# Second and third order dispersion effects analyzed by the split-step Fourier method for soliton propagation in optical fibers

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We present the Split-Step Fourier Method to analyze second and third order dispersion effects in optical fibers. The numerical method was used to simulate solitons propagation and their potential application in the high-speed transmission of information (Tbit/s). Its use in this type of systems is very useful but it is needed to make major changes in the system design. In this paper the NLSESolver Program was used to simulate the soliton propagation.We also discuss higher order solitons that can be used for nonlinear pulse compression, but this lead to a critical choice of the pump wavelength. It is well known that a Sech-shaped pulse with a suitable energy, injected into an optical fiber with anomalous dispersion, can evolve as a higher-order soliton and after certain propagation distance the pulse duration can be substantially decreased. Higher order soliton can break up into fundamental soliton, process that can be used in supercontinuum generation in photonic crystal fibers with applications ranging from sensors, waveguide devices to lasers.

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## 1. Introduction

In the last 35 years numerous studies were reported about soliton propagation and their applications in nonlinear optics [1-8], DNA research [9], solar physics [10], and in other fields like particle physics, oceanography, mathematics [11,12] etc.

It is well known that under certain conditions optical pulses can propagate inside fibers in the form of solitons. The use of soliton in optical communication was first mentioned in 1973 and in 1980 the optical solitons were first observed experimentally [1].

Optical solitons are localized electromagnetic waves that propagate steadily in nonlinear media resulting from a robust balance between nonlinearity and linear broadening due to dispersion and/or diffraction [13,14].

Optical fiber solitons are pulses of light which propagate without dispersion, even though the medium they are traveling through is naturally dispersive. Solitons in optical fibers are the result of the balance between the group velocity dispersion (GVD) and the self-phase modulation (SPM). GVD and SPM effects limit the performance of optical communications when acting independently on optical pulses propagating inside fibers [1]. The solution to solve the dispersion problem in high bit rate in optical communication systems is to use optical solitons-pulses that preserve their shape over long distances [15-17]. The propagation of soliton inside a fiber is well described by the nonlinear Schrödinger equation (NLSE) which is very hard to solve analytically, so there different numerical methods were introduced [1-4], [7]. Also there are other types of studies regarding the importance of numerical simulations in optical fibers and amplifiers [18-22].

In this paper we used the NLSESolver program in order to solve the nonlinear Schrödinger equations. These equations are derived from Maxwell's equations for a nonlinear medium using paraxial approximation [5].

## 2. Theory

Optical solitons can be described mathematically with the nonlinear Schrödinger equation (NLSE) [3]. This equation is given by:

$$\frac{\partial A}{\partial z} = \left(\hat{D} + \hat{N}\right)A, \tag{1}$$

where $\hat{D}$ is the dispersion and absorption term and $\hat{N}$ is an nonlinear operator:

$$\hat{D} = -\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2}A \tag{2}$$

$$\hat{N} = i\gamma\left(|A|^2 + \frac{i}{\omega_0}\frac{\partial}{A\partial T}(|A|^2 A) - T_R\frac{\partial |A|^2}{\partial T}\right) \tag{3}$$

In our simulations we will neglect the last two terms in $\hat{N}$, so that the equation reduces to:

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} = i\gamma|A|^2 A \tag{4}$$

where A (z,T) is the complex envelope of the pulse, $T = t - \beta_1 z$ represents the normalized time, $\beta_1$ the inverse of the group velocity, $\beta_2$ is the quadric dispersion coefficient which is responsible for second order dispersion effects, $\beta_3$ is the cubic dispersion coefficient which is responsible for third order dispersion effects, α is the fiber loss coefficient and γ is the nonlinear parameter. The nonlinear parameter is defined as:

$$\gamma = \frac{2\pi n_2}{\lambda A_{eff}} \quad (5)$$

where $n_2$ is the nonlinear-index coefficient, $\lambda$ is the optical wavelength, and $A_{eff}$ is the fiber effective core area.

