Study of transport phenomena in plasma by extended irreversible thermodynamics

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One of most intellectually interesting problems in plasma physics is the problem of turbulence and the associated transport of the plasma properties including density, temperature and momentum. The aim of this paper is to determine the transport coefficients in plasma particularly electrical and thermal conductivities by Extended Irreversible Thermodynamics (EIT). Transport coefficients are determined for different types of particles electrons and ions.

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1. Introduction

Understanding turbulent transport in magnetized plasma is a subject of utmost importance for comprehending and optimizing experiments in present fusion devices and for designing future reactors, too [1-2].

The fundamental properties of plasma are markedly dependent upon the interactions of the plasma particles with the force fields existing inside it. Processes related to the transport of mass, momentum, energy and charges in plasma are generally called transport phenomena. There exist general equations describing these different phenomena, and the special effects are characterized by coefficients generally called transport coefficients [3].

Understanding and controlling the rate at which particles and heat escape from the reactor chamber is critical to the successful design and operation of a magnetic fusion device. In the early days of the fusion program, estimates of particle and heat transport based on simple collisional diffusion were made. However, these estimates were found to drastically under-predict the observed in experiments and the large measured transport was labeled “anomalous”. Understanding this anomalous transport has been a primary goal of the fusion program ever since [4-5].

Recent descriptions of heat and particles transport in plasma have opened a promising field of application for Extended Irreversible Thermodynamics.

It has been shown that the so-called extended irreversible thermodynamics (EIT) attempts to cover those nonequilibrium situations which are supposed as not covered by local equilibrium assumption [6–8].

The basic features of this formalism and several applications are reviewed. Extended irreversible thermodynamics includes dissipative fluxes (heat flux, viscous pressure tensor, electric current) in the set of basic independent variables of the entropy [9].

This is achieved in EIT by enlarging the space of fundamental independent variables, such as the dissipative fluxes; say the heat flux density, dissipative stress tensor etc [10-11]

A new formulation of nonequilibrium thermodynamics is based on the postulate that the entropy density (S) is a function of both ordinary thermodynamic variables and certain additional variables (heat flux, particle flux, etc.) [12-14]

The purpose of this paper is to determine the parallel and the perpendicular transport coefficients in plasma. In the second section we write the fundamental hypotheses of EIT and the corresponding evolution equations for the fluxes. In the third section, we develop the transport equations of particle and heat fluxes and we determine the parallel and perpendicular transport coefficients for multispecies of plasma (electrons and ions).

In the last section we comment the result obtained from graphical representation.

2. Generalized Gibbs equation

As in classical irreversible thermodynamics (CIT), the entropy and the Gibbs equation play a central role in extended irreversible thermodynamics (EIT). Here, it is assumed that the entropy will not only depend on the classical variable, namely the specific internal energy u, but in addition on the dissipative flux, so the generalized Gibbs equation takes the form [15-17]

$$dS^a = \frac{dU^a}{T^a} - \frac{\mu_e^a}{T^a} de_a - \frac{1}{\rho_a T^a} \left[\left(\alpha_{11}^a \mathbf{q}^a + \alpha_{12}^a \mathbf{j}^a \right) . dq^a + \left(\alpha_{21}^a \mathbf{q}^a + \alpha_{22}^a \mathbf{j}^a \right) d\mathbf{j}^a \right] \quad (1)$$

Where U is the internal energy, μ_e the chemical potential, α_{ij} phenomenological coefficient, e_a total electric charge contributed by particle a, j^a particle flux, q^a heat flux and T_a is the absolute temperature of particle α.

The balance equations of total electric charge contributed by particle a and internal energy are given by:

$$\rho^a q_t e_a = -\nabla.\mathbf{j}^a \quad (2)$$

With ρ^a is the total mass density of particle a

$$\rho^a \partial_t U^a = -\nabla.\mathbf{q}^a + \mathbf{j}^a.E \quad (3)$$

Here E and j^a . E are respectively the electric field and the joule heating term.

In virtue of the balance equations (2) and (3) for U and e^a , one obtains for the entropy balance

$$\rho^a \partial_t S + \nabla.\left(\frac{1}{T_a}\mathbf{q}^a - \frac{\mu_e^a}{T_a}\mathbf{j}^a\right) = \mathbf{q}^a.\left(\nabla T_a^{-1} - \frac{\alpha_{11}^a}{T_a}\frac{d\mathbf{q}^a}{dt} - \frac{\alpha_{21}^a}{T_a}\frac{d\mathbf{j}^a}{dt}\right) + \quad (4)$$
$$\mathbf{j}^a.\left(\frac{\mathbf{E}^a}{T_a} - \nabla\frac{\mu_e^a}{T_a} - \frac{\alpha_{12}^a}{T_a}\frac{d\mathbf{q}^a}{dt} - \frac{\alpha_{22}^a}{T_a}\frac{d\mathbf{j}^a}{dt}\right)$$

This equation can be cast in the general form of a balance equation

$$\rho S = -\nabla.\mathbf{j}^a + \sigma^a \quad (5)$$

Where the quantity σ^a is the entropy production $\left(\sigma^a > 0\right)$ and j^a is the entropy flux.

