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Resonant tunneling properties of one-dimensional photonic crystals containing negative refractive index materials

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The resonant tunneling properties of one-dimensional photonic crystal composed of alternately arranged PIM-PIM and PIM-NIM are studied theoretically by using transfer matrix method. Transmittance and band structure properties of the structure are analyzed for different values of electric permittivity ($\pm\varepsilon_i$) and magnetic permeability ($\pm\mu_i$) of the materials. The PIM layers are replaced by NIM layers ($-\varepsilon_2$ and $-\mu_2$) in the periodic structure, we have found some peculiar band-gap structure and sharp transmittance properties. The transmission bands of the structure are found very sharp when refractive index and impedance of the NIM are very low. Such resonant tunneling property of the one dimensional photonic crystals containing NIMs may be utilized to make a very narrow filter.

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1. Introduction

In 1967, Veselago [1-2], Russian physicist, first formally considered a negative index material (NIM) medium with simultaneously negative the electric permittivity and magnetic permeability at a certain frequency from a theoretical point of view, and concluded that the phase velocity and energy velocity of such media would point in opposite directions. The most fundamental optical effect of the negative refraction is that the bending of light occurs in negatively direction when the light crosses the interface between two the materials [3-4]. The principle of negative refraction can be applied to all electromagnetic waves to study the wave propagation. According to Snell's law when an electromagnetic waves traverse the interfaces from a refractive index n_1 to refractive index n_2, the change in its trajectory can be determined from the ratio of refractive index n_2 / n_1. For the negative refractive index (NIM), the Snell's law shows that the electromagnetic wave would refract in negative angle due to the value of the ε and μ is simultaneously negative i.e. $n = -\sqrt{\varepsilon\mu}$.

Photonic Crystals fully employ the coherent nature of the light, and open a new area for the development of all optical integrated circuits. The idea of photonic crystals is given by John [5] who studied the anomalous absorption of light in a disordered medium, where he noticed the role of localization in anomalous absorption and Yablonovitch [6] proposed and fabricated the first photonic crystal, or Photonic Band Gap (PBG) material. The size of unit cells of the photonic crystals is comparable to the wavelength of the light. When light travels in the crystal, the Bragg reflected waves interfere with the original beam. The coherence completely changed the homogeneous distribution of the light field in the crystal, and the dispersion relation of the propagating waves.

Recently Silvestre et al. [7] have studied the role of the dispersion on zero-refractive index bandgaps in periodic multilayers combining ordinary positive index materials (PIMs) and dispersive metamaterials (NIMs) with negative index in certain frequency ranges. The production of zero-average-index bandgaps has been investigated by changing the dispersion models of the metamaterial's constituents. The dispersion relation and associated electric fields of one-dimensional photonic crystal which is composed of alternating layers of right-handed materials (RHMs) and left-handed materials (LHMs) have been investigated for oblique propagation by Dios-Leyva and González-Vasquez [8]. The calculations are performed by assuming that the dielectric permittivity and magnetic permeability are constant in the RHM, whereas both parameters follow plasma like dispersion in the LHM. The dispersion curves and associated electric fields are shown to exhibit the interesting features around both the magnetic plasma frequency and the central frequency (ω_0) at which the spatial average of the wave vector component perpendicular to the layers is vanished. The features around magnetic plasma frequency are determined by the occurrence of a strong coupling between the radiation field and the "magnetic plasma" contained in the LHM. The broadband omni-directional reflection properties of single negative index materials have been calculated by using a dispersion relation with parameters taken from experimental data [9]. The reflecting band does not shift in frequency but actually widens with increasing angles of incidence. The operational bandwidth can be found 100% by increasing the separation between the electric and magnetic plasma frequencies [10]. By using the Lorentz and Drude medium models for double-negative slab, Sabah and Uçkun [11] have studied the electromagnetic wave propagation through the frequency-dispersive and lossy double-negative slab embedded between two different semi-infinite media. The reflected, transmitted and loss powers for the frequency-dispersive and lossy double-negative slab are determined using reflection and transmission coefficients for both TE and TM waves.

