Role of noise-binding energy interplay on Stark shift and dipole polarizabilities of impurity doped quantum dots

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Current work explores the influence of *noise-binding energy (BE) interplay* on three important electrical properties of *GaAs* quantum dot (QD) containing impurity. The said properties are *Stark shift (SS)*, *static dipole polarizability (SDP)* and *dynamic dipole polarizability (DDP)*. The study exploits *Gaussian white noise* and as a dopant we invoke Gaussian impurity. The route of introduction of noise to the system noticeably affects the said interplay giving rise to important characteristics in the manifestation of above properties. Only in case of DDP, the control of external photon energy also appears to be significant. On the whole, the study reveals that by delicate adjustment of several control parameters it is indeed feasible to fine-tune the noise-BE interplay and consequently the above three electrical properties of doped QD system.

(Received January 14, 2019, accepted August 20, 2019)

Keywords: Quantum dot, Binding energy, Stark shift, Static dipole polarizability, Dynamic dipole polarizability, Gaussian white noise

1. Introduction

Over the last few decades we envisage a great deal of research involving low-dimensional semiconductor systems (LDSS) e.g. quantum wells (QWLs), quantum wires (QWRs) and quantum dots (QDs). The said research has gained so much momentum because of two principal reasons; the first one being technology-driven and the second one is somewhat academic. The technology-driven aspect stems from the nanoscale extensions of LDSS which result into augmented quantum effects (in comparison with the bulk materials) reflected in various physical properties of LDSS (electrical, optical, magnetic etc.). Moreover, LDSS based devices also enjoy high degree of flexibility in their designing. In consequence, LDSS have become inalienable building blocks of high-performance microelectronic and optoelectronic devices such as QD lasers, solar cells, single electron transistors and quantum computers. And, from an academic perspective, study of LDSS physics grossly refreshes many important concepts of quantum mechanics. Inclusion of impurity (dopant) to LDSS brings about immediate interplay between the intrinsic confinement potential of LDSS with the dopant potential. In consequence, various physical properties of LDSS (electronic, magnetic, optical etc.) undergo discernible changes from that of a dopant free condition and the said changes possess important implications from a technological perspective. Naturally, there are ample studies on LDSS physics with due emphasis on dopant contributions [1-33].

External electric field *(F)* holds a special status in examining a number of physical properties of LDSS. *F* usually induces polarization of electronic distribution thereby disrupting the symmetry of the system. Such disruption enforces substantial change in the energy spectra of LDSS which ultimately tailors the intensity output of LDSS-based devices. One of the important physical properties of LDSS is *Stark shift (SS)* that can be envisaged under an applied electric field. Study of SS provides us an improved vision of the alterations in the internal charge distribution and spatial separation of the carriers in semiconductor heterostructures [34-37]. This actually leads to change in the binding energy (BE) of excitons and impurities in LDSS [38]. Aforesaid separation becomes large if SS is large and favors nonlinear optical phenomena like emission and absorption [39]. Thus, greater magnitude of SS of LDSS emerges as the cornerstone for the fabrication of novel optoelectronic devices, e.g. optical modulators and optical bistable devices [40, 41]. By and large, SS fine-tunes and modulates the electronic states of LDSS which is undoubtedly significant in the context of quantum information technology [42]. Naturally, the importance of studying SS in LDSS has been exhibited through a number of notable works [34-55].

Dipole polarizability (DP) is another important physical property of LDSS that manifests in presence of electric field. Exposure of LDSS to external electric field causes shift of the center of negative charge distribution and the resulting induced dipole moment gets linked with the electric field via electronic dipole polarizability. From a physical perspective, DP signifies the lowest-order (linear) response of LDSS to the external electric field reflected via distortion of the electron cloud [56, 57]. Mathematically, DP is given by the second-order derivative of total energy with the variation of external homogeneous electric field. DP possesses sufficient importance owing to its utility in a number of physical problems of LDSS such as examining the pressure-effect, understanding various interactions, studying scattering phenomenon and various optical properties etc. Moreover, in general, DP is connected with various physical quantities e.g. dielectric constant, ion mobility in gas, the van-der-Waals constant, the long range electron-atom interaction etc. [57, 58]. DP can be termed as *static dipole polarizability (SDP)* if the external electric field does not depend on time [48, 59] and *dynamic dipole polarizability (DDP)* in case of a time-dependent electric field of frequency *(ν)* [48, 59-62]. We, therefore, come across significant volume of works on DP (both static and dynamic) which have been carried out mainly for confined hydrogen atom under diverse conditions [56-67]. However, the work of Çakir et al. [48] on DP has come out to be prominently relevant in the field of LDSS physics.