The entropy production obeys

$$\sigma^a = \mu_{11}\mathbf{q}^a.\mathbf{q}^a + \mu_{12}\mathbf{j}^a.\mathbf{q}^a + \mu_{21}\mathbf{q}^a.\mathbf{j}^a + \mu_{22}\mathbf{j}^a.\mathbf{j}^a \quad (6)$$

The coefficient $\mu ij > 0$ is a consequence of the entropy production with σ^a is positive $\left(\sigma^a > 0\right)$.

Considering equations (4) and (5), we obtain the simplest evolution equations for $\mathbf{q}^a$ and $\mathbf{j}^a$ compatible with a definite positive entropy production, one assumes linear relations between the thermodynamic forces and the fluxes $\mathbf{q}^a$ and $\mathbf{j}^a$. This result in

$$\frac{\mathbf{E}^a}{T_a} - \nabla\frac{\mu_a^a}{T_a} - \frac{\alpha_{12}^a}{T_a}\frac{d\mathbf{q}^a}{dt} - \alpha_{22}^a\frac{d\mathbf{j}^a}{dt} = \mu_{21}^a\mathbf{q}^a + \mu_{22}^a\mathbf{j}^a \quad (7)$$

$$\nabla T_a^{-1} - \frac{\alpha_{11}^a}{T_a}\frac{d\mathbf{q}^a}{dt} - \frac{\alpha_{21}^a}{T_a}\frac{d\mathbf{j}^a}{dt} = \mu_{11}^a\mathbf{q}^a + \mu_{12}^a\mathbf{j}^a \quad (8)$$

Let us assume that ∇T_a^{-1} and $\frac{E^a}{T_a} - \nabla\frac{\mu_a^a}{T_a}$ vanish in system of equation (7) and (8), so that they refer to fluctuations near an equilibrium state. The equations (7) and (8) become:

$$-\frac{\alpha_{12}^a}{T_a}\frac{d\mathbf{q}^a}{dt} - \frac{\alpha_{22}^a}{T_a}\frac{d\mathbf{j}^a}{dt} = \mu_{21}^a\mathbf{q}^a + \mu_{22}^a\mathbf{j}^a \quad (9)$$

$$-\frac{\alpha_{11}^a}{T_a}\frac{d\mathbf{q}^a}{dt} - \frac{\alpha_{21}^a}{T_a}\frac{d\mathbf{j}^a}{dt} = \mu_{11}^a\mathbf{q}^a + \mu_{12}^a\mathbf{j}^a \quad (10)$$

After resolution of equations (9) and (10) we have

$$\begin{pmatrix}\partial_t\mathbf{q}^a \\ \partial_t\mathbf{j}^a\end{pmatrix} = -T\left(\left(\alpha^a\right)^T\right)^{-1}.\mu^a.\begin{pmatrix}\mathbf{q}^a \\ \mathbf{j}^a\end{pmatrix} \quad (11)$$

This is equivalent to

$$\partial_t\mathbf{j}^a = -T\left(\frac{\alpha_{11}\mu_{21} - \alpha_{12}\mu_{11}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}\right).\mathbf{q}^a - T\left(\frac{\alpha_{11}\mu_{22} - \alpha_{12}\mu_{12}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}\right).\mathbf{j}^a \quad (12)$$
$$= k_1.\mathbf{q}^a + k_2.\mathbf{j}^a$$

$$\partial_t\mathbf{q}^a = -T\left(\frac{\alpha_{22}\mu_{11} - \alpha_{21}\mu_{21}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}\right).\mathbf{q}^a - T\left(\frac{\alpha_{22}\mu_{12} - \alpha_{21}\mu_{22}}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}}\right).\mathbf{j}^a \quad (13)$$
$$= k_3.\mathbf{q}^a + k_4.\mathbf{j}^a$$

3. Parallel transport coefficients of Plasma

We Consider plasma where the particles (electrons (e) , ions (i), move parallel to the magnetic fields.

The generalized Gibbs equation (1) becomes:

$$dS = \sum_{a=e,i}\frac{dU^a}{T_a} - \sum_{a=e,i}\frac{\mu_e^a}{T_a}de_a - \sum_{a=e,i}\frac{1}{\rho_a T_a} \quad (14)$$
$$\left[\sum_{b=e,i}\left(\left(\alpha_{11}^{ab}\mathbf{q}^a + \alpha_{12}^{ab}\mathbf{j}^a\right).d\mathbf{q}^b + \left(\alpha_{21}^{ab}\mathbf{q}^a + \alpha_{22}^{ab}\mathbf{j}^a\right).d\mathbf{j}^b\right)\right]$$

And the evolutions equations (12) and (13) of the fluxes can be written

$$\partial_t\mathbf{j}^a = \sum_{b=e,i}\left(k_1^{ab}.\mathbf{q}^b + k_2^{ab}.\mathbf{j}^b\right) \quad (15)$$

$$\partial_t\mathbf{q}^a = \sum_{b=e,i}\left(k_3^{ab}.\mathbf{q}^b + k_4^{ab}.\mathbf{j}^b\right) \quad (16)$$