In this communication we study theoretically the band structure and the transmission properties of the one-dimensional photonic crystal composed of alternately arranged PIM-PIM and PIM-NIM by using simple transfer matrix method [12-14]. The band structure and the transmittance properties are analyzed for different values of electric permittivity ($\pm\varepsilon_i$) and magnetic permeability ($\pm\mu_i$) of the materials where i=1,2. The refractive index of the material is $n_i^2=\varepsilon_i\mu_i$. By introducing $-\varepsilon_2$ and $-\mu_2$ of the NIM layers in the periodic structure, we have found some peculiar band-gap structure and sharp transmittance properties compare to conversional photonic crystal composed of alternately arranged PIM-PIM. The forbidden bands are extremely wide for same refractive index of PIM-PIM and same surface impedance of PIM-NIM materials. The transmission bands are found very sharp without oscillation for the structure with PIM-NIM materials. The physical significance of the peculiar band gap and photon tunneling modes has been explained on the basis of the discrete modes in the periodic structure of PIM-NIM layers. Such photon tunneling property one dimensional photonic crystals containing NIM may be utilized to make a very narrow filter.

2. Theory

For study of dispersion and transmittance binary one-dimensional periodic structure with alternating layer (ε_1, μ_1) and (ε_2, μ_2) as shown in the d_1 and d_2 are the width of the two layers respectively. The corresponding refractive index is given by $n_i = \pm\sqrt{\varepsilon_i\ \mu_i}$. The periodic structure of (ε_1, μ_1) and (ε_2, μ_2) is periodic frequency dependent permittivity $\varepsilon(x+d)=\varepsilon(x)$ and permeability $\mu(x+d)=\mu(x)$ [12,13]. We consider normal propagation of monochromatic electromagnetic field (with time dependence $e^{-j\omega t}$) in a periodic structure with oblique wave vector β=0 along the y-axis. For the E- polarization case, one has chosen the following way [14-17];

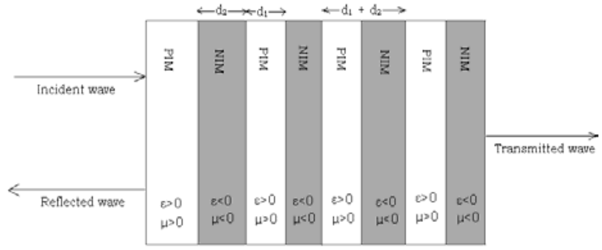

Fig. 1. Schematic diagram of periodic structure containing PIM-NIM.

$$\left.\begin{aligned}
E_{1z} &= e^{j\beta y}\left[A e^{jk_{1x}x} + B e^{-jk_{1x}x}\right] \\
H_{1y} &= -\frac{k_{1x}}{\omega\mu_1} e^{j\beta y}\left[A e^{jk_{1x}x} - B e^{-jk_{1x}x}\right] \\
H_{1x} &= -\frac{\beta}{\omega\mu_1} e^{j\beta y}\left[A e^{jk_{1x}x} + B e^{-jk_{1x}x}\right] \text{in region I}
\end{aligned}\right\} \quad (1)$$

and

$$\begin{aligned}
E_{2z} &= e^{j\beta y}\left[C e^{jk_{2x}x} + D e^{-jk_{2x}x}\right] \\
H_{2y} &= -\frac{k_{2x}}{\omega\mu_2} e^{j\beta y}\left[C e^{jk_{2x}x} - D e^{-jk_{2x}x}\right] \quad (2) \\
H_{2x} &= -\frac{\beta}{\omega\mu_2} e^{j\beta y}\left[A e^{jk_{2x}x} + B e^{-jk_{2x}x}\right] \quad \text{in region II}
\end{aligned}$$

where k_{ix} is the component of the wave vector along x-axis is region i = 1,2 i.e. $k_i^2 = \varepsilon_i\mu_i\frac{\omega^2}{c^2} - \beta^2$. Here c is the speed of light in vacuum and β is the z-component wave vector. Imposing the periodicity constant E[(x+d), y]=E(x,0) exp(i Kd + iβy), where $d=d_1+d_2$ is the periodic thickness of unit cell. K is the dimensionless Bloch wave vector of the periodic unit cell, which define the transmission across the layer and its dependence on the wave vector component along the β can be found explicitly for two-layer periodic structure.