Inclusion of *noise* influences the functioning of LDSS-based devices. There are some external means (also called 'modes' or 'pathways') via which noise may enter the system. However, the impact of noise becomes different for different pathways. Two such modes are usually termed as *additive* and *multiplicative* based on the type of attachment of noise to the system coordinates. Thus, application of noise affects the physical properties of the system which manifestly depends on the said pathways. Therefore, understanding the noise effects on physical properties of LDSS is quite demanding.

In this communication we focus on analyzing how the *interplay between noise and binding energy (BE)* influences the *SS, SDP and DDP* of 2-d *GaAs* QD. Any alteration in BE of LDSS has immediate impacts on its properties which ultimately influences the design of novel optoelectronic devices. The *x - y* confinement is depicted by the harmonic oscillator potential and the *z*-confinement is made by a perpendicular magnetic field. In addition, the system is subjected to an external electric field of strength *F* along *x* and *y*-directions. The QD contains *Gaussian impurity* as dopant and at the same time is fed with *Gaussian white noise* applied via *additive* and *multiplicative* pathways (modes). The study unfolds how the interplay between BE and noise modulates the aforesaid electrical properties with special reference to the role played by the noise mode.

2. General formalism

The Hamiltonian (H_0) of the system can be written as

$$H_0 = H_0' + V_{imp} + |e| F(x+y) + V_{noise}. \quad (1)$$

H_0' is the dopant-free Hamiltonian and e is the electronic charge. Consideration of effective mass approximation further gives

$$H_0' = \frac{1}{2m^*}\left[-i\hbar\nabla + \frac{e}{c}\mathbf{A}\right]^2 + \frac{1}{2}m^*\omega_0^2\left(x^2+y^2\right). \quad (2)$$

m^* and ω_0 denote the effective mass of the electron and the harmonic confinement frequency, respectively. **A** is the vector potential given by $A = (By, 0, 0)$, where B is the strength of the magnetic field. H_0' may be transformed to

$$H_0' = -\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2}m^*\omega_0^2 x^2 + \frac{1}{2}m^*\Omega^2 y^2 - i\hbar\omega_c y\frac{\partial}{\partial x}, \quad (3)$$

where $\Omega\left(=\sqrt{\omega_0^2+\omega_c^2}\right)$ and $\omega_c\left(=\frac{e\mathrm{B}}{m^*c}\right)$ are the effective confinement frequency in the *y*-direction and the cyclotron frequency, respectively. V_{imp} refers to the potential that describes the impurity (dopant) and reads $V_{imp} = V_0 e^{-\gamma\left[(x-x_0)^2+(y-y_0)^2\right]}$. Here (x_0, y_0), V_0 and $\gamma^{-1/2}$ stand for the dopant site (coordinate), dopant potential strength, and the spatial zone over which the impurity potential is effective, respectively. V_{noise} of eqn (1) being the noise part of the Hamiltonian. In the present work Gaussian white noise has been exploited having features like *zero average* and *spatial δ-correlation*. Moreover, introduction of noise to the system is carried out in two different routes (called additive and multiplicative) which actually guide the size of system-noise interplay. Now, the construction of Hamiltonian matrix (H_0) has been carried out using the direct product basis of the harmonic oscillator eigenstates. The energy levels and the eigenstates of the system have been obtained by diagonalizing H_0. The routine convergence test has been done during diagonalization.

The energy difference with and without the electric field comes out as the measure of SS and can be written as [36, 37, 43].

$$\Delta E_{SS} = \left[E(F\neq 0) - E(F=0)\right]. \quad (4)$$

Following Çakir et al. [45], SDP for LDSS can be represented by

$$\alpha_D = 2e^2\sum_{j=1}\frac{\left|\left\langle\psi_j^{(0)}\right|x+y\left|\psi_0^{(0)}\right\rangle\right|^2}{E_j^{(0)}-E_0^{(0)}}, \quad (5)$$

where, e is the electronic charge, $\psi_j^{(0)}$ and $\psi_0^{(0)}$ are the $|j\rangle$-th and $|0\rangle$-th eigenstates without electric field and $E_j^{(0)}$ and $E_0^{(0)}$ are the respective energies. In the current study we take into account the first five eigenstates (i.e. $|j\rangle = |1\rangle$ to $|5\rangle$) in the above summation because of their considerable transition dipole moment values. Consideration of further terms in the above summation does not significantly alter the result.