If we consider that $U = A(z,T)/\sqrt{P_0}$ , where $P_0$ is the peak value power of initial pulse, equation (4) can be written as follows:

$$\frac{\partial U}{\partial z} + \frac{\alpha}{2}U + \frac{i\beta_2}{2}\frac{\partial^2 U}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 U}{\partial T^3} = i\gamma |U|^2 U \quad (6)$$

where the intensity is defined as $I = |U|^2$.

The parameters used to analyze the soliton propagation in optical fibers are:

$$L_D = \frac{T_0^2}{|\beta_2|} \text{ and } L_{NL} = \frac{1}{\gamma P_0} \quad (7)$$

where $L_D$ is the dispersive length, $L_{NL}$ is the nonlinear length of the optical soliton, and $T_0$ is the initial pulse length.

The soliton order (N) and the soliton period are defined as:

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \text{ and } T = \frac{\pi}{2}\frac{T_0^2}{|\beta_2|} \quad (8)$$

## 3. Results

In this paper we will use NLSE Solver program developed by Godvind P. Agrawal. As an input field we will consider that the shape of our pulse is defined as:

$$Secant = \sqrt{P_0} \times Sech\left(\frac{T}{T_0}\right)^{1+i*C} \quad (9)$$

where C is the chirp parameter, which will be equal to zero in our case.

For this type of pulses with $T_0$ and $P_0$ chosen in order to make N = 1 (first order soliton), the pulse will propagate without distortions on long distances. These types of solitons are attractive for optical communications systems. Figure 1 presents the simulation of the first order soliton with $T_0 = 3$ s, $\beta_2 = -20(s^2/m)$, $\beta_3 = 0\ s^2/m$, $\gamma = 4\ W^{-1}m^{-1}$. $P_0 = 0.5$ W, and D = 4m, in order to make N = 1.

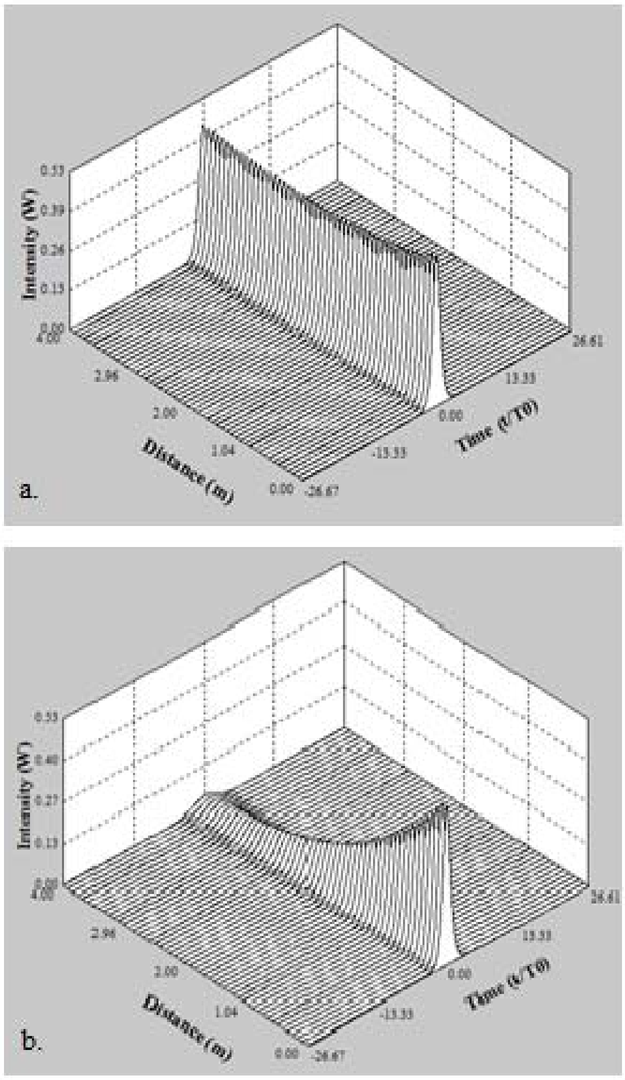


*Fig. 1. Evolution of the first order temporal soliton: a. without fiber losses α = 0 $m^{-1}$; b. with fiber losses α = 0.4 $m^{-1}$*