The tangential electric and magnetic field should be continuous at x=0 and x=d_2 i.e.

$$\left.\begin{aligned}
&E_{1z}(x=0^-) = E_{2z}(x=0^+) \\
&H_{1y}(x=0^-) = H_{2y}(x=0^+) \\
\text{and}\quad & \\
&E_{2z}(x=d_2^-) = E_{1z}(x=d_2^+) \\
&H_{2y}(x=d_2^-) = H_{1y}(x=d_2^+)
\end{aligned}\right\} \quad (3)$$

To obtain the dispersion relation for this one-dimensional photonic crystal, we have to use the following periodic conditions according to the Bloch's theorem;

$$E_{2z}(x=d_2) = E_{1z}(x=-d_1)\,e^{iKd},\ H_{2y}(x=d_2) = H_{1y}(x=-d_1)\,e^{iKd} \quad (4)$$

where K is the first Brillouin zone $-\pi/d \le K \le \pi/d$. Substituting equations (1) and (2) into equation (3) and (4) we obtain the matrix M_{ij} characterizing the wave scattering in the periodic structure.

$$\begin{pmatrix} A_{n-1} \\ B_{n-1} \end{pmatrix} = M \begin{pmatrix} A_n \\ B_n \end{pmatrix} \quad (5)$$

where matrix is represented by $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$, with the matrix elements,

$$M_{11} = e^{ik_{ix}d_1}\left[\mathrm{Cos}(k_{2x}d_2) + \frac{i}{2}\left(\frac{Z_1}{Z_1} - \frac{Z_2}{Z_1}\right)\sin(k_{2x}d_2)\right],$$

$$M_{12} = e^{-ik_{ix}d_1}\left[\frac{i}{2}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_1}\right)\sin(k_{2x}d_2)\right],$$

$$M_{21} = e^{ik_{ix}d_1}\left[-\frac{i}{2}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\right)\sin(k_{2x}d_2)\right],$$

$$M_{22} = e^{-ik_{ix}d_1}\left[\cos(k_{2x}d_2) - \frac{i}{2}\left(\frac{Z_1}{Z_2} - \frac{Z_2}{Z_1}\right)\sin(k_{2x}d_2)\right]$$

The dispersion relation of the periodic structure of the unit cell,

$$\mathrm{Cos}\,(Kd) = \frac{1}{2}Tr\left[M(\omega)\right] \quad (6)$$

where $M(\omega)$ is a 2×2 matrix given by equation (5). For a binary unit cell i.e. PIM-NIM-PIM-NIM... N^{th} stacks, we have the band structure for the considered structure;

$$\mathrm{Cos}\,(Kd) = \cos(k_{1x}d_1)\cos(k_{2x}d_2) - \frac{1}{2}\left[\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right]\sin(k_{1x}d_1)\sin(k_{2x}d_2) \quad (7)$$

The transmission (t) and reflection (r) coefficients can be related easily between the plane wave amplitudes in the left, right and back is given as follow:

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = M \begin{pmatrix} 1 \\ r \end{pmatrix} \quad (8)$$

where $M_{11}=M_{11}U_{N-1}-U_{N-2}$, $M_{12} = M_{12}U_{N-1}$, $M_{22}=M_{22}U_{N-1}-U_{N-2}$, $M_{21}=M_{12}U_{N-1}$ and $U_N=\sin[(N+1)K(\omega)d]/\sin[K(\omega)d]$

So, that the reflection coefficient and transmission coefficients 'r' and 't' are given by,

$$r = -\frac{M_{21}}{M_{22}}$$

and

$$t = M_{11} - \frac{M_{21}\,M_{21}}{M_{22}} \quad (9)$$

The reflectance (R) and transmittance (T) of the considered structure are given by;

