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Surface carriers' concentration dynamics caused by a small alternating applied voltage

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One of our previous papers was devoted to threshold voltage in MOSFETs and MODFETs viewed as a problem of nonlinear dynamics. The behavior of surface carriers' concentration under D.C. (direct current) applied voltage has been investigated in details. In this paper we went a step further and investigated the behavior of the same quantity under combined D.C. and A.C. (alternating current). As a main result emerged that it was impossible to cause small harmonic oscillations of surface carriers' concentration around some equilibrium value regardless of applied D.C. voltage and thus imposed operating regime.

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1. Introduction

In each text-book concerning linear electronics, the following fact is accepted and afterwards used without reluctance [1, 2]. By application of D.C. voltage one imposes the operating conditions (the mean values of relevant physical quantities – currents, voltages, concentrations etc.). We were also taught that around arbitrary value of this D.C. voltage it was always possible to impose time dependent A.C. voltage of much smaller magnitude and so far cause the non-vanishing harmonic oscillations of other investigated quantities regardless of dynamical properties of exploited devices [3]. This is the so called static picture. The aim of the effort to be presented in this paper is to investigate the validity of this belief, i.e. to find out if dynamical properties of our devices can play any significant role relevant to this problem [1]. The standard combination of D.C. and A.C. voltage [2]:

$$V_g(t) = (V_0 + V_m \cos wt) \cdot h(t); \quad V_m << V_0 \quad (1)$$

used in linear electronics has been applied to nonlinear dynamical models developed in the previous paper [1]. The results are exposed in subsequent sections.

2. The analysis of MOSFET dynamics

If the model that described MOSFET (Metal Oxide Semiconductor Field Effect Transistor) dynamics developed in [1] was supposed to be valid even if voltage (1) was applied, the corresponding equation could be written as followed [1]:

$$\begin{aligned}\dot{n}_s &= \lambda^2 \left[(V_0 + V_m \cos wt) - V_T \right] \cdot n_s - \beta^2 n_s^{\,2} = \\ &= \lambda^2 \left[(V_0 - V_T) + V_m \cos wt \right] \cdot n_s - \beta^2 n_s^{\,2}\end{aligned} ; \quad t>0 \quad (2)$$

together with the initial condition $n_s(0)=k>0$; λ^2, β^2 denoted positive constants described in [1] and V_T was the extrapolated threshold voltage. The equation (2) is recognized as Bernoulli's differential equation and can be solved analytically [4]:

$$n_s(t) = \frac{e^{\lambda^2 \left[(V_0 - V_T) \cdot t + \frac{V_m}{w} \sin wt \right]}}{\frac{1}{k} + \beta^2 \int_0^t e^{\lambda^2 \left[(V_0 - V_T) \cdot \tau + \frac{V_m}{w} \cdot \sin w\tau \right]} d\tau} \quad (3)$$

The integral in the denominator of equation (3) can't be evaluated in a closed form, therefore perturbative expansion must be used; the first correction term will imply [5]:

$$n_s(t) = \frac{e^{\lambda^2 (V_0 - V_T) t} \cdot \left(1 + \frac{V_m}{w} \sin wt \right)}{\frac{1}{k} + \beta^2 \int_0^t e^{\lambda^2 (V_0 - V_T) \cdot \tau} \cdot \left(1 + \frac{V_m}{w} \sin w\tau \right) d\tau} \quad (4)$$

what finally gives:

$$n_s(t)=\frac{e^{\lambda^2(V_0-V_T)\cdot t}\cdot\left(1+\frac{V_m}{w}\sin wt\right)}{\frac{1}{k}+\beta^2\frac{e^{\lambda^2(V_0-V_T)}-1}{\lambda^2(V_0-V_T)}+\beta^2\frac{\lambda^2V_m}{w}\cdot F}\quad(5)$$

where F is:

$$F=\frac{\lambda^2(V_0-V_T)\sin wt-w\cos wt}{\left[\lambda^2(V_0-V_T)\right]^2+w^2}\cdot\left(e^{\lambda^2(V_0-V_T)}+w\right)\quad(5a)$$

The analysis of relation (5) appears to be very interesting:

- if $(V_0\text{-}V_T)<0$ the asymptotic solution $n_{as}(t)$ for $t\to\infty$ tends to zero exponentially, i.e. $n_s(t)$ vanishes and no harmonic oscillations appear.
- if $(V_0\text{-}V_T)>0$ the influence of initial condition disappears and the nonvanishing asymptotic solution can be written as follows:

$$\underset{t\to\infty}{n_s(t)}\cong\frac{\lambda^2(V_0-V_T)}{\beta^2}\left\{1+\lambda^2V_m\frac{\lambda^2(V_0-V_T)\cos wt+w\sin wt}{\left[\lambda^2(V_0-V_T)\right]^2+w^2}\right\}\quad(6)$$

This nonvanishing solution obviously has two terms: the first one is recognized as D.C. component already obtained in [1] and the second one is A.C. component that oscillates round the first term (it also exibits a phase shift in comparison to the starting A.C. voltage $V_m\cos wt$):

$$n_{s0}=\frac{\lambda^2(V_0-V_T)}{\beta^2}\quad(7a)$$

$$\delta n_s(t)=\frac{\lambda^2(V_0-V_T)}{\beta^2}\cdot\frac{\lambda^2V_m}{\sqrt{\left[\lambda^2(V_0-V_T)\right]^2+w^2}}\cdot\cos(wt+\eta)\quad(7b)$$

The condition $V_m<<V_0$ provides that $|\delta n_s(t)|<<n_{s0}$ and $\delta n_s(t)$ can therefore be understood as a first order correction. The same result can be obtained without solving the exact differential equation, but exploiting the expansion method from the very beginning [5]. This procedure has appeared convenient for the generalization of the problem and understanding its solution. The equation (2) can be rewritten in the following form:

$$\dot{n}_s=\left[\lambda^2(V_0-V_T)\cdot n_s-\beta^2n_s^{\,2}\right]+\lambda^2n_sV_m\cos wt=f(n_s)+g(n_s)\cdot V_m\cos wt\quad(8)$$

with explicitly separated stationary and perturbative terms. The intention is to linearize equation (8) in the vicinity of different fixed points n_s* and in each case search for the first order correction $\delta n_s(t)$:

$$n_s(t)=n_s*+\delta n_s(t)\quad(9)$$

Exploiting Taylor's expansion equation (8) turns into:

$$\frac{d}{dt}\delta n_s=\sum_{k=0}^{+\infty}f^{(k)}(n_s*)\cdot\frac{(\delta n_s)^k}{k!}+\left(\sum_{k=0}^{+\infty}g^{(k)}(n_s*)\frac{(\delta n_s)^k}{k!}\right)\cdot V_m\cos wt\quad(10a)$$

what, with the second order correction ($(\delta n_s)^2$, $\delta n_s\cdot V_m$,…) neglected, gives:

$$\frac{d}{dt}\delta n_s\cong f(n_s*)+f'(n_s*)\cdot\delta n_s+g(n_s*)V_m\cos wt\quad(10b)$$

Assuming only the first order correction it has already been supposed that δn_s is proportional to V_m (which itself is small enough).

- The analysis of the fixed point $n_s*=0$ $(V_0<V_T)$ implies:

$$f(n_s*)=0,\quad g(n_s*)=0,\quad f'(n_s*)=\lambda^2(V_0-V_T)\quad(11a)$$

and the equation (10b), together with its solution, reads:

$$\frac{d}{dt}\delta n_s=\lambda^2(V_0-V_T)\cdot\delta n_s,\quad \delta n_s(0)\neq0$$

$$\delta n_s(t)=\delta n_s(0)\cdot e^{\lambda^2(V_0-V_T)\cdot t}\quad(11b)$$

Obviously, the allowed D.C. voltage causes that the first order correction is limited to the vanishing solution, i.e. no harmonic oscillation is possible.