Considering equation (12) and (13) the coefficients k_1^{ab}, k_2^{ab}, k_3^{ab}, k_4^{ab} are given by

$$k_1^{ab} = -T_a\left(\frac{\alpha_{11}^{ab}\mu_{21}^b - \alpha_{12}^{ab}\mu_{11}^b}{\alpha_{11}^{ab}\alpha_{22}^{ab} - \alpha_{12}^{ab}\alpha_{21}^{ab}}\right)$$

$$k_2^{ab} = -T_a\left(\frac{\alpha_{11}^{ab}\mu_{22}^b - \alpha_{12}^{ab}\mu_{12}^b}{\alpha_{11}^{ab}\alpha_{22}^{ab} - \alpha_{12}^{ab}\alpha_{21}^{ab}}\right)$$

$$k_3^{ab} = -T_a\left(\frac{\alpha_{22}^{ab}\mu_{11}^b - \alpha_{21}^{ab}\mu_{21}^b}{\alpha_{11}^{ab}\alpha_{22}^{ab} - \alpha_{12}^{ab}\alpha_{21}^{ab}}\right)$$

$$k_4^{ab} = -T_a\left(\frac{\alpha_{21}^{ab}\mu_{22}^b - \alpha_{22}^{ab}\mu_{12}^b}{\alpha_{11}^{ab}\alpha_{22}^{ab} - \alpha_{12}^{ab}\alpha_{21}^{ab}}\right)$$

The coefficients μ_{ij}^b are given by

$$\mu_{11}^b = \frac{1}{\lambda^b T_b^2} \quad , \quad \mu_{11}^b = \frac{-\left(\Pi^b + \mu^b\right)}{\lambda^b T_b^2}$$

$$\mu_{11}^b = \frac{\varepsilon T^b - \mu^t}{\lambda^b T_b^2} \quad , \quad \mu_{22}^b = \frac{\left(\Pi^b + \mu_e\right)\left(\mu_e - T^b\right)}{\lambda^b T_b^2} + \frac{1}{\sigma^b T_b^2}$$

With

$$\lambda^b = \frac{5 n_b k_b^2 T_b}{2 m_b}\tau^b \quad , \sigma^b = \frac{n_b e_b^2 \tau^b}{m_b}$$

$$\Pi^b = \frac{\pi^2 k_B T_b^2}{2 e_b^2 \varepsilon_F^b} \quad , \varepsilon^b = \frac{\pi^2 \tau_b k_B^2}{2 e_b^2 \varepsilon_F^b}$$

$$\mu^b = k_B T_b \log\left(\frac{1}{n_b}\left(\frac{2\pi k_B T_b}{m_b}\right)^{\frac{3}{2}}\right)$$

Where τ_b n_b and k_B are respectively the relaxation time of particle b, the particle b density and the Boltzmann constant.

Determination of the coefficients α_{ij}^{ab} [18]

$$\alpha_{11}^{ab} = \frac{k_B}{\langle \delta \mathbf{q}^a \delta \mathbf{q}^b \rangle} \quad ; \quad \alpha_{12}^{ab} = \frac{k_B}{\langle \delta \mathbf{q}^a \delta \mathbf{j}^b \rangle}$$
$$\alpha_{21}^{ab} = \frac{k_B}{\langle \delta \mathbf{j}^a \delta \mathbf{q}^b \rangle} \quad ; \quad \alpha_{22}^{ab} = \frac{k_B}{\langle \delta \mathbf{j}^a \delta \mathbf{j}^b \rangle} \tag{17}$$

With $\partial \mathbf{q}^a$ is the fluctuation of the heat flux, $\partial \mathbf{j}^a$ is the fluctuation of the particle flux.

Determination of α_{ij}^{ab} :

The fluctuations of the heat flux are given

$$\delta \mathbf{q}^a = \int \left(\frac{1}{2} m C^2 - \frac{5}{2} k_B T\right) C \delta f^a \tag{18}$$

Using this expression, frequently called the subtracted heat flux, to compute the second moments of the fluctuations, one finds that:

$$\langle \delta \mathbf{q}^a \delta \mathbf{q}^b \rangle = \int dc \int dc' \left(\frac{1}{2} m_a C^2 - \frac{5}{2} k_B T_a\right) C \left(\frac{1}{2} m_b C'^2 - \frac{5}{2} k_B T_b\right) C' \langle \delta f^a (C) \delta f^b (C') \rangle \tag{19}$$

$$\langle \delta \mathbf{j}^a \delta \mathbf{q}^b \rangle = \int dc \int dc' (\frac{1}{2} C \left(\frac{1}{2} m_b C'^2 - \frac{5}{2} k_B T_b\right) C C' \langle \delta f^a (C) \delta f^b (C') \rangle \tag{20}$$

$$\langle \delta \mathbf{q}^a \delta \mathbf{j}^b \rangle = e_b \int dc \int dc' \left(\frac{1}{2} C \left(\frac{1}{2} m_b C'^2 \frac{5}{2} K_b T_b\right) C C'\right) \langle \delta f^a (C) \delta f^b (C') \rangle \tag{21}$$

$$\langle \delta \mathbf{q}^a \delta \mathbf{j}^b \rangle = e_a e_b \int dc \int dc' C C' \langle \delta f^a (C) \delta f^b (C') \rangle \tag{22}$$