$$R = |r|^2 \text{ and } T = |t|^2 \quad (10)$$

3. Results and discussion

For TE mode, the band structure and the transmittance properties are analyzed for different value of $+\varepsilon_1$ and $+\mu_1$ for PIM and $-\varepsilon_2$ and $-\mu_2$ for NIM materials. For the numerical calculations, firstly we have taken the periodic structure with PIM-PIM of the different combinations of electric permittivity and magnetic permeability i.e. (a) $\varepsilon_1=2$, $\mu_1=1$ and $\varepsilon_2=1$ $\mu_2=1$; (b) $\varepsilon_1=2$, $\mu_1=1$ and $\varepsilon_2=1$ $\mu_2=2$ which is shown in the figures 3 and 4 respectively. The thicknesses of the materials are $d_1=\lambda_0/4n_1$ and $d_2=\lambda_0/4n_2$ respectively where $\lambda_0=800\mu m$. The refractive index of the material is $n_i^2=\varepsilon_i\mu_i$. The figure 3 is depicted the cosine wave, band structure and transmittance of the structure containing $\varepsilon_1=2$, $\mu_1=1$, and $\varepsilon_2=1$ $\mu_2=1$ versus normalized frequency. The cosine wave shows forbidden gap at the normalized frequency of 1, 3 and 5 due to its value is large than one. The allowed band gaps of the periodic structure containing $\varepsilon_1=2.0$, $\mu_1=1.0$ and $\varepsilon_2=1.0$, $\mu_2=1.0$ have also found at the normalized frequency of 2, 4 and 6 due to its value is small than one. The forbidden and allowed band gaps are matched with the band structure and the transmittance of the periodic structure having $\varepsilon_1=2.0$, $\mu_1=1.0$ and $\varepsilon_2=1.0$, $\mu_2=1.0$.

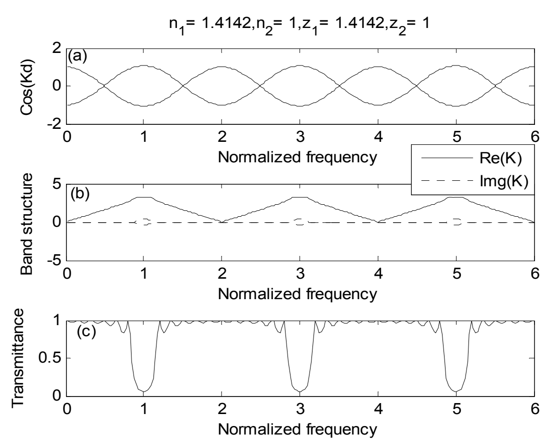

Fig. 2. Cos wave, band structure and transmittance for $\varepsilon_1=2.0$, $\mu_1=1.0$ and $\varepsilon_2=1.0$, $\mu_2=1.0$ for photonic crystal containing PIM–PIM.

The transmittance of the same structure has found 100% at the frequency ranges i.e. 0.0-0.5, 1.5-2.5, 3.5-4.5 and 5.5-6.0 which is totally verified with band structure as well as cosine wave of the structure for same normalized frequency. The forbidden band gaps of the periodic structure have 0% transmittance at the 1, 3 and 5 of the normalized frequency but for the allowed band gaps of the structure have 100% transmittance at 2, 4 and 6 of the normalized frequency ranges. The first positive index material (PIM)-second positive index material (PIM**)** with ε_1=2 and μ_1=1, ε_2=1, μ_2=1 of the periodic structure have refractive indices of PIM n_1=1.4142, n_2=1.00 and surface impedances of PIM z_1=1.4142 and z_2=1.00 respectively.

Similar calculation has been performed for second combination of periodic structure with ε_1=2, μ_1=1 and ε_2=1, μ_2=2 which is shown in the figure 3. In this case the refractive indices of the PIMs are same (n=1.4142) but the surface impedance is changed and becomes z_1=1.4142 and z_2=0.70711 for first and second PIMs respectively. The figure 3 shows same results as depicted in the figure 1. The figure 3 obtained large band gaps in compression to figure 1 due change in the refractive index. The structure has very interesting observations that the band gaps are become large when the refractive index is same but surface impedance is changed. Such band gap is obtained in the magnetic photonic crystals.