Only the fundamental soliton (N=1) maintains its shape and remains chirp-free during its propagation inside optical fibers. This property makes the temporal soliton an ideal candidate for optical communications. The effects of fiber dispersion are exactly compensated by the fiber nonlinearity when the input pulse has a "sech" shape [16]. It is also known that optical solitons of first order are stable against perturbations. In soliton communication systems it is necessary to introduce an optical source capable to produce chirp free pico-second pulses at high repetition rates and to operate at wavelength 1.55 µm. Also there were papers that reported the use of mode-locked semiconductor lasers, in soliton communication systems, with pulses of 12-18 ps and 40 Gb/s repetition rate [16].

Two neighboring solitons should be separated and the spacing between them should exceed a few times their Full

Width at Half Maximum (FWHM). The bits near solitons perturb them because the combined optical field is not a solution of NLSE. If the separation between them is enough then the bit rate of the soliton communication system will drop.

Another problem in optical communication systems is the losses of power due to fiber losses. This problem can be solved if amplifiers are introduced in the system.

Pulses corresponding to other integer values of N represent higher-order temporal solitons. To obtain, for the same parameters, the second order soliton (N = 2) we must rise $P_0$ to 1.12 W. Second order soliton evolution is presented in Fig. 2.

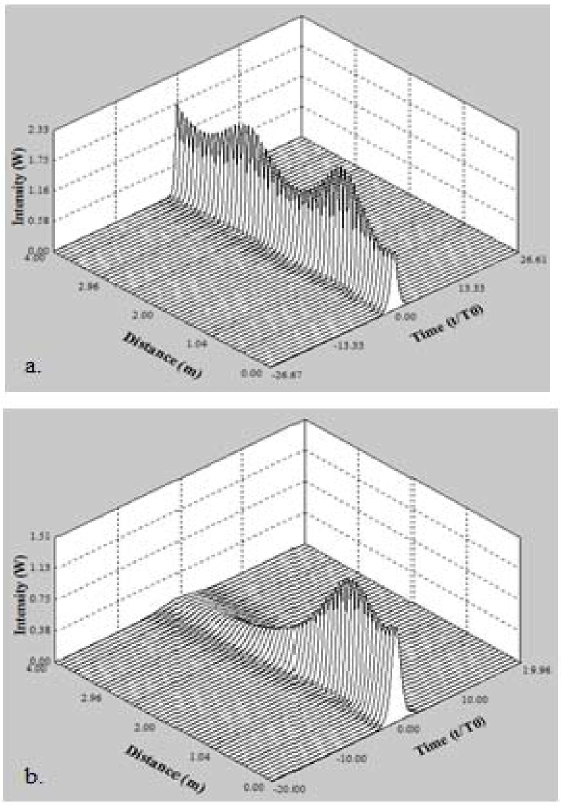


*Fig. 2. Evolution of the second order temporal soliton: a. without fiber losses $\alpha = 0\ m^{-1}$; b. with fiber losses $\alpha = 0.4\ m^{-1}$*

In Fig. 2a, the second order soliton first compresses at the half period and then returns to the initial pulse shape at the full period if the loss of the optical fiber is neglected. In the case of soliton losses Fig. 2b, the second order soliton compresses less at the half period and it can not return to the initial pulse shape at the full period.

For higher values of N the solitons exist and they are all periodic. Since high intensity is necessary to generate solitons, if the field increases its intensity even further the medium could be damaged.

Figure 3 presents the evolution of the third order soliton (N = 3, $P_0$ = 1.67 W) in optical fibers without loss (Fig. 3a) and with loss (Fig. 3b).

Higher-order solitons can be used for nonlinear pulse compression which means that after some distance the pulse duration can be substantially decreased, but the choice of the pump wavelength will be critical.

For each $N > 1$, the higher order soliton is really a nonlinear superposition of $N$ fundamental solitons. The energies are $N^2$ higher than that of fundamental soliton. The higher order solitons, unlike the fundamental soliton, are not stable in the face of perturbations and they generally break up and are diverging into fundamental solitons.