Where $C = c - v$ the particle velocity relative to the mean motion, with c is the velocity of a particle, v is the mean velocity.

$$\langle \delta f^a (C) \delta f^b (C') \rangle = \frac{1}{v} f_{eq}(C) \delta (C - C) \tag{23}$$

Where V is the volume of the system

The local equilibrium Maxwell-Boltzmann distribution function f_{eq}

$$f_{eq} = n\left(\frac{m}{2\pi K_b T}\right)^{3/2} \exp\left(-\frac{mC^2}{2 K_B T}\right)$$

After calculations, we found that

$$\alpha_{11}^{ab} = \frac{\left(48 V \pi^3 K_B^{1/2}\right)\left(5 n_a n_b T_a T_b \sqrt{\pi}\right)}{\left[\frac{3\beta_b^5 / \beta_a^2}{\left(1+\left(\frac{\beta_b}{\beta_a}\right)^2\right)^{\frac{7}{2}}} - \frac{3\beta_b^3}{\left(1+\left(\frac{\beta_b}{\beta_a}\right)^2\right)^{\frac{5}{2}}} - \frac{3\beta_b^5 / \beta_a^2}{\left(1+\left(\frac{\beta_b}{\beta_a}\right)^2\right)^{\frac{5}{2}}} + \frac{5\beta_b^3}{\left(1+\left(\frac{\beta_b}{\beta_a}\right)^2\right)^{\frac{3}{2}}}\right]}$$

$$\alpha_{12}^{ab} = \frac{24 V \pi^3 K_B^{3/2}}{\left(e_b n_a n_b T_a \sqrt{\pi} \beta_b^3\right)\left[\frac{3}{\left(1+\left(\frac{\beta_b}{\beta_a}\right)^2\right)^{\frac{5}{2}}} - \frac{5}{\left(1+\left(\frac{\beta_b}{\beta_a}\right)^2\right)^{\frac{3}{2}}}\right]}$$

$$\alpha_{22}^{ab} = \frac{24 V \pi^3 K_B^{3/2}}{\left(e_a n_a n_b T_b \sqrt{\pi} \beta_a^3\right)\left[\frac{3}{\left(1+\left(\frac{\beta_a}{\beta_b}\right)^2\right)^{\frac{5}{2}}} - \frac{5}{\left(1+\left(\frac{\beta_a}{\beta_b}\right)^2\right)^{\frac{3}{2}}}\right]}$$

$$\alpha_{22}^{ab} = \frac{12 V K_B}{\left(e_a n_b n_a T_b \sqrt{\pi} \beta_a^3\right)}\left(\frac{2\pi^2 T_b}{m_b}\right)^{\frac{3}{2}}\left(1 + \left(\frac{\beta_a}{\beta_b}\right)^2\right)^{\frac{3}{2}}$$

With: $\beta_a = \left(\frac{m_a}{2T_a}\right)^{1/2}$ and $\beta_b = \left(\frac{m_b}{2T_b}\right)^{1/2}$

Here m_a, n_a, V and e_a, are respectively the mass, the density of particle a, volume of the system and the charge of particle a .

The state of the plasma after a short transition time remains close to the local plasma equilibrium. For this reason, the local plasma equilibrium will be a reference state. The distribution function can the conveniently be written in the form: [17]

$$f^a = f_0^a + f_1^a \tag{24}$$

With f_1^a is a deviation of distribution function, and then can be expanded in a series of irreducible Hermite polynomial as

$$f_1^a = \sum_{n=0} h^{a(2n+1)} H_r^{(2n+1)}(v) f_0^a(v) \quad (25)$$

With $h^{a(2n+1)}$ the hermitien moment and $H_r^{(2n+1)}$ the irreducible Hermite polynomials

We limited in the 13 Moment approximation (n=1) so

$$f_1^a = \left(h_r^{a(1)} H_r^{a(1)} + h_r^{a(3)} H_r^{a(3)}\right) f_0^a \quad (26)$$

With $H_r^{a(1)} = \sqrt{2}\beta_a C$

And $H_r^{a(a)} = \sqrt{\frac{1}{5}}\beta_a C\left[2(\beta_a C)^2 - 5\right]$

We derive a relation between the heat flux and the hermitien moments in order to determine the dimensionless equations of fluxes

$$h_r^{a(1)} = \frac{1}{n_a}\left(\frac{m_a}{T_a}\right)^{\frac{1}{2}} j_r^a \quad (27)$$

$$h_r^{a(1)} = \sqrt{\frac{2}{5}}\frac{1}{n_a T_a}\left(\frac{m_a}{T_a}\right)^{\frac{1}{2}} q_r^a \quad (28)$$

Where $h_r^{a(1)}$ and $h_r^{a(3)}$ are respectively the dimensionless particle flux and the dimensionless heat flux of particle a ($a = e, i, I$).

