

# Research on light propagation property of one-dimension photonic crystal

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The property of light propagation in one-dimension photonic crystal has been discussed by transfer matrix method (TMM) in this paper. The influence of changing permittivity ratio for photonic crystal band gap has also been researched. And the holophote in 1D photonic crystal for the wavelength  $\lambda=830$  nm has been designed. Two specific photonic crystal filters have been designed and optimized, which allow light of the wavelength  $\lambda=800$  nm or  $\lambda=830$  nm to pass respectively.

(Received June 19, 2012; accepted October 30, 2012)

*Keywords:* Photonic crystal, Photonic band gap, Transfer matrix method, Filter

## 1. Introduction

Photonic crystals are artificial structures in which the dielectric constant change periodically. The concept of photonic crystal was first proposed separately by Eli Yablonovitch [1] and Sajeev John [2] in 1987. Photonic band gap is the distinct character of photonic crystal. Photons whose frequency is between the photonic band gap are forbidden to travel through the crystal in any direction of propagation. The particularity of this band gap gives photonic crystal a broad application prospects. Photonic crystal can be used to make optical holophote, optical filter and many other optical devices [3].

This paper, which adopts transfer matrix method, aims to calculate the band structure of one-dimensional photonic crystal and study the effects differently structured parameters have on the band structure. Afterwards, a holophote has been designed which works at the wave length 830nm and optimal design of the holophote has also been processed. At last, this paper introduces the design of two photonic crystal filters that respectively have narrow-band high transmittance at the wavelength 830nm and 800nm.

## 2. The transfer matrix method of one-dimension photonic crystals

In general, when light of wavelength  $\lambda$  incident upon the photonic crystal with the incident angle  $\theta_0$ , the transfer matrix of layer i is [4]:

$$T_i = \begin{bmatrix} \cos \sigma_i & \frac{j}{\eta_i} \sin \sigma_i \\ j\eta_i \sin \sigma_i & \cos \sigma_i \end{bmatrix} \quad (1)$$

where

$$\begin{cases} \sigma_i = \frac{2\pi}{\lambda} n_i h_i \cos \theta_i \\ \eta_i = \sqrt{\frac{\epsilon_0}{\mu_0}} \sqrt{\epsilon_i} \cos \theta_i \quad (\text{for TE wave}) \\ \eta_i = \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{\sqrt{\epsilon_i}}{\cos \theta_i} \quad (\text{for TM wave}) \end{cases} \quad (2)$$

For the 1D photonic crystals with N layers, the transfer matrix is:

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix} = T_1 T_2 \cdots T_N \begin{bmatrix} E_{N+1} \\ H_{N+1} \end{bmatrix} = M \begin{bmatrix} E_{N+1} \\ H_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_{N+1} \\ H_{N+1} \end{bmatrix} \quad (3)$$

where  $E_1, H_1$  are respectively electric vector and magnetic vector at the left of the first interface,  $E_{N+1}, H_{N+1}$  are respectively electric vector and magnetic vector at the right of the Nth interface. For TE waves, the reflection coefficient can be gotten by the equation as follow:

$$r_{TE} = \frac{A\eta_0 + B\eta_0\eta_1 - C - D\eta_{N+1}}{A\eta_0 + B\eta_0\eta_1 + C + D\eta_{N+1}} \quad (4)$$

The reflectance is:

$$R = r \cdot r^* \quad (5)$$

The transmission coefficient is:

$$t_{TE} = \frac{2\eta_0}{A\eta_0 + B\eta_0\eta_1 + C + D\eta_{N+1}} \quad (6)$$

The transmittance is:

$$T = t \cdot t^* \quad (7)$$

The treatment of the TM waves is similar. Through a simple substitution, we can get the reflectance and transmittance.