Figures 4 and 5 present a fifth order soliton that breaks-up during its propagation. In this case the simulation parameters are $T_0$ = 3 s, $\beta_2 = -2(s^2/m)$, $\beta_3 = 0\ s^2/m$, $\gamma = 4\ W^{-1}m^{-1}$, $P_0$ = 0.83 W, and D = 4 m in order to make N = 5.

If we increase the order of the soliton we can see that the dispersion is higher and that the soliton splits in several parts. The soliton break-up play an essential role in the supercontinuum generation in photonic crystal fibers and in soliton fiber lasers.

This break up effect, into diverging fundamental solitons, is due to the presence of absorption or to numerous additional nondissipative mechanisms. The break-up mechanism can be described as a result of the energy loss due to the nonlinear absorption. [13]

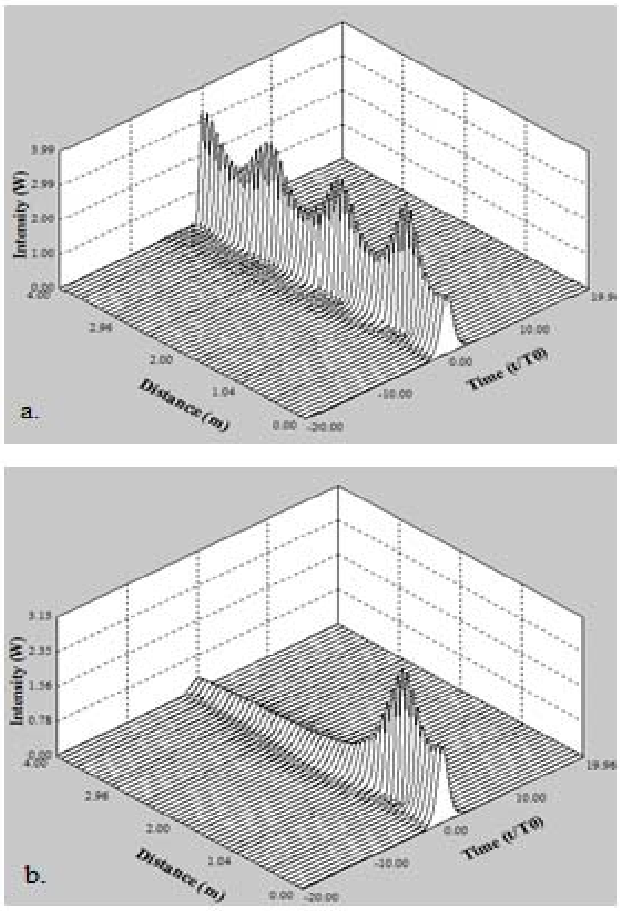


*Fig. 3. Evolution of the third order temporal soliton: a. without fiber losses $\alpha = 0\ m^{-1}$; b. with fiber losses $\alpha = 0.4\ m^{-1}$*

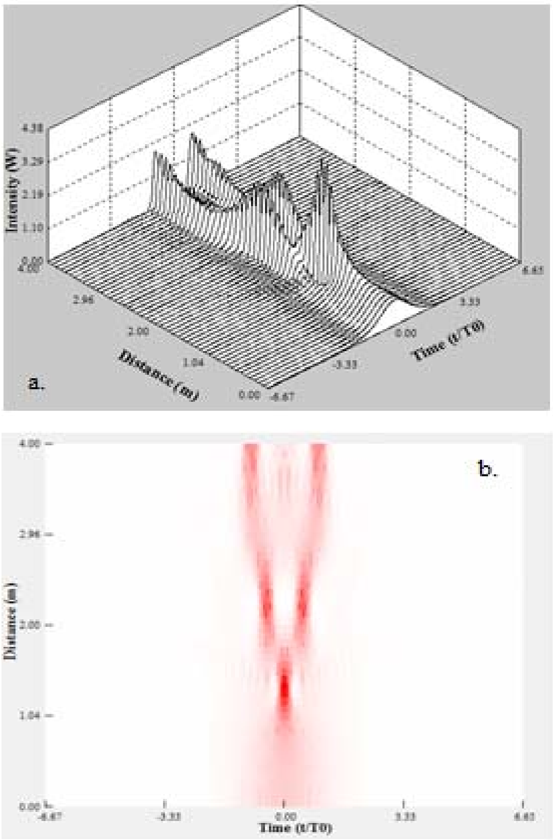


*Fig. 4. Evolution of the fifth order temporal soliton: a. without fiber losses α = 0 $m^{-1}$; b. projection of time vs. distance axes for a. case.*