In order to study DDP, the QD is subjected to an electric field of strength *F* with oscillation frequency *ν*. The time-dependent and frequency-dependent electrically induced perturbation to QD is given by [59, 60]

$$\mathrm{V}(\nu,t) = F\left[\exp(-2\pi i\nu t) + \exp(2\pi i\nu t)\right]. \quad (6)$$

According to Çakir et al. DDP is given by [48]

$$\alpha_D(h\nu) = \sum_{j=1} \frac{f_{j0}}{\left(E_j^{(0)} - E_0^{(0)}\right)^2 - (h\nu)^2}, \qquad (7)$$

Where $h\nu$ and f_{j0} are the photon energy and the *oscillator strength (OS)* for the $|0\rangle \to |j\rangle$ transitions. Pursuing Çakir et al. [48] the OS for the dipole allowed transitions is given by:

$$f_{j0} = \frac{2}{3}\Delta E_{j0} \left| \left\langle \psi_j^{(0)} \middle| x + y \middle| \psi_0^{(0)} \right\rangle \right|^2$$

Present study considers only $|0\rangle \to |1\rangle$ transition to calculate DDP owing to its substantial transition dipole moment value. Following Çakir et al. [48], eqn(5) reveals occurrence of *singular points* as soon as $h\nu = E_1^{(0)} - E_0^{(0)}$ where *sign-inversion* of polarizability takes place.
The ground state binding energy E_B can be written as

$$E_B = E_0 - E, \qquad (8)$$

where E and E_0 are the ground state energies with and without impurity, respectively.

3. Results and discussion

We have used ε = 12.4 and m^* = 0.067m_0 (m_0 is the mass of electron in vacuum).Moreover, during the study a few relevant parameters assume following fixed values: $\hbar\omega_0$ = 250.0 meV, V_0 = 280.0 meV, B = 20.0 T, F = 100 kV/cm, r_0 = 0.0 nm and ζ = 1.0 x 10^{-4}, where ζ is the noise strength.

A. Stark shift (SS):

We begin with the profile of SS (absolute value) against BE in absence of noise [fig. 1(i)]and when noise enters the system via additive [fig. 1(ii)] and multiplicative [fig. 1(iii)]routes, respectively. The plot reveals steady rise of SS as BE increases under all conditions.

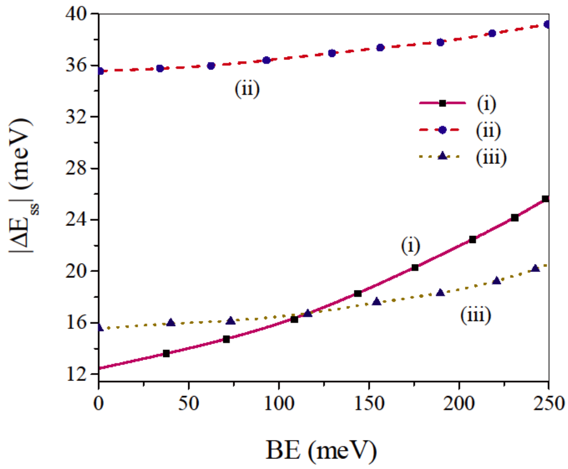

Fig. 1. Plots of $|\Delta E_{ss}|$ against BE: (i) noise-free state, (ii) additive noise operates and (iii) multiplicative noise operates

However, the extent of enhancement comes out to be more in absence of noise than in its presence. The observation suggests that, irrespective of presence of noise, an increase in BE of the system promotes greater asymmetric arrangement of electron probability density. In view of SS profile the noise-BE interplay does not play any notable role as the qualitative features remain more or less unchanged.

B. Static dipole polarizability (SDP):

Fig. 2 depicts the profiles of SDP against BE under noise-free state [fig. 2(i)] and under the governance of additive [fig. 2(ii)] and multiplicative [fig. 2(iii)] noise, respectively. The plots reveal similar pattern of SDP profiles without noise and under additive noise whence we observe steady decline of SDP as BE of the system increases. The observation again suggests steady amplification of the overall system confinement and consequent quenching of the system leading to drop in SDP [48, 58, 61, 63, 64]. Thus, the absence of noise and the presence of additive noise exhibit similarity with respect to the manner in which the spatial confinement of the system depends on BE. Presence of multiplicative noise causes a prominent deviation of SDP profile from previous two cases, as the BE varies. We now observe a modest but steady rise of SDP with increase in BE. The observation suggests multiplicative noise induced mild diminish of system confinement as BE increases.