The evolution equations of the fluxes **j** and **q** have the forms

$$\partial_t j^a = \partial_t\left(e_a \int C f^a dC\right) \quad (29)$$

$$\partial_t q^a = \partial_t\left(\int\left(\frac{1}{2}m_a C^2 - \frac{5}{2}K_B T\right) C f^a dC\right) \quad (30)$$

By considering the equations (26) and (27), the equations (28) and (29) can be written

$$\tau^a \partial_t h_r^{a(1)} + h_r^{a(1)} = G_r^{a(1)} + \frac{e_a \tau^a}{m_a c}\varepsilon_{rmn} h_r^{a(1)} B \quad (31)$$

$$\tau^a \partial_t h_r^{a(a)} + h_r^{a(1)} = G_r^{a(a)} + \frac{e_a \tau^a}{m_a c}\varepsilon_{rmn} h_r^{a(a)} B \quad (32)$$

Where τ^a the relaxation time of particlea, B the magnetic field.

With $G_r^{a(a)}$ and $G_r^{a(3)}$ are the source term related to the thermodynamic forces by relation

$$G_r^{a(1)} = \tau_a \frac{1}{n_a}\left(\frac{m_a}{T_a}\right)^{1/2}\left(\frac{1}{m_a}\nabla_r(n_a T_a) + \frac{e_a n_a}{m_a}\mathbf{E}_r\right) \quad (33)$$

$$G_r^{a(a)} = -\tau_a \sqrt{\frac{5}{2}}\left(\frac{m_a}{T_a}\right)^{1/2}\frac{1}{m_a}\nabla_r(T_a) \quad (34)$$

Considering equations (12) and (13) the evolution equations of particle and heat flux has the form

$$\partial_t \mathbf{j}^a = \sum_{b=e,i}\left(k_1^{ab}.\mathbf{q}^b + k_2^{ab}\mathbf{j}^b\right) \quad (35)$$

$$\partial_t \mathbf{q}^a = \sum_{b=e,i}\left(k_1^{ab}.\mathbf{q}^b + k_4^{ab}\mathbf{j}^b\right) \quad (36)$$

So the dimensionless equations of particle and heat fluxes in parallel direction become:

$$\partial_t h_{//}^{b(1)} = \sum_{b=e,i}\left(k_1^{'ab} h_{//}^{b(3)} + k_4^{ab} h_{//}^{b(1)}\right) \quad (37)$$

$$\partial_t h_{//}^{a(3)} = \sum_{b=e,i}\left(k_1^{'ab} h_{//}^{b(3)} + k_4^{'ab} h_{//}^{b(1)}\right) \quad (38)$$

With:

$$k_1^{'ab} = \sqrt{\frac{5}{2}}\frac{n_b}{n_a}T_b\left(\frac{m_a T_b}{m_b T_a}\right)^{\frac{1}{2}} k_1^{ab}$$

$$k_2^{'ab} = \frac{n_b}{n_a}\left(\frac{m_a T_b}{m_b T_a}\right)^{\frac{1}{2}} k_2^{ab}$$

$$k_3^{'ab} = \sqrt{\frac{5}{2}}\frac{n_b}{n_a}T_b\left(\frac{m_a T_b}{m_b T_a}\right)^{\frac{1}{2}} k_3^{ab}$$

$$k_4^{'ab} = \frac{n_b}{n_a}\left(\frac{m_a T_b}{m_b T_a}\right)^{\frac{1}{2}} k_4^{ab}$$

The dimensionless evolution equation (31) of particle flux in parallel direction became

$$\tau^a \partial_t h_{//}^{a(1)} + h_{//}^{a(1)} = G_{//}^{a(1)} \quad (39)$$

Where the term of magnetic field vanish

For electrons (e), the equation (39) become

$$\tau^e \partial_t h_{//}^{e(1)} + h_{//}^{e(1)} = G_{//}^{e(1)} \quad (40)$$

The expression of electron relaxation time τ^e is [19]

$$\tau^e = \frac{3 m_e^{1/2} T_e^{3/2}}{4\sqrt{2\pi} Z^2 e^4 n_i \ln\Lambda} \quad (41)$$

Where $\ln\Lambda = \ln\frac{3/2\,(T_e + T_i)\lambda_D}{Ze^2}$ the coulomb logarithm, and λ_D is the Debye length.

We replace $\tau^e \partial_t h_r^{e(1)}$ in equation (40) by its expression from equation (37), we find:

$$\tau^e \sum_{b=e,i} (k_1^{'ab} h_{//}^{b(3)} + k_2^{'ab} h_{//}^{b(1)}) + h_{//}^{e(1)} = G_{//}^{e(1)} \quad (42)$$

This is equivalent to:

$$\tau^e k_1^{'ee} h_{//}^{e(3)} + \tau^e k_2^{'ee} h_{//}^{e(1)} + \tau^e k_1^{'ei} h_{//}^{i(3)} + \tau^e k_2^{'ei} h_{//}^{i(1)} + h_{//}^{e(1)} = G_{//}^{e(1)} \quad (43)$$

Considering Fourier transformed

$$\partial_t h_{//}^{b(1)} = i\omega h_{//}^{b(1)}$$

$$\partial_t h_{//}^{b(3)} = i\omega h_{//}^{b(3)}$$

We place in the asymptotic limit where we neglect $\partial_t h_r$, with we take $\omega = 0$, the evolutions equations (43) reduces to

$$h_{//}^{e(1)} = L_{11}^{ee} G_{//}^{e(1)} + L_{13}^{ee} G_{//}^{e(3)} + L_{11}^{ei} G_{//}^{i(1)} + L_{13}^{ei} G_{//}^{i(3)} \quad (44)$$

Similar for ions (i) we determine $h_{//}^{i(1)}$

$$h_{//}^{i(1)} = L_{11}^{ie} G_{//}^{e(1)} + L_{13}^{ie} G_{//}^{e(3)} + L_{11}^{ii} G_{//}^{i(1)} + L_{13}^{ii} G_{//}^{i(3)} \quad (45)$$

The L_{ij}^{aa} coefficients with (a=e,i and i=1 j=1,3) are the pure transport coefficients and L_{ij}^{ab} (a, b= e, i) with (a ≠ b)) the mixed transport coefficients.