- The situation near the fixed point $n_s*=\frac{\lambda^2(V_0-V_T)}{\beta^2}$ is quite different $(V_0>V_T)$; the equation (10b) then becomes:

$$f(n_s*)=0,\quad g(n_s*)=\lambda^2\cdot\frac{\lambda^2(V_0-V_T)}{\beta^2},\quad f'(n_s*)=-\lambda^2(V_0-V_T)\quad(12a)$$

$$\frac{d}{dt}\delta n_s=-\lambda^2(V_0-V_T)\cdot\delta n_s+\lambda^2\frac{\lambda^2(V_0-V_T)}{\beta^2}\cdot V_m\cos wt\quad(12b)$$

the solution of this first order inhomogeneous equation can be immediately written:

$$\delta n_s(t) = C \cdot e^{-\lambda^2(V_0-V_T)t} + \lambda^2 \frac{\lambda^2(V_0-V_T)}{\beta^2} \cdot V_m \frac{\lambda^2(V_0-V_T)\cos wt + w\sin wt}{\left[\lambda^2(V_0-V_T)\right]^2 + w^2} \quad (13)$$

As an asymptotic solution survives only the second term; it exibits a harmonic oscillation of shifted phase whose amplitude is:

$$(\delta n_s)_{\max} = \lambda^2 \frac{\lambda^2(V_0-V_T)}{\beta^2} \cdot \frac{V_m}{\sqrt{\left[\lambda^2(V_0-V_T)\right]^2 + w^2}} \quad (14)$$

The expectation that the correction was proportional to V_m has also been confirmed. The essence of the conclusion of our analysis was not surprising; the fixed point n_s*=0 allows no harmonic oscillations (device is off), while another fixed point $n_s * = \frac{\lambda^2(V_0 - V_T)}{\beta^2}$ allows them (device is on).

3. The Analysis of HEMT Dynamics

As suggested in [1], the dynamical equation in the case of HEMT (High Electron Mobility Transistor) subjected to voltage (1) becomes:

$$\dot{n}_s = \alpha^2 n_{s0}^{\;2}(X_0 + X_m \cos wt)\cdot n_s - \alpha^2 n_{s0}(1 + X_0 + X_m \cos wt)\cdot n_s^{\;2} + \alpha^2 n_s^{\;3} \quad (15)$$

with dimensionless parameters:

$$X_0 = \frac{V_0 - V_T}{V_M - V_T}, \quad X_m = \frac{V_m}{V_M - V_T}, \quad V_m << V_0 - V_T \quad (X_m << X_0)$$

The Abellian differential equation (15) has no solution in the closed term, so it is necessary to start immediately with the Taylor's expansion [4]:

$$\dot{n}_s = \alpha^2 n_s (n_s - n_{s0}X_0)(n_s - n_{s0}) - \alpha^2 n_{s0} \cdot n_s (n_s - n_{s0}) \cdot X_m \cos wt$$

or:

$$\dot{n}_s = f(n_s) + g(n_s)\cdot X_m \cos wt \quad (16)$$

The linearization of the equation (16) around arbitrary fixed point, together with the cut-off of higher order correction terms, gives:

$$n_s(t) = n_s * + \delta n_s(t)$$

and:

$$\frac{d}{dt}\delta n_s = f(n_s *) + f'(n_s *)\cdot \delta n_s + g(n_s *)\cdot X_m \cos wt \quad (17)$$

Proceeding in the same way as in the case of MOSFET, we analyze equation (17) for different fixed points [1]:

- Subthreshold region: $X_0<0$, $n_s*=0$

$$f(n_s *) = 0 \quad g(n_s *) = 0, \quad f'(n_s *) = \alpha^2 n_{s0}^{\;2} \cdot X_0 \cdot \delta n_s \quad (18a)$$

and therefore equation (17) becomes:

$$\frac{d}{dt}\delta n_s = \alpha^2 n_{s0}^{\;2} \cdot X_0 \cdot \delta ns \quad (18b)$$

what gives:

$$\delta n_s(t) = \delta n_s(0)\cdot e^{\alpha^2 n_{s0}^{\;2} X_0 t} \quad (18c)$$

According to our expectations only vanishing solution appears, i.e. no oscillations are produced again (the device is off).