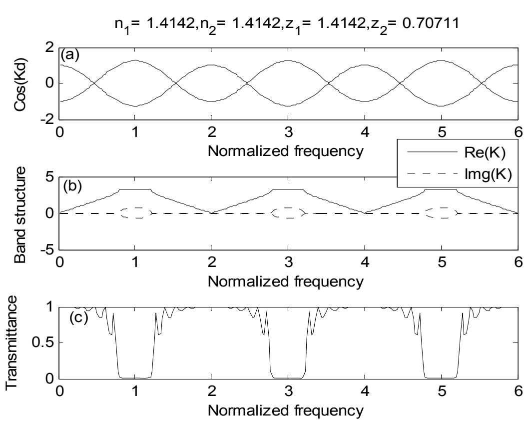

Fig. 3. Cos wave, band structure and transmittance for ε_1=2.0, μ_1=1.0 and ε_2=1.0, μ_2=2.0 for photonic crystal containing PIM–PIM.

The transmittance of the structure become large even refractive index contrast (Δn) is zero but due to the changed surface impedance of the structure, the band structure and the transmittance is become enlarge compare to the previous structure. As we know that the band gaps of the photonic crystal become large when the refractive index contrast (Δn) is large. But the changed surface impedance of the materials can also enlarge the band gaps. The study of the band gap and the transmittance of the considered structure may reveal the enlarged band gaps due to change of the surface impedance at interfaces of the two positive materials. The transmittance of the structure versus normalized frequency for n_1=1.4142 and different values of the n_2=1.00, 1.4142, 3.7417 is shown in the figure 4. The surface impedances of the structure are z_1=1.4142 and z_2=1.00, 0.7071, 0.2673. The reflectance is become enlarge when contrast of the refractive index is large and surface impedance is very low.

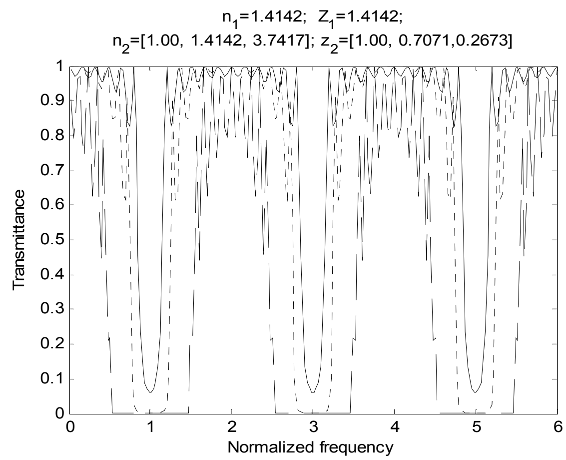

Fig. 4. Transmittance versus normalized frequency for n_1=1.4142 and n_2=1.00 (solid), 1.4142 (dot), 3.7417 (dash) lines

Secondly we have taken the periodic structure with positive index material (PIM)-negative index material (NIM**)** of the different combinations of the electric permittivity and the magnetic permeability i.e. (a) ε_1=2, μ_1=1 and ε_2=-1, μ_2=-1; (b) ε_1=2, μ_1=1 and ε_2=-1 μ_2=-2 for TE mode. The abnormal phenomena such as spurious modes with complex ω, discrete modes and photon tunneling modes by analyzing the explicit dispersion relation and cosine wave for a 1-D periodic structure with alternating PIM and NIM layers. The first positive index material (PIM)-second negative index material (NIM**)** with ε_1=2, μ_1=1 and ε_2=-1, μ_2=-1 of the periodic structure having refractive indices of PIM and NIM become n_1=1.4142, n_2=-1.00 and surface impedances of PIM z_1=1.4142 and z_2=1.00 respectively. When the PIM layers are replaced by NIM layers (-ε_2 and -μ_2) in the periodic structure, we have found some peculiar band-gap structure and sharp transmittance properties which is shown in Fig. 5. The cosine wave is nearly equal to one for all normalized frequency ranges. The transmission bands of the structure are found sharp at the normalized frequency of 0, 2, 4 and 6 due to the negative refractive index of the NIM. By introducing -ε_2 and -μ_2 of the NIM layers in the periodic structure, we have found some peculiar band-gap structure and sharp transmittance properties compare to conversional photonic crystal composed of alternately arranged PIM-PIM.