Similar of particle flux we develop now the dimensionless evolution equation of heat flux of multispecies of plasma so we have

$$h_{//}^{e(3)} = L_{31}^{ee} G_{//}^{e(1)} + L_{33}^{ee} G_{//}^{e(3)} + L_{31}^{ei} G_{//}^{i(1)} + L_{33}^{ei} G_{//}^{i(3)} \quad (46)$$

$$h_{//}^{i(3)} = L_{31}^{ie} G_{//}^{e(1)} + L_{33}^{ie} G_{//}^{e(3)} + L_{31}^{ii} G_{//}^{i(1)} + L_{33}^{ii} G_{//}^{i(3)} \quad (47)$$

Identification of the transport coefficients

The transport matrix L_{ij}^{ab} (whose coefficients are the transport coefficients) has the characteristic Onsager symmetry, which reduces here to the simple matrix symmetry $L_{ij}^{ab} = L_{ji}^{ab}$ so

The pure transport coefficients have the form

$$L_{11}^{ee} = \frac{1}{1+\tau^e k_2^{'ee}} \quad , \quad L_{13}^{ee} = -\frac{\tau^e k_1^{'ee}}{1+\tau^e k_2^{'ee}}$$

$$L_{33}^{ii} = -\frac{1}{1+\tau^i k_2^{'ii}} \quad , \quad L_{13}^{ii} = -\frac{\tau^i k_1^{'ii}}{1+\tau^i k_2^{'ii}} ,$$

$$L_{33}^{ee} = -\frac{1}{1+\tau^e k_2^{'ee}}$$

With the expression of ion relaxation time is

$$\tau^i = \frac{3 m_i^{1/2} T_i^{3/2}}{4\sqrt{2\pi} Z^2 e^4 n_i \ln \Lambda} \quad (48)$$

Where m_i, Z_i and T_i are respectively the mass of ion, the charge and the ion temperature.

The mixed transport coefficients

$$L_{11}^{ei} = -\frac{\tau^e k_2^{'ei}}{1+\tau^e k_2^{'ee}} \quad , \quad L_{31}^{ei} = -\frac{\tau^e k_4^{'ei}}{1+\tau^e k_3^{'ee}}$$

$$L_{33}^{ei} = -\frac{\tau^e k_3^{'ei}}{1+\tau^e k_3^{'ee}}$$

4. The perpendicular transport coefficients of plasma

In this paragraph we study the transport phenomena in presence of a constant magnetic field (**B**) where the particles (electrons (e), ions (i)), move perpendicular to the magnetic fields.

B is parallel to the Z axis so we project the dimensionless evolution equations in the x and y direction

For particle fluxes

We determine now the dimensionless evolution equation of particle flux of multispecies of plasma so we have

$$h_x^{e(1)} = L_{11}^{ee} G_x^{e(1)} + L_{13}^{ee} G_x^{e(3)} + L_{11}^{ei} G_x^{i(1)} + L_{13}^{ei} G_x^{i(3)} + \left(\frac{-eB\tau_e}{m_e c}\right) h_y^{e(1)} \quad (49)$$

$$h_y^{e(1)} = L_{11}^{ee} G_y^{e(1)} + L_{13}^{ee} G_y^{e(3)} + L_{11}^{ei} G_y^{i(1)} + L_{13}^{ei} G_y^{i(3)} + \left(\frac{-eB\tau_e}{m_e c}\right) h_x^{e(1)} \quad (50)$$

We suppose

$$x_a = \frac{e_a B \tau_a}{m_a c} \quad , \quad x_a = \Omega_a \tau_a \qquad \text{With } \Omega_a = \frac{e_a B}{m_a c}$$

is the Larmor frequency of species a, a=(e,i) and B is the magnetic field

After introducing (50) in (49) we use

For electrons

$$h_x^{e(1)} = \frac{L_{11}^{ee}}{1+x_e^2} G_x^{e(1)} + \frac{L_{13}^{ee}}{1+x_e^2} G_x^{e(3)} + \frac{L_{11}^{ei}}{1+x_e^2} G_x^{i(1)} + \frac{L_{13}^{ei}}{1+x_e^2} G_x^{i(3)} + x_e \frac{L_{11}^{ee}}{1+x_e^2} G_y^{e(1)} + x_e \frac{L_{13}^{ee}}{1+x_e^2} G_y^{e(3)} + x_e \frac{L_{11}^{ei}}{1+x_e^2} G_y^{i(1)} + x_e \frac{L_{13}^{ei}}{1+x_e^2} G_y^{i(3)} \quad (51)$$

$$h_y^{e(1)} = \frac{L_{11}^{ee}}{1+x_e^2} G_y^{e(1)} + \frac{L_{13}^{ee}}{1+x_e^2} G_y^{e(3)} + \frac{L_{11}^{ei}}{1+x_e^2} G_y^{i(1)} + \frac{L_{13}^{ei}}{1+x_e^2} G_y^{i(3)} - x_e \frac{L_{11}^{ee}}{1+x_e^2} G_x^{e(1)} - x_e \frac{L_{13}^{ee}}{1+x_e^2} G_x^{e(3)} - x_e \frac{L_{11}^{ei}}{1+x_e^2} G_x^{i(1)} - x_e \frac{L_{13}^{ei}}{1+x_e^2} G_x^{i(3)} \quad (52)$$