- Above threshold region: $0<X_0<1$, $n_s*=n_{s0}X_0$

$$f(n_s *) = 0, \quad g(n_s *) = \alpha^2 n_{s0}^{\;3} \cdot X_0(1 - X_0), \quad f'(n_s *) = \alpha^2 n_{s0}^{\;2} \cdot X_0(X_0 - 1) \quad (19a)$$

what turns equation (17) into [1]:

$$\frac{d}{dt}\delta n_s = \alpha^2 n_{s0}^{\;2} X_0(X_0 - 1)\cdot \delta n_s + \alpha^2 n_{s0}^{\;3}(1 - X_0)\cdot X_0 \cdot X_m \cos wt \quad (19b)$$

This equation (19a) is not as simple as equation (18a), but simple enough to be solved analytically:

$$\delta n_s(t) = C \cdot e^{-\alpha^2 n_{s0}^{\;2} X_0(1-X_0)t} + \alpha^2 n_{s0}^{\;3} X_0(1 - X_0)\frac{\alpha^2 n_{s0}^{\;2} X_0(1 - X_0)\cos wt + w\sin wt}{\left[\alpha^2 n_{s0}^{\;2} X_0(1 - X_0)\right]^2 + w^2} \quad (19c)$$

For the allowed values of parameter X_0 the first term vanishes and the only one that survives as a stationary solution is a second one. It straightforward leads to the conclusion that in this region it's possible to produce harmonic oscillations of the surface carriers' concentration around some arbitrary chosen fixed point:

- Saturation region: $X_0>1$, $n_s*=n_{s0}$

$$f(n_s*)=0,\quad g(n_s*)=0$$
$$f'(n_s*)=\alpha^2 n_{s0}{}^2(1-X_0) \tag{20a}$$

In this region the equation (17) can be rewritten as follows [1]:

$$\frac{d}{dt}\delta n_s=\alpha^2 n_{s0}{}^2(1-X_0)\cdot\delta n_s \tag{20b}$$

together with its straightforward solution:

$$\delta n_s(t)=\delta n_s(0)\cdot e^{\alpha^2 n_{s0}{}^2(1-X_0)t} \tag{20c}$$

In the allowed range of parameter X_0, the only existing solution is the vanishing one. According to our modest opinion that seems surprising (one cannot say the device is off, we rather say it is saturated - therefore we find it unexpected).

4. Generalization

The question that naturally arises is why in some operating regimes (i.e. for specific D.C. gate voltages) it appears impossible to impose harmonic oscillations of much smaller amplitude, while in the other ones it becomes possible. The proper answer to this question demands more detailed analysis. Starting from the equation (10b), one inevitably notices the following fact: if the specified fixed point n_s* has a property that $f(n_s*)$ and $g(n_s*)$ are zero simultaneously, than it's impossible to cause small magnitude harmonic oscillations (because $f(n_s*)=0$ by itself if n_s* is a fixed point, the condition reduces to $g(n_s*)=0$); on the contrary, when $g(n_s*)\neq 0$ ($f(n_s*)=0$) these small oscillations happen. Up to this moment only the first order correction in V_m (X_m) is considered. In order to investigate if upper conclusions hold even for higher order corrections we use perturbation technique to investigate our equation of interest in its most general form [5]:

$$\frac{d}{dt}\delta n_s=\sum_{k=0}^{+\infty}\frac{1}{k!}f^{(k)}(n_s*)\cdot(\delta n_s)^k+\left(\sum_{k=0}^{+\infty}\frac{1}{k!}g^{(k)}(n_s*)(\delta n_s)^k\right)\cdot X_m\cos wt \tag{21}$$

The excess surface concentration $\delta n_s(t)$ has been expanded in terms of dimensionless small parameter X_m deeply connected with applied A.C. gate voltage [5]:

$$\delta n_s(t)=\sum_{p=1}^{+\infty}(X_m)^p\cdot\delta n_{sp}(t) \tag{22}$$

The index p starts with $p=1$ because $f(n_s*)$ equals zero (the crucial property of fixed points). The incorporation of (22) into equation (1) gives:

$$\sum_{p=1}^{+\infty}(X_m)^p\frac{d}{dt}\delta n_{sp}(t)=$$
$$=\sum_{k=0}^{+\infty}\left\{\frac{1}{k!}\left[f^{(k)}(n_s*)+g^{(k)}(n_s*)\cdot Xm\cos wt\right]\cdot\left[\sum_{p=1}^{+\infty}(X_m)^p(\delta n_{sp}(t))\right]^k\right\} \tag{23}$$