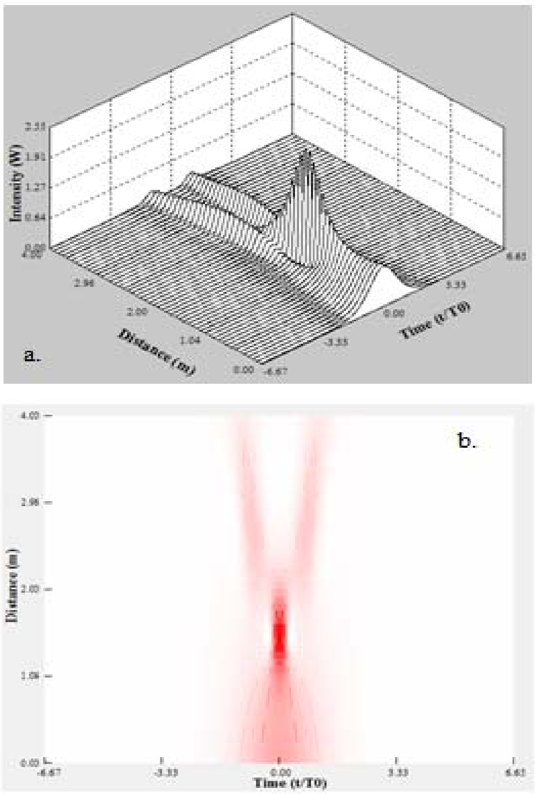


*Fig. 5. Evolution of the fifth order temporal soliton: a. with fiber losses α = 0.4 $m^{-1}$; b. projection of time vs. distance axes for a. case.*

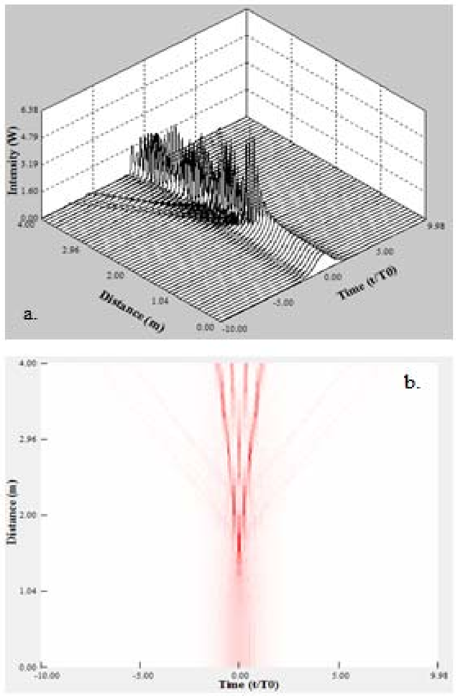


*Fig. 6. Evolution of the eight order temporal soliton: a. without fiber losses α = 0 $m^{-1}$; b. projection of time vs. distance axes for a. case.*

It was reported that the eight-order soliton (Fig. 6 and Fig. 7) can be obtained experimental by nonlinearly soliton compression of the chirp-compensated semiconductor optical amplifier fiber laser (SOAFL) pulse in a 112 m long single-mode fiber at an input peak power of 51 W, providing the pulse width, the line width, and the nearly transform-limited time-bandwidth product are <200 fs, 13.8 nm, and 0.34, respectively [17].