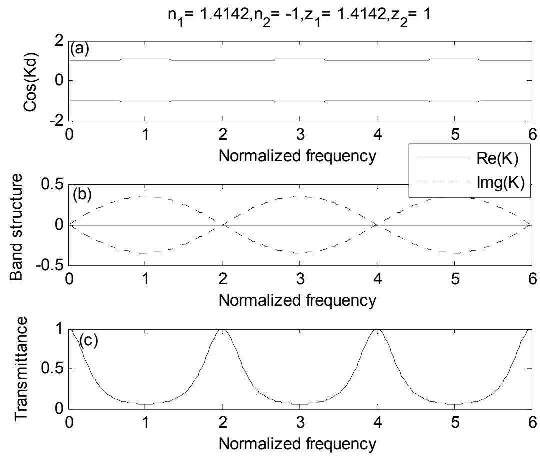

Fig. 5. Cos wave, band structure and transmittance for ε_1=2.0, μ_1=1.0 and ε_2=-1.0, μ_2=-1.0 for photonic crystal containing PIM–NIM.

The periodic structure with positive index material (PIM)-negative index material (NIM**)** of the different combinations of the electric permittivity and the magnetic permeability i.e. ε_1=2, μ_1=1 and ε_2=-1, μ_2=-2 for TE mode. The first positive index material (PIM)-second negative index material (NIM**)** with ε_1=2, μ_1=1 and ε_2=-1, μ_2=-2 of the periodic structure having refractive indices of PIM and NIM become n_1=1.4142, n_2=-1.4142 and surface impedances of PIM z_1=1.4142 and z_2=0.70711 respectively. When NIM layers are taken large value than previous one in the periodic structure, we have found same peculiar band-gap structure with very sharp transmittance properties which is shown in figure 6. The cosine wave is also nearly equal to one for all normalized frequency ranges. The transmission bands of the structure are found very sharp at the normalized frequency of 0, 2, 4 and 6 due to the negative refractive index of the NIM compare to the earlier.

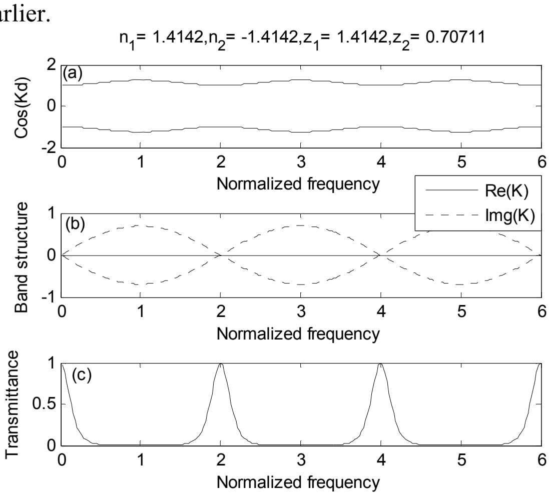

Fig. 6. Cos wave, band structure and transmittance for ε_1=2.0, μ_1=1.0 and ε_2=-1.0, μ_2=-2.0 for photonic crystal containing PIM–NIM.

The sharp transmission is obtained when the refractive index and surface impedance both are very low. It may be revealed from study that the sharp transmittance property can be achieved when both the negative refractive index and the surface impedance are very low. The transmission bands are found very sharp without oscillation for the structure with PIM-NIM materials due to NIM. The physical significance of the peculiar band gap and photon tunneling modes has been obtained due to the discrete modes in the periodic structure of PIM-NIM layers. Such photon tunneling property one dimensional photonic crystals containing NIM may be utilized to make a very narrow filter. The forbidden bands of considered structure are extremely wide and the transmission bands are very sharp without oscillation. There exist some photons tunneling propagation modes in which some propagating modes cannot propagate inside the periodic structure. The transmittance of the structure versus normalized frequency for n_1=1.4142 and different values of the n_2=-1.00, -1.4142, -3.7417 is shown in the figure 7. The surface impedances of the structure are z_1=1.4142 and z_2=1.00, 0.7071, 0.2673. The transmittance is become very sharp when the negative refractive index and surface impedance are very low. Such photon tunneling property one dimensional photonic crystals containing low refractive index of NIM and low impedance may be utilized to make a very narrow filter.