The next step is to form a chain of first order differential equations with respect to $\delta n_{sp}(t)$ by making equal the left-side and right-side equation's terms multiplying X_m^p for each integer p separately. It turned out very difficult (und inconvenient also) to develop the general expression for an arbitrary integer p. Instead of that, several starting equations of this chain are written:

$p=1$:
$$\frac{d}{dt}\delta n_{s1}=f'(n_s*)\cdot\delta n_{s1}+g(n_s*)\cdot\cos wt \tag{24a}$$

$p=2$:
$$\frac{d}{dt}\delta n_{s2}=f'(n_s*)\cdot\delta n_{s2}+\frac{1}{2}f''(n_s*)(\delta n_{s1})^2+ +g'(n_s*)\cdot\delta n_{s1}\cdot\cos wt \tag{24b}$$

$p=3$:
$$\frac{d}{dt}\delta n_{s3}=f'(n_s*)\cdot\delta n_{s3}+g'(n_s*)\cdot\delta n_{s2}\cdot\cos wt+ +\frac{1}{2}g''(n_s*)(\delta n_{s1})^2\cdot\cos wt+f''(n_s*)\cdot\delta n_{s1}\cdot\delta n_{s2}+ +\frac{1}{6}f'''(n_s*)\cdot(\delta n_{s1})^3 \tag{24c}$$

etc.

The previous set of equations (24) enables us to assume the general shape of these equations for an arbitrary integer p (this is so obvious and doesn't need any further argumentation):

$$\frac{d}{dt}\delta n_{sp}=f'(n_s*)\cdot\delta n_{sp}+\varphi(\delta n_{s1},\delta n_{s2},...,\delta n_{s(p-1)},\cos wt) \tag{25}$$

with an important feature $f'(n_s*)<0$. This equation has a solution in a closed form:

$$\delta n_{sp}(t) = e^{f'(ns^*)t} \cdot \left[C_p + \int \varphi\left(\delta n_{s1},\ldots,\delta n_{s(p-1)},\cos wt\right) e^{-f'(n_s^*)t} \cdot dt\right] \tag{26}$$

with C_p being an integration constant to be determined from initial conditions. The fact $f'(n_s^*)<0$ implies that after a long period of time, the term containing C_p vanishes, so the only term that survives as an asymptotic solution is the second one. Therefore, in each power of perturbative series the nonvanishing A.C. component may appear, if only the inhomogeneous part of equation (25) is nonzero.

The several starting equations of this perturbative expansion, together with their analytical solutions, are listed bellow:

1. up to X_m^1:

$$\frac{d}{dt}\delta n_{s1} = f'(n_s^*)\cdot \delta n_{s1} + g(n_s^*)\cos wt \tag{27a}$$

$$\delta n_{s1}(t) = C_1 e^{f'(n_s^*)t} + g(n_s^*)\frac{w\sin wt - g(n_s^*)\cos wt}{w^2 + g^2(n_s^*)} \tag{27b}$$

2. up to X_m^2:

$$\frac{d}{dt}\delta n_{s2} = f'(n_s^*)\cdot \delta n_{s2} + \left[\frac{f''(n_s^*)}{2}(\delta n_{s1})^2 + g'(n_s^*)\cdot \delta n_{s1}\cdot \cos wt\right] \tag{28a}$$

$$\delta n_{s2}(t) = C_1 e^{f'(n_s^*)t} \cdot \left\{C_2 + \int e^{-f'(n_s^*)t}\cdot \left[\frac{f''(n_s^*)}{2}(\delta n_{s1})^2 + g'(n_s^*)\cdot \delta n_{s1}\cdot \cos wt\right]dt\right\} \tag{28b}$$