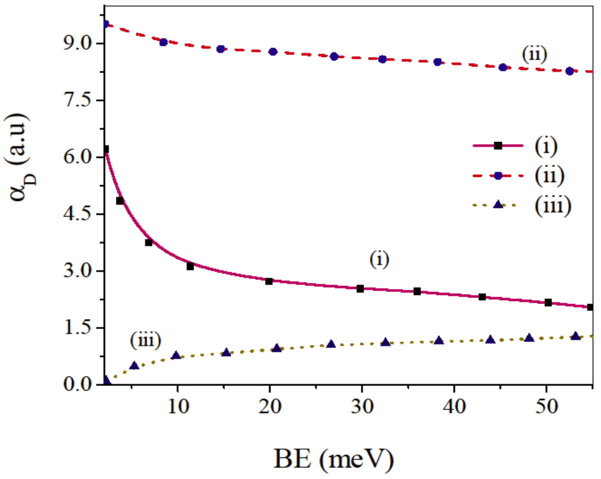

Fig. 2. Plots of α_D against BE: (i) noise-free state, (ii) additive noise operates and (iii) multiplicative noise operates

C. Dynamic dipole polarizability (DDP):

We now look at the plots of DDP against oscillation frequency (v) when BE takes on different values (given in the figure caption) without noise [Fig. 3 a], with additive noise [Fig. 3 b] and with multiplicative noise [Fig. 3c], respectively. It has been observed that under all situations DDP undergoes enhancement as v increases [48, 62] for given values of BE. In addition to this, a shift of singular point towards greater photon energy [48] is also observed with increase in BE. Moreover, as a common outcome (i.e. both without and with noise), we also envisage positive (negative) values of DDP as the oscillation frequency stays to the left (right) of the singular point. Above observations can be mathematically justified on the basis of eqn. (7). This is because of the fact that in the present study hv has been varied in the close vicinity of $\Delta E_{01} = E_1^{(0)} - E_0^{(0)}$ energy interval. Such variation originates from our concern about $|0\rangle \rightarrow |1\rangle$ transition which is the largest contributing term in the summation of eqn. (7).