For TM wave, the reflection coefficient is:

$$r_{TM} = \frac{A\eta_0 + B\eta_0\eta_1 - C - D\eta_{N+1}}{A\eta_0 + B\eta_0\eta_1 + C + D\eta_{N+1}} \quad (8)$$

The reflectance is:

$$R = r \cdot r^* \quad (9)$$

The transmission coefficient is:

$$t_{TM} = \frac{2\eta_0}{A\eta_0 + B\eta_0\eta_1 + C + D\eta_{N+1}} \frac{\cos\theta_0}{\cos\theta_{N+1}} \quad (10)$$

The transmittance is:

$$T = t \cdot t^* \quad (11)$$

### 3. Analyzing the photonic band gap of one-dimension photonic crystals

Now, only consider the situation of the incident angle  $\theta=0$ . Under this situation, there is no difference between TE waves and TM waves. The dispersion relation of one-dimension photonic crystal is [5]:

$$\cos kd = \cos\sigma_1 \cos\sigma_2 - \frac{1}{2} \left( \frac{\eta_h}{\eta_l} + \frac{\eta_l}{\eta_h} \right) \sin\sigma_1 \sin\sigma_2 \quad (12)$$

where  $k$  is the Bloch wave vector.

Suppose that the one-dimension photonic crystal showed in Fig.1 is in the air and all media are non-magnetic materials, then  $\mu_0 = \mu_{N+1} = 1$ ,  $\eta_0 = \eta_{N+1} = 1$ . Assume the higher refractive index of dielectric A is 3.16, the lower refractive index of dielectric B is 1.414 and  $n_1h_1 = n_2h_2 = 207.5nm$ . The number of period is 20 and the number of dielectric slab is 41. The fundamental

frequency  $\omega_0 = c\pi / (n_1h_1 + n_2h_2)$ . Based on these known parameters and Eqs.(3), (6), (7) and (12), the dispersion relation of one-dimension photonic crystal is shown in Fig.1, and the relation between transmittance  $T$  and normalized frequency  $\omega/\omega_0$  is shown in Fig. 2.

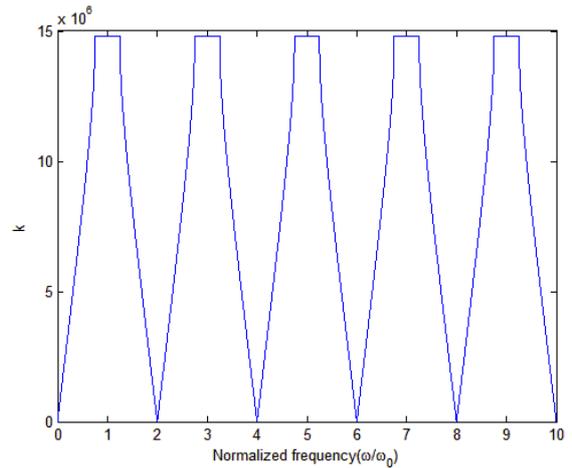


Fig. 1. The dispersion relation ( $n_1h_1 = n_2h_2$ )

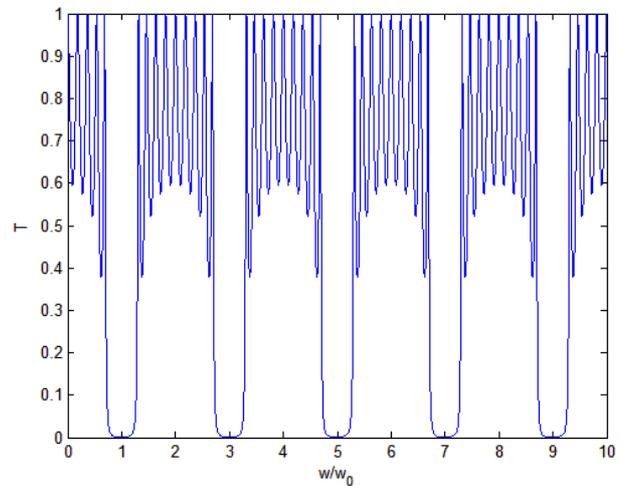


Fig. 2. Transmittance vs. normalized frequency ( $n_1h_1 = n_2h_2$ )

From Fig.1 and Fig.2, it is easy to find that the forbidden gap disappear at even multiples of the fundamental frequency when  $n_1h_1 = n_2h_2$ .