For ions

$$h_x^{i(1)} = \frac{L_{11}^{ie}}{1+x_i^2} G_x^{e(1)} + \frac{L_{13}^{ie}}{1+x_i^2} G_x^{e(3)} + \frac{L_{11}^{ii}}{1+x_i^2} G_x^{i(1)} + \frac{L_{13}^{ii}}{1+x_i^2} G_x^{i(3)} + x_i \frac{L_{11}^{ie}}{1+x_i^2} G_y^{e(1)} + x_i \frac{L_{13}^{ie}}{1+x_i^2} G_y^{e(3)} + x_i \frac{L_{11}^{ii}}{1+x_i^2} G_y^{i(1)} + x_i \frac{L_{13}^{ii}}{1+x_i^2} G_y^{i(3)} \quad (53)$$

$$h_y^{i(1)} = \frac{L_{11}^{ie}}{1+x_i^2} G_y^{e(1)} + \frac{L_{13}^{ie}}{1+x_i^2} G_y^{e(3)} + \frac{L_{11}^{ii}}{1+x_i^2} G_y^{i(1)} + \frac{L_{13}^{ii}}{1+x_i^2} G_y^{i(3)} - x_i \frac{L_{11}^{ie}}{1+x_i^2} G_x^{e(1)} - x_i \frac{L_{13}^{ie}}{1+x_i^2} G_x^{e(3)} - x_i \frac{L_{11}^{ii}}{1+x_i^2} G_x^{i(1)} - x_i \frac{L_{13}^{ii}}{1+x_i^2} G_x^{i(3)} \quad (54)$$

For heat fluxes

Similar of particle flux we develop now the dimensionless evolution equation of heat flux of multispecies of plasma so we have

$$h_x^{e(3)} = L_{31}^{ee} G_x^{e(1)} + L_{33}^{ee} G_x^{e(3)} + L_{31}^{ei} G_x^{i(1)} + L_{33}^{ei} G_x^{i(3)} + \left(\frac{-eB\tau_e}{m_e c}\right) h_y^{e(3)} \quad (55)$$

$$h_y^{e(3)} = L_{31}^{ee} G_y^{e(1)} + L_{33}^{ee} G_y^{e(3)} + L_{31}^{ei} G_y^{i(1)} + L_{33}^{ei} G_y^{i(3)} + \left(\frac{-eB\tau_e}{m_e c}\right) h_x^{e(3)} \quad (56)$$

After introducing (56) in (55) we use

For electrons

$$h_x^{e(3)} = \frac{L_{31}^{ee}}{1+x_e^2} G_x^{e(1)} + \frac{L_{33}^{ee}}{1+x_e^2} G_x^{e(3)} + \frac{L_{31}^{ei}}{1+x_e^2} G_x^{i(1)} + \frac{L_{33}^{ei}}{1+x_e^2} G_x^{i(3)} + x_e \frac{L_{31}^{ee}}{1+x_e^2} G_y^{e(1)} + x_e \frac{L_{33}^{ee}}{1+x_e^2} G_y^{e(3)} + x_e \frac{L_{31}^{ei}}{1+x_e^2} G_y^{i(1)} + x_e \frac{L_{33}^{ei}}{1+x_e^2} G_y^{i(3)} \quad (57)$$

$$h_y^{e(3)} = \frac{L_{31}^{ee}}{1+x_e^2} G_y^{e(1)} + \frac{L_{33}^{ee}}{1+x_e^2} G_y^{e(3)} + \frac{L_{31}^{ei}}{1+x_e^2} G_y^{i(1)} + \frac{L_{33}^{ei}}{1+x_e^2} G_y^{i(3)} - x_e \frac{L_{31}^{ee}}{1+x_e^2} G_x^{e(1)} - x_e \frac{L_{33}^{ee}}{1+x_e^2} G_x^{e(3)} - x_e \frac{L_{31}^{ei}}{1+x_e^2} G_x^{i(1)} - x_e \frac{L_{33}^{ei}}{1+x_e^2} G_x^{i(3)} \quad (58)$$

