_{1z}^{eu})$ with the transmission-

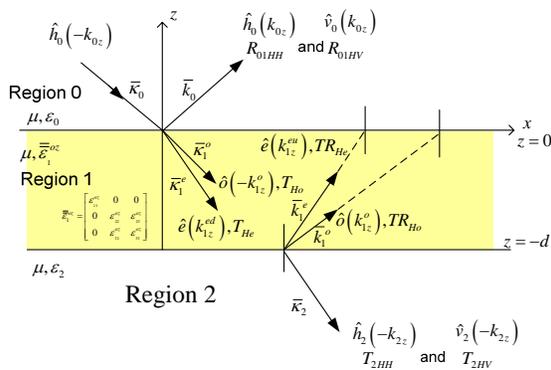


Fig. 10 Reflection and transmission of TE waves in two-layered anisotropic structure with a tilted optic axis.

reflection coefficients TR_{Ho} and TR_{He} . The electric field vector in Region 1 can be written as

$$\bar{E}_1 = \left[\begin{matrix} T_{Ho}\hat{o}(-k_{1z}^o)e^{i(\bar{k}_1^o \cdot \bar{r})} + TR_{Ho}\hat{o}(k_{1z}^o)e^{i(\bar{k}_1^o \cdot \bar{r})} \\ + T_{He}\hat{e}(k_{1z}^{ed})e^{i(\bar{k}_1^e \cdot \bar{r})} + TR_{He}\hat{e}(k_{1z}^{eu})e^{i(\bar{k}_1^e \cdot \bar{r})} \end{matrix} \right] e^{-i\omega t} \quad (102)$$

The transmitted wave in Region 2 has both horizontally and vertically polarized waves due to anisotropic-isotropic interface. The transmitted wave is represented with transmission coefficients T_{2HH} and T_{2HV} . The field vector in Region 2 is

$$\bar{E}_2 = \left[\hat{h}_2(-k_{2z})T_{2HH}e^{i(\bar{k}_2 \cdot \bar{r})} + \hat{v}_2(-k_{2z})T_{2HV}e^{i(\bar{k}_2 \cdot \bar{r})} \right] e^{-i\omega t} \quad (103)$$

Wave component accompanied with the transmission coefficient T_{2HV} and unit vector \hat{v} is referred as cross polarized term. The Fresnel coefficients are found by application boundary conditions using equations (51)-(54) as described in the previous sections. TM wave incidence can be carried out similarly by considering the incident wave as a vertically polarized wave for the layered structure.

6. Conclusion

In this paper, wave propagation in two-layered uniaxially anisotropic media is given in detail. The significance of the contribution of the cross polarized waves due to tilted optic axis is illustrated by a numerical example. The reflection and transmission coefficients for the two-layered anisotropic structure when its optic is vertically oriented and tilted are derived using the boundary conditions across the interface between isotropic and anisotropic media. Application example is given using the results in this paper with a microstrip structure. Our results can be used in electromagnetic radiation and scattering problems and analysis of the material properties for the composite structures involving anisotropic medium.

References

- [1] J. A. Kong, Theory of Electromagnetic Waves, Wiley-Interscience, New York (1975).
- [2] A. Eroglu, J. K. Lee, IEEE Trans. Antennas Propag., 53 (2005).
- [3] J. K. Lee, J. A. Kong, Electromagnetics, 3, 111 (1983).
- [4] Y. H. Xu, K. Li, W. Ren, IEEE Antennas Wireless and Propag Letters., 6, 427 (2007).
- [5] Q. X. Sheng, Z. Peng, IET Microwaves, Antennas & Propagation, 4, 492 (2010).
- [6] D. Gajewski, Geophysical Journal International 113, 299 (2007).
- [7] F. Cakoni, D. Colton, P. Monk, J. Sun, Inverse Problems, 26, 1 (2010).
- [8] J. Yuan, C. Gu, Z. Li, International Journal of RF and Microwave Computer-Aided Engineering 20, 416, (2010).
- [9] Y. Geng, X. Wu, B. Guan PIERs, 1, 403 (2005).
- [10] K. Wong, H. Chen IEE Pt-H, 139, 314 (1992).
- [11] V.V. Varadan, A. Lakhtakia, V. K. Varadhan, IEEE Trans. Antennas and Propagation, 37, 800 (1989).

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