If let  $h_2$  change and meets  $n_2h_2 = 507.5nm$ , the dispersion relation of one-dimension photonic crystal is

shown in Fig.3 and the relation between transmittance  $T$  and normalized frequency  $\omega/\omega_0$  is shown in Fig.4.

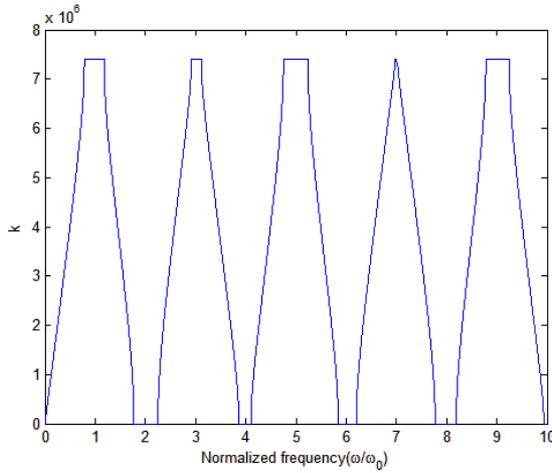


Fig. 3. The dispersion relation ( $n_1 h_1 \neq n_2 h_2$ )

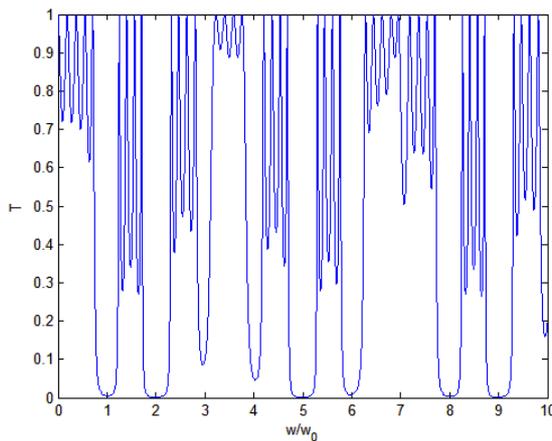


Fig. 4. Transmittance vs. normalized frequency ( $n_1 h_1 \neq n_2 h_2$ )

From Fig. 3 and Fig. 4, the forbidden gap appear at even multiples of the fundamental frequency when  $n_a h_1 \neq n_b h_2$ . This phenomenon can be explained by Eqs. (12). When the frequency  $\omega = 2N\omega_0$  ( $N$  is integer), if  $n_1 h_1 = n_2 h_2$ , from Eqs. (12), we can get  $\cos kd = 1$ , so the solution of  $k$  exists and the forbidden gap disappear. If  $n_1 h_1 \neq n_2 h_2$ , we can get  $\cos kd > 1$  or  $\cos kd < -1$ , so the solution of  $k$  does not exist and the forbidden gap appear.

Now, the influence of changing permittivity ratio for

photonic crystal band gap is analyzed. The structure parameters are given as following:

$$h_1 : h_2 = 1.55 : 2.8, \quad a = 435 \text{ nm}, \quad \varepsilon_1 : \varepsilon_2 = 2 : 1 \quad \text{or}$$

$$\varepsilon_1 : \varepsilon_2 = 3 : 1 \quad \text{or} \quad \varepsilon_1 : \varepsilon_2 = 4 : 1.$$

Fig. 5 shows the host band-gap width changing with permittivity ratio.