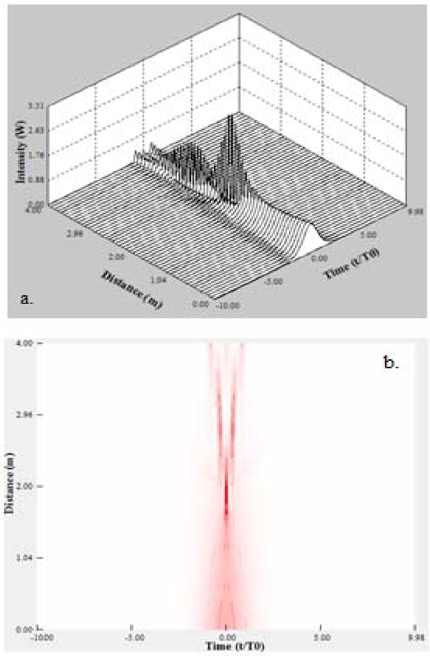


*Fig. 7. Evolution of the eight order temporal soliton: a. without fiber losses α = 0 $m^{-1}$; b. projection of time vs. distance axes for a. case.*

The effects mentioned above where due to the second order dispersion. We can see that when $\beta_2 = 0$, the oscillations are intense and the intensities drop down to zero. In this case the field intensity propagates linearly with the distance. When $\beta_2$ has high values, the third order effects can be ignored and the intensity of the field drops down.

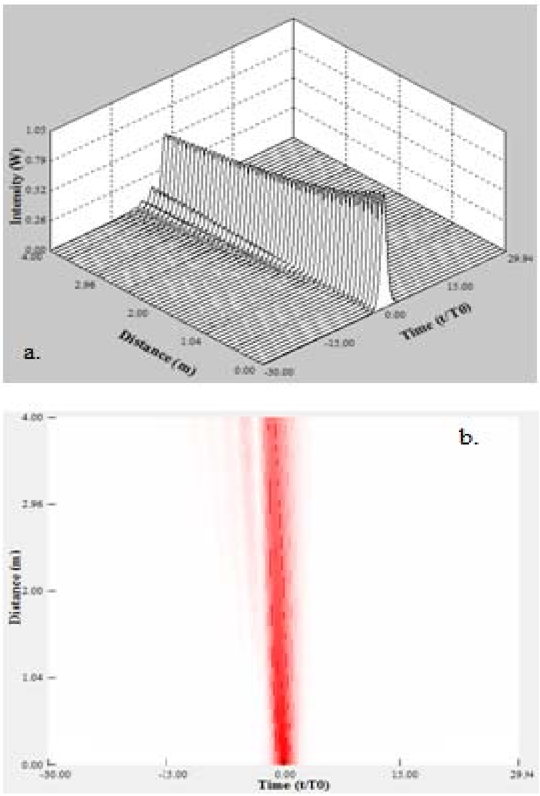


*Fig. 8. Evolution of the soliton with third order dispersion effect: a. $\beta_2$= -2 $s^2$/m; b. projection of time vs. distance axes for a) case.*

In the case of the third order effects $\beta_2$ will be negligible and the third order dispersion effects will become relevant. If $\beta_3$ is positive the oscillations will occur near the end of the pulse and if $\beta_3$ is negative the end of the pulse will develop the oscillations. In the following simulation $\beta_2 = 0$ ($s^2$/m), $\beta_3 = -2$ $s^2$/m, $\gamma = 1$ $W^{-1}m^{-1}$ and $P_0 = 1W$ (Fig 8).

We can see that though we didn't introduce losses in our simulations, the third order dispersion coefficient introduces some losses during the propagation.