For ions

$$h_x^{i(3)} = \frac{L_{31}^{ie}}{1+x_i^2} G_x^{e(1)} + \frac{L_{33}^{ie}}{1+x_i^2} G_x^{e(3)} + \frac{L_{31}^{ii}}{1+x_i^2} G_x^{i(1)} + \frac{L_{33}^{ii}}{1+x_i^2} G_x^{i(3)} + x_i \frac{L_{31}^{ie}}{1+x_i^2} G_y^{e(1)} + x_i \frac{L_{33}^{ie}}{1+x_i^2} G_y^{e(3)} + x_i \frac{L_{31}^{ii}}{1+x_i^2} G_y^{i(1)} + x_i \frac{L_{33}^{ii}}{1+x_i^2} G_y^{i(3)} \quad (59)$$

$$h_y^{i(3)} = \frac{L_{31}^{ie}}{1+x_i^2} G_y^{e(1)} + \frac{L_{33}^{ie}}{1+x_i^2} G_y^{e(3)} + \frac{L_{31}^{ii}}{1+x_i^2} G_y^{i(1)} + \frac{L_{33}^{ii}}{1+x_i^2} G_y^{i(3)} - x_i \frac{L_{31}^{ie}}{1+x_i^2} G_x^{e(1)} - x_i \frac{L_{33}^{ie}}{1+x_i^2} G_x^{e(3)} - x_i \frac{L_{31}^{ii}}{1+x_i^2} G_x^{i(1)} - x_i \frac{L_{33}^{ii}}{1+x_i^2} G_x^{i(3)} \quad (60)$$

5. Result and discussion

In Figs. 1-4 we plotted the perpendicular electrical conductivity, the thermoelectric conductivity, and electron and ion thermal conductivities as function of x_s $(s = e, i)$.

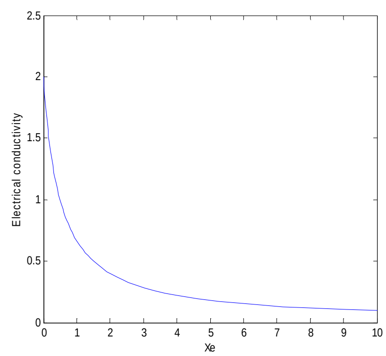

Fig. 1. Perpendicular electrical conductivity as function of x_e.

This plot show that the electrical conductivity decreases with increasing of x_e

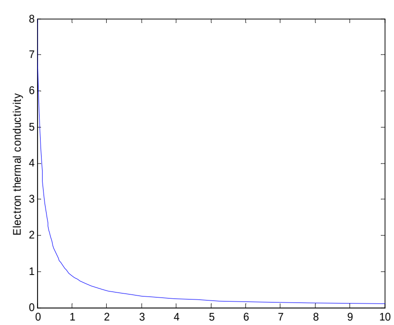

Fig. 2. Perpendicular electron thermal conductivity as function of x_e.

We notice that the perpendicular electron thermal conductivity decreases with increasing of x_e.

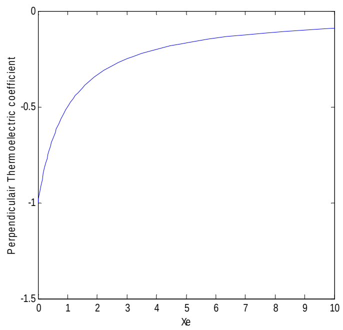

Fig. 3. Perpendicular thermoelectric conductivity as function of x_e.

We notice that the perpendicular thermoelectric conductivity increases with increasing of x_e.

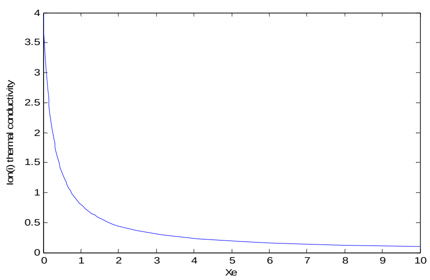

Fig. 4. Perpendicular ions thermal conductivity as function of x_i.

In this curve we plotted the perpendicular ions thermal conductivity as function x_i of. This plot shows that the perpendicular ions thermal conductivity decreases with increasing of x_i

The perpendicular transport coefficients are monotonously decreasing function of x_a . So for a very strong magnetic field, the particles would stick to the field lines, and there would be no transport in any direction perpendicular to (**B**). This situation is opposed by the collisions the latter make the particles jump from one field line to another, thus making a perpendicular transport possible. In the perpendicular direction the collisions favor the transport thus for large values of the parameter $x_a = \Omega_a \tau_a$ i.e. for large magnetic field the coefficients are decreasing functions of x_a. This situation is clearly illustrated in Figs. 1-4.

6. Conclusion

In this paper we have been interested to calculate by Extended Irreversible Thermodynamics the transport coefficients like electrical and thermal conductivities after developing transport equations of dissipative fluxes like particles and heat fluxes of multispecies plasma (electron and ions).

In absence of magnetic field all the transport coefficient are proportional to the relaxation time this important characteristic can be easily understood. It implies that as the collision frequency increases the transport coefficients decrease in order words, the collisions tend to oppose the transport of matter and energy; they act as an obstacle to the free flow of these quantities. This result is in perfect agreement with kinetic theory result [20-21].

The asymptotic perpendicular transport coefficients are proportional to the collision frequency this property vividly illustrates that the collisions oppose the parallel transport but favor the perpendicular one. It may be said that in plasma in presence of a constant magnetic field, when the collision frequency is increased

And the equality such coefficients, which has been obtained here from purely EIT, is supported by kinetic theory.

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