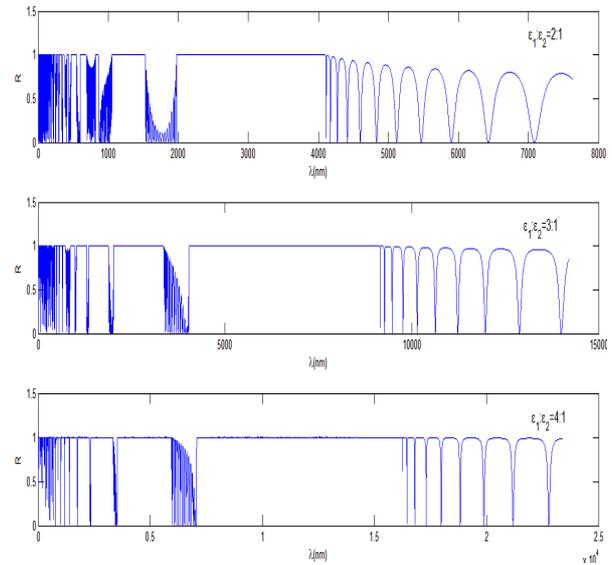


Fig. 5. The variance of photonic band gap with different permittivity ratio of two media.

From Fig. 5, it is concluded that band gap becomes wide when the permittivity ratio  $\varepsilon_1 / \varepsilon_2$  or the thickness ratio  $h_1 / h_2$  and the host period  $a$  respectively becomes large. And the number of band gaps increase when the permittivity ratio becomes large. Similarly, the influence of changing other parameters such as the thickness ratio and the host period for photonic crystal band gap can be analyzed. However, from the result of simulation, the influences of changing the thickness ratio and host period to the band gap are relatively small. Hence, when we need larger bandwidth, it is prior to increase permittivity ratio of two media.

#### 4. Design of holophote

Here we design 1D photonic crystal holophote by TMM. Construct one-dimension photonic crystal with

three alternate layers, as Fig. 6 shows.

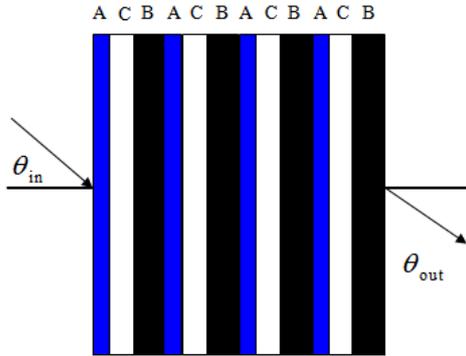


Fig. 6. The one-dimension photonic crystal with three alternate layers

The media parameters can be designed as:  $n_a = 3.16$ ,

$n_b = 1.414, n_c = 2.3, N=20$ . Where  $n_a, n_b, n_c$  are the

corresponding refractive indexes of  $A, B, C$ .  $N$  is the number of host period. Suppose all media are non-magnetic materials and the one-dimension crystal is in the air, hence Permeability  $\mu = 1$ .

When the values of  $n_a, n_b$  and  $n_c$  are settled, adjust the thickness of the media slab to satisfy the following equation:

$$n_a d_a = n_b d_b = n_c d_c = (1/4)\lambda_0 = \pi c / 2\omega_0$$

Choose  $\lambda_0 = 830nm$ , Fig. 7 shows the result of simulation by TMM.

In the Fig. 7, (a) corresponds to the situation of TE waves; (b) corresponds to the situation of TM waves. For TE waves, it is easy to see if incident angle become large, the band gap become wide. However, for TM waves, when incident angle become large, the band gap become narrow. Beside this, the band gap of one-dimension photonic crystal in TE modes is wider compared with it in TM modes. At last, there is another point which is needed to notice in Fig. 7. No matter in TE modes or in TM modes and no matter what the incident angle is, there is a

wavelength range that electromagnetic wave can not pass. This range constitutes the complete band gap. This range is about(1060nm,1200nm).