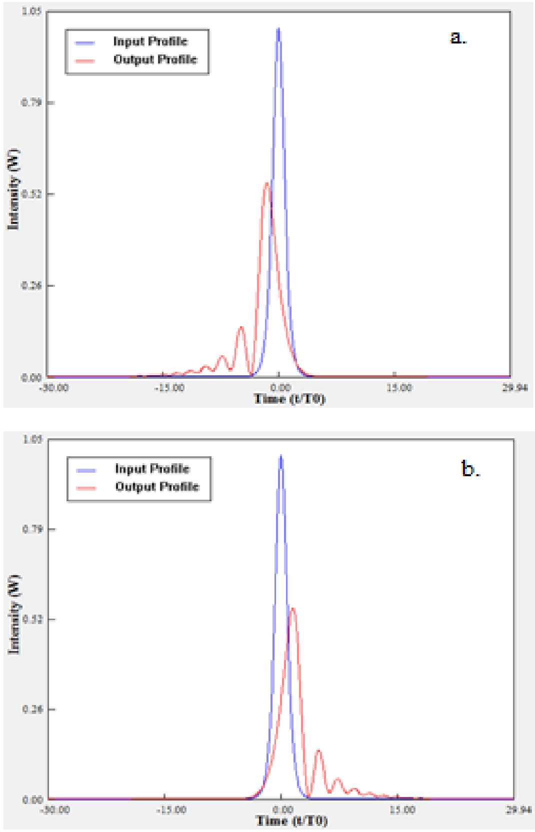


*Fig.9. Evolution of the: a. input and output profiles for $\beta_2$= -2 $s^2$/m; b. input and output profiles for $\beta_2$= 2 $s^2$/m .*

The input and the output profiles for this simulation are given in Fig. 9. We can see clearly that if $\beta_3$ is positive oscillations will occur near the end of the pulse and if $\beta_3$ is negative the end of the pulse will develop the oscillations.

If we increase the gamma parameter, responsible for nonlinearities in optical fibers we can observe a high dispersion and attenuation of the initial pulse (Fig. 10).

The third-order dispersion slows down the soliton and as a result, the soliton peak is delayed by an amount that increases linearly with distance.

The Split-step Fourier method was used to numericaly solve the nonlinear Schrödinger equation, describing the soliton propagation with third-order dispersion. Our numerical simulation show that third order dispersion can change the behavior of soliton. This may cause the compresion of the initial pulse in dispersion-shifted fibers. Also we can observe that the resultant soliton deviates more from its starting point.

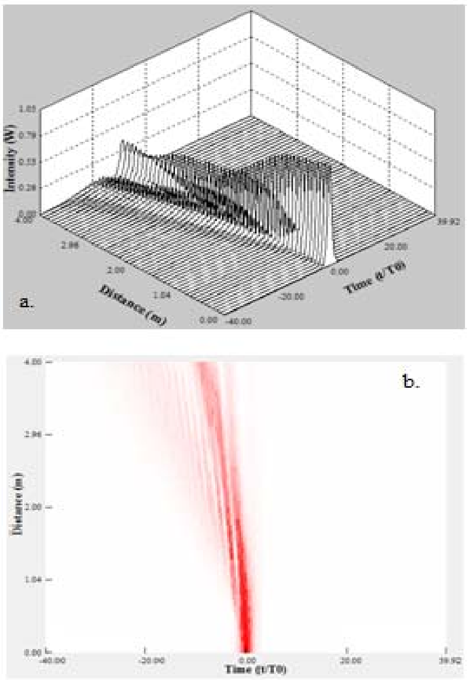


*Fig.10. Evolution of the soliton with third order dispersion effect: a. for $\beta_3 = -2\ s^2/m$, $\gamma = 5\ W^{-1}m^{-1}$; b. projection of time vs. distance axes for a. case;*

## 4. Conclusions

In system design it is very important to develop studies on optical solitons and their evolution, using different numerical methods. NLSESolver Program was used to simulate the soliton propagation in optical fibers and analyze second and third order dispersion effects.

Practical implementation of soliton has been a real challenge because of the higher bit rate and longer distances transmission demands.

Temporal solitons are very attractive for optical communications because they are capable to maintain their shape even in the presence of fiber dispersion. Solitons application in communication systems opens the way to high-speed transmission of information (for first order soliton). Their use in this type of systems is very useful but major changes in system design are necessary to be made.

Higher order soliton can break up into fundamental soliton, process that can be used in supercontinuum generation in photonic crystal fibers and in other novel applications, but the pump wavelength would be critical.

It was shown that in the case of second order dispersion effects, for higher order soliton, the break-up into multiple parts occur and the soliton losses its energy. This process is resulting from the interplay of diffraction, dispersion and nonlinear effects. We observed that for third order effects the soliton deviates from the initial position and the peak is delayed.

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