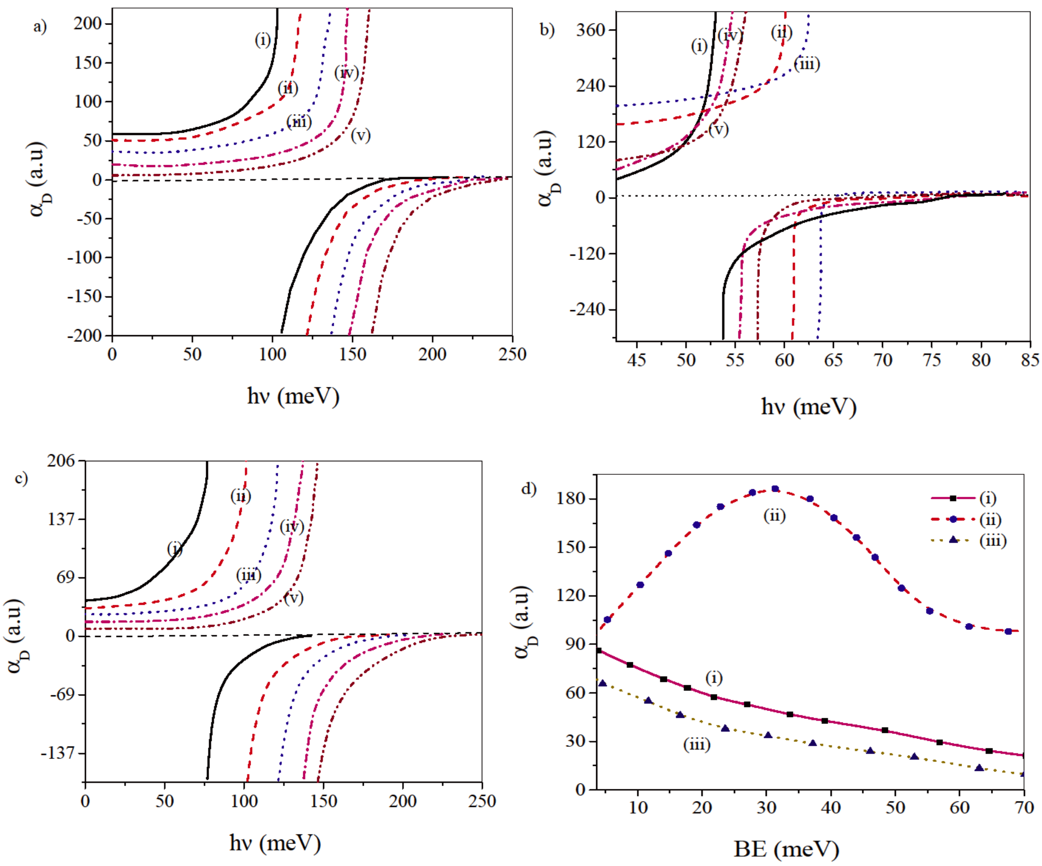

Fig. 3. Plots of α_D vs hv for different values of BE: (a) without noise, (b) when additive noise operates and (c) when multiplicative noise operates. The BE values are (i) 4.0 meV, (ii) 5.0 meV,(iii) 10.0 meV, (iv) 20.0 meV and (v) 70.0 meV. (d) Plot of α_D against BE at hv = 50 meV: (i) without noise, (ii) when additive noise operates and (iii) when multiplicative noise operates

For a better understanding of noise-BE interplay we plot DDP as a function of BE at hv = 50 meV without noise [Fig. 3d(i)], under applied additive noise [Fig. 3d(ii)] and under applied multiplicative noise [Fig. 3d(iii)], respectively. Now, we observe similar DDP profiles without noise and when multiplicative noise operates whereas noticeable departure occurs when additive noise becomes present. In the first two cases as mentioned above a steady fall of DDP can be evidently observed as BE increases. Gradual increase in BE results into augmentation of effective system confinement. In consequence, energy separation between the eigenstates also increases and OS is diminished. As a resultant of all these changes DDP undergoes a steady decline [48, 62]. Deviation in DDP profile in presence of additive noise is revealed by the appearance of conspicuous *maximization* at *BE*~ 30.0 meV. Such maximization indicates additive noise-induced sharp enhancement in the spatial stretch of the system at this typical BE.

The features of the DDP profiles as outlined above possess a mathematical justification [cf. eqn (7)]. Eqn (7) contains a summation and each term belonging to the summation is jointly contributed by a number of factors viz. the energy interval among the quantum states, the relevant oscillator strength and the impinging photon energy. It is the resultant effect of relative magnitudes of all these factors that ultimately fix the gross magnitude of DDP over the entire summation. Variation of BE, presence/absence of noise, the route via which noise is applied to the system, and the impinging photon frequency, in combination, affect the individual terms of the above summation inviting contrasting features in the gross DDP profile.

Discussions hitherto made leaves an impression that the noise-BE interplay indeed influences the profiles of the important electrical properties of doped QD viz. SDP and DDP. Interestingly, the route of introduction of noise markedly affects the said interplay. A difference in the pathway simply means a change in the way noise couples with the system co-ordinates and thus affects the BE of the system differently. In case of SDP, presence of additive noise fails to produce any new qualitative features from that of a noise-free state. It is multiplicative noise which brings about some noticeable deviation from a noise-free condition. However, in case of DDP, the role played by additive and multiplicative noise is exactly reversed with respect to the noise-free state. This is because of additional subtlety residing in the expression of DDP thanks to the incoming photon frequency term.

4. Conclusion

The influence of noise-BE interplay on three important electrical properties of impurity doped QD system has been thoroughly studied. The said properties are SS, SDP and DDP. We have found increase in magnitude of SS as BE increases under all conditions. Absence of noise and presence of additive noise produce similar features in case of SDP revealed through the fall of the quantity as BE increases. However, in presence of multiplicative noise, SDP increases as BE increases. DDP, on the other hand, decreases with increase in BE without noise and with multiplicative noise. Applied additive noise causes maximization of DDP at a BE value of ~ 30 meV. The investigation throws light on the possibility of controlling above three electrical properties of doped QD by wise adjustment of noise-BE interplay. And the said adjustment can be achieved by regulating the BE of the system and by introducing noise to the system via some desired mode. Only in case of DDP, additional control of incoming photon energy also appears to be crucial.

Acknowledgements

The authors AB., AG and MG thank DST-FIST (Government of India) and UGC-SAP (Government of India) for support. Special thanks to Sk. Md. Arif for his sincere cooperation.

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