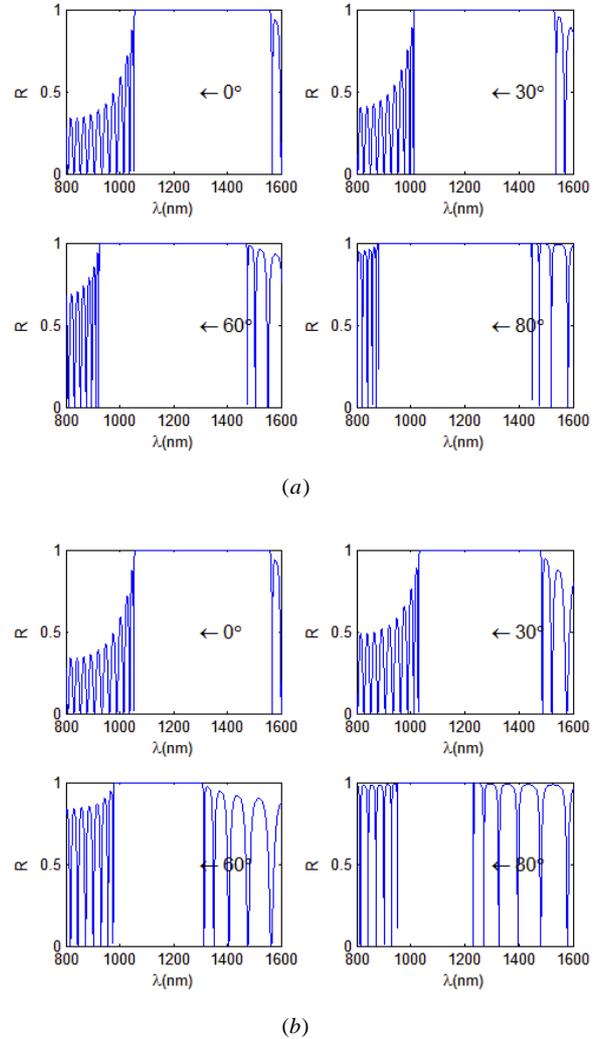


Fig. 7. The relation among reflectance, wavelength and incident angle

### 5. Designing and optimizing photonic crystal filters

After discussion above, we can select proper materials and parameters to design photonic crystal filters according to demands. In this part, we design the one-dimension photonic crystal shaped like  $(AB)^m B^n (BA)^m$ .  $A$  and  $B$  are different dielectrics. The corresponding refractive indexes are  $n_a, n_b$  and their thickness are  $d_a, d_b, m$  and  $n$  are the number of period. To simplify, assume all media are non-magnetic materials, the  $\mu = 1$ . All incident waves are TE waves and the incident angle  $\theta = 0$ . The relation between

$n_a d_a$  and  $n_b d_b$  satisfies:  $n_a d_a = n_b d_b$ . With the methods above, proper filters is easily designed by computer simulation.

During the wavelength range  $780nm-900nm$ , design the photonic crystal filter, which allows only light of the wavelength and  $\lambda = 800nm$  to pass. By computer simulation, the parameters of filter are selected as follows:

$$n_a = 3.16, n_b = 1.414, \quad n_a d_a + n_b d_b = 400nm,$$

$$m = 5, n = 30$$

The relation between Transmittance and wavelength is shown in Fig. 8.