3. up to X_m^3:

$$\frac{d}{dt}\delta n_{s3} = f'(n_s^*)\cdot \delta n_{s3} + \left[\begin{array}{l}\dfrac{f'''(n_s^*)}{6}(\delta n_{s1})^3 + f''(n_s^*)\cdot \delta n_{s1}\cdot \delta n_{s2} + \\ + \dfrac{g''(n_s^*)}{2}(\delta n_{s1})^2\cos wt + g'(n_s^*)\cdot \delta n_{s2}\cdot \cos wt\end{array}\right] \tag{29a}$$

$$\delta n_{s3}(t) = C_1 e^{f'(n_s^*)t} \cdot \left\{C_3 + \int e^{-f'(n_s^*)t}\left[\begin{array}{l}\dfrac{f''(n_s^*)}{6}(\delta n_{s1})^3 + f''(n_s^*)\delta n_{s1}\delta n_{s2} + \\ + \left(\dfrac{g''(n_s^*)}{2}(\delta n_{s1})^2 + g'(n_s^*)\cdot \delta n_{s2}\right)\cdot \cos wt\end{array}\right]dt\right\} \tag{29b}$$

These solutions (27b), (28b) and (29b) are inserted into relation (22) in order to achieve the exact expression for $\delta n_s(t)$ as far as possible. Higher order perturbative terms ($p\geq 2$) are undesirable because they introduce higher order harmonics and the signal amplification that depends on the signal magnitude itself. Only the first order perturbative nonzero term is convenient because it provides amplification independent of the signal itself and retains only first harmonic with phase shift only (27b). If only the first order term in perturbative power series is considered, the general picture reduces to the already mentioned condition for the appearance of harmonic oscillations: $g(n_s^*)\neq 0$; otherwise, the harmonic oscillations are absent.

5. Discussion and conclusions

So much attention has been paid to the behavior of surface carriers' concentration because it appears as a crucial physical quantity that governs lateral electron transport in unipolar devices. This lateral transport is caused by an applied D.C. drain-source voltage and time-dependent drain current can be written in the following form [2]:

$$i_D(t) = eW\mu \cdot (n_s^* + \delta n_s)\cdot \frac{dV}{dx} \tag{30}$$

where μ denotes the carriers' mobility in the channel, W the width of the channel and $V(x)$ is the quasi-Fermi potential for electrons.

The standard procedure in unipolar structures is to integrate the expression (30) along the channel [6]:

$$i_D(t) = \frac{eW}{L}\cdot \int_0^{V_{DS}} \mu \cdot (n_s^* + \delta n_s(t))dV = I_D + \delta i_D(t) \tag{31}$$

The purpose of this section is not to examine the relation (31) in details, but to roughly describe the mechanism of producing alternating drain current component $\delta i_D(t)$. Obviously, the variation of surface carriers' concentration $\delta n_s(t)$ causes the appearance of drain current $\delta i_D(t)$ proportional to $\delta n_s(t)$. It finally means that $\delta i_D(t)$ will also be a harmonic (sine or cosine) function of time (probably with a phase shift included). Furthermore, one could extract transconductance g_m (presumably defined as $\partial i_D/\partial V_g$) and all other small signal parameters relevant for electronics [2].

The paper has inevitably shown how the incorporation of dynamical treatment of relevant variables could affect the whole situation and lead to some (un)expected conclusions. The main feature of this influence is confined in the statement, together with a plenty of proof, that in some regions of operation it's possible to transfer small

A.C. signal from gate to drain terminal and in other regions of operation it becomes impossible.

References

[1] R. M. Šašić, P. M. Lukić, R. M. Ramović, S. M. Osojić, J. Optoelectron. Adv. Mater. **9**(9), 2703 (2007).
[2] Y. P. Tsividis, ″Operation and Modeling of MOS Transistors″, McGraw Hill, (1988)
[3] P. Roblin, H. Rohdin, ″High-speed Heterostructure Devices″, Cambridge University Press, (2004)
[4] Roel Snieder, ″Guided Tour of Mathematical Methods for the Physical Sciences″, 2nd edition, Cambridge University Press, (2004)
[5] S. H. Strogatz, ″Nonlinear Dynamics and Chaos (with Applications to Physics, Biology, Chemistry and Engineering)″, Westview Press, (USA), (1994)
[6] R. M. Šašić, P. M. Lukić, R. M. Ramović, J. Optoelectron. Adv. Mater. **8**(1), 324 (2006).

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