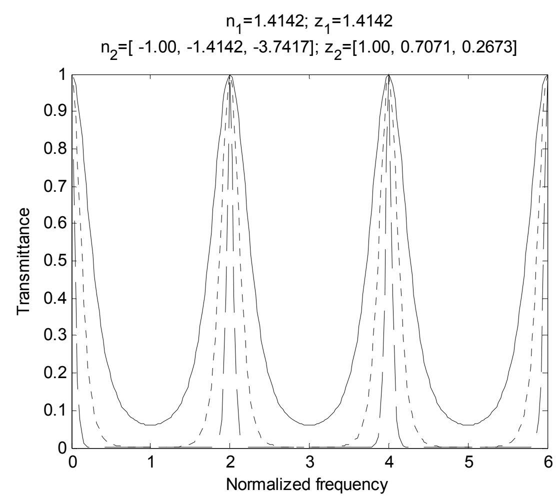

Fig. 7. Transmittance versus normalized frequency for n_1=1.4142 and n_2=-1.00 (solid), -1.4142 (dot), -3.7417 (dash) lines

4. Conclusion

We have studied the resonant properties of the one-dimensional photonic crystal composed of alternately arranged PIM-PIM and PIM-NIM by using transfer matrix method and Bloch's theorem. The structure containing PIM-PIM has very interesting observations that the bands are broaden when the refractive index is same but the surface impedance is changed. The transmittance of the structure become large even refractive index contrast (Δn) is zero but due to the changed surface impedance of the structure, the band structure and the transmittance is become large. As we know that the bandgaps of the photonic crystal become large when the refractive index contrast (Δn) is large. But the changed surface impedance of the materials can also enlarge the bandgaps. The study of the bandgap and the transmittance of the PIM-PIM periodic structure may reveal the enlarged band gaps due to change of the surface impedance at interfaces of the two positive materials. The sharp transmission of the PIM-NIM period structure is obtained when the refractive index and surface impedance both are very low. It may be revealed from study that the sharp transmittance property can achieve when both the refractive index of the NIM and the surface impedance are very low. The transmission bands are found very sharp without oscillation for the structure with PIM-NIM materials due to NIM. The physical significance of the peculiar bandgap and photon tunneling modes has been obtained due to the discrete modes in the periodic structure of PIM-NIM layers. Such photon tunneling property one dimensional photonic crystals containing NIM may be utilized to make a very narrow filter.

References

[1] V. G. Veselago, Sov. Phys- Usp. **10**, 509 (1968).
[2] V. G. Veselago, L. Braginsky, V. Schklover, C. Hafner, J. Comput. Theory Nanoscience **3,** 01 (2006).
[3] R. A. Shelby, D.R. Smith, S.C. Nemat-Nasser, S. Schultz, Appl. Phys. Lett. **78**, 489 (2001).
[4] N. Wongkasem, A. Akyurtlu, K. A. Marx, Progress In Electromagnetics Research, PIER **63**, 295 (2006).
[5] S. John, Phys. Rev. Lett. **58,** 2486 (1987).
[6] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[7] E. Silvestre, R. A. Depine, María L. Martínez-Ricci, J. A. Monsoriu, J. Opt. Soc. Am. B **26**, 581 (2009).
[8] M. de Dios-Leyva and O. E. González-Vasquez, Phys. Rev. B **77**, 125102 (2008).
[9] M. Bloemer, G. D'Aguanno, M. Scalora, Appl. Phys. Lett. **87**, 261921 (2005).
[10] K. B. Thapa, A. Vishwakarma, R. Singh, S. P. Ojha, J. Ovonic Research **06**, 105 (2010).
[11] C. Sabah, S. Uçkun, Opto-electronics Rev., **15**, 133 (2007).
[12] M. Born, E. Wolf, Principles of Optics (Oxford: Pergamon, 1989).
[13] P. Yeh, Optical Waves in Layered Media (John Wiley & Sons, 1984).
[14] R. Srivastava, K. B. Thapa, S.Pati, S. P. Ojha,, Progress In Electromagnetics Research, PIER, **81**, 225 (2008).
[15] X. Shen, X. Chen, M. Jiang, D. Shi, Optoelectronics Letter, **01**, 0201 (2005).
[16] L. Wu, S. He, L. Shen, arXiv:Cod-mat/ 0210328 V2-9 (2002).
[17] R. Kumar, Dissertation of M.Phil. "One dimensional periodic structure containing left-handed materials", UIET,CSJM University, Kanpur (2009).

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