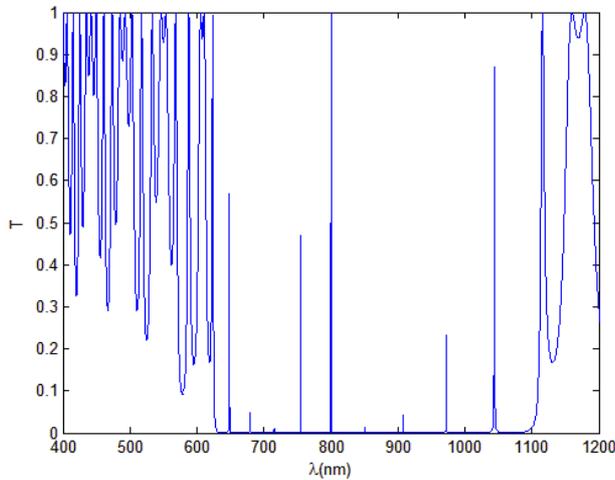


Fig. 8. Transmittance vs. wavelength (800nm)

From Fig. 8, it is easy to conclude that during the wavelength range  $780nm-900nm$ , the filter has narrow-band high transmittance at the wavelength 800nm. Similarly, other filters which allow only a fixed wavelength to pass can be easily designed.

If only light of the wavelength  $\lambda = 830nm$  is allowed to pass, the parameters of filter are:

$$n_a = 3.16, n_b = 1.414, \quad n_a d_a + n_b d_b = 415nm, \\ m = 5, n = 30.$$

Now, add a dielectric slab C between A and B in every period, the correspond refractive index is  $n_c = 2.3$ . So the structure of photonic crystal is changed to  $(ACB)^m B^n (BCA)^m$ . Adjust the thickness of the media slab to satisfy the following equation:

$$n_a d_a = n_b d_b = n_c d_c = (1/6)\lambda_0$$

Choose  $\lambda_0 = 830nm$  and Keep other parameters unchanged and then we get the filter in which only wavelength  $\lambda = 830nm$  could pass. Comparison between the structure  $(ACB)^m B^n (BCA)^m$  and the structure  $(AB)^m B^n (BA)^m$  is shown in Fig. 9.

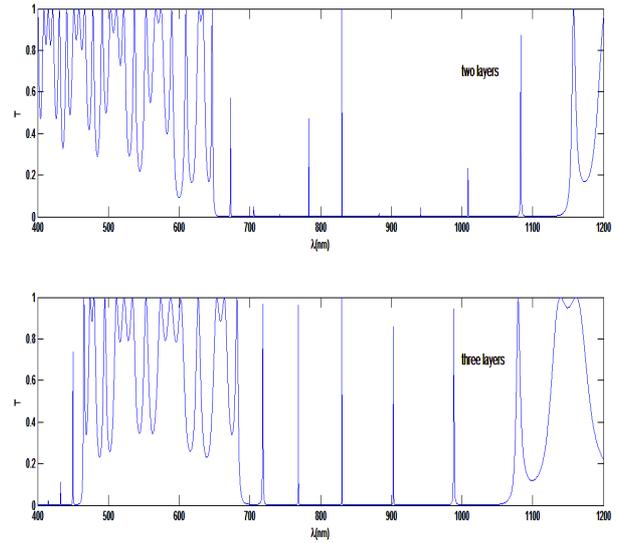


Fig. 9. Transmittance vs. wavelength (830nm)

From Fig. 9, we know that these two structures both have narrow-band high transmittance at the wavelength 830nm. But the photonic crystal with two alternate layers system has a wider band gap than the photonic crystal with three alternate layers system.

By the result of computer simulation, it is concluded that the demanded photonic crystal filter could be obtained through adjusting the structure and parameters of photonic crystal.

## 6. Summary

In this paper, the dispersion relation and the width and distribution of band gap of one-dimension photonic crystal has been discussed by the TMM. By computer simulation, it is concluded that band gap becomes wide when the permittivity ratio or the thickness ratio or the host period becomes large. Among them, the change of permittivity ratio is the most obvious impact to the width of band gap. Then, design holophotes which work at the wave length 830nm and optimize them. At last, this paper introduces

the design of two photonic crystal filters that respectively have Narrow-band high transmittance at the wavelength 800 nm and 830 nm.

#### **Acknowledgement**

The author acknowledges the financial support of the National College Student Innovation Foundation.

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