Symmetric structure response of long range polariton plasmons

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The present work deals with the study a symmetric structure response generating a plasmons polariton of long range using a simplified geometrical structure consisting on a gold thin film set between two layers of PMMA. Surface plasmons are considered as typical mode of the metal-dielectric interface. This aims an optimization of the parameters affecting the surface plasmons ranges, such as the multilayer structure thickness, incidence angle and the wave length.

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1. Introduction

Since the discovery of the surface plasmon optical excitation phenomenon by Kretschmann and Otto in 1968, the plasmonics technology has been rapidly developed; especially the plasmon wave guides under the strip form of finite width and nanometeric thickness become a promising issue for applications dedicated to miniaturized optical devices. The configuration allowing surface plasmon generation of long range seems a breakthrough for lithographic applications of higher resolution.

The surface plasmons are considered as typical modes of an interface metal-dielectric [1]. They are characterized by the propagation, localization and resonance, which make them suitable for nano-optics applications [2]. Among these properties, only the transverse magnetic property (TM) can induce a dipolar momentum and hence, excite a plasmon-polariton. The TM wave which obeys Maxwell equations and satisfies the continuity conditions at the interface. It propagates along the dielectric-metal interface and its amplitude decreases exponentially as one move away; perpendicularly, from the interface. The mediums are supposed isotropic, homogeneous and linear [3]. The plasmons range can be greatly increased in a symmetrical configuration of a gold layer having a dielectric constant ε_m set between two PMMA layers of dielectric constant ε_d.

2. Field equation description

The considered mediums are assumed isotropic, homogeneous and linear. The plasmon range width can be considerably increased in a symmetrical configuration such as a gold layer of dielectric constant ε_2, inserted between two PMMA layers of dielectric constant ε_i

In this configuration, the expression of the magnetic field associated to the plasmon mode is given by.

$$\mathbf{H}=H_0 \exp[i(\omega t-k_r x)]\, f(z)=\mathbf{H}_y \tag{1}$$

With: H_0 the normalizing constant and f(z) expresses the dependence of $\mathbf{H}$ as a function of z such that f(z =0)=1
The z decay is expressed by

$$f(z) = \exp(-a_d z)\ z>0 = \exp(a_m z)\ z<0 \tag{2}$$

and

$$\begin{aligned} a_d^2 &= k_{ps}^2 - (w/c)^2\, \varepsilon_d \\ a_m^2 &= k_{ps}^2 - (w/c)^2\, \varepsilon_m \qquad R(a_i) > 0 \qquad i = \{m, d\} \end{aligned} \tag{2'}$$

a_i (i=m, d) are the propagation constants.

Considering that the system is invariant if translated with respect to y and the continuity conditions valid a $z=0$ we get:

$$\varepsilon_d\, a_m + \varepsilon_m(w)\, a_d = 0 \tag{3}$$

Condition of Eq.4 can only be satisfied if mediums m and d have opposite dielectric constants.

As ε_d is positive for a dielectric, the second medium must be a metal of dielectric expression given by:

$$\varepsilon_m(w) = \varepsilon_r + \varepsilon_i\, j \text{ such that } \varepsilon_r < 0.$$

The model used to determine the gold dielectric expression is the Drude [4] model shown in Fig.1.

Gold dielectric expression is complex and the wave vector is also complex as expressed by:

$$\begin{aligned} \varepsilon_m &= \varepsilon_r + \varepsilon_i\, j. \\ k_{ps} &= k_r + j\, k_i \end{aligned} \tag{4}$$

After identifying the real and imaginary parts and considering that $|\varepsilon_r|>>\varepsilon_i$ and $\varepsilon_r<0$, we get:

$$\mathrm{k_r} = k_0\sqrt{\frac{\varepsilon_r\varepsilon_d}{\varepsilon_r+\varepsilon_d}}$$
$$k_i = k_0\sqrt{\frac{\varepsilon_r\varepsilon_d}{\varepsilon_r+\varepsilon_d}}\left(\frac{\varepsilon_r}{2\varepsilon_i^2}\right) \tag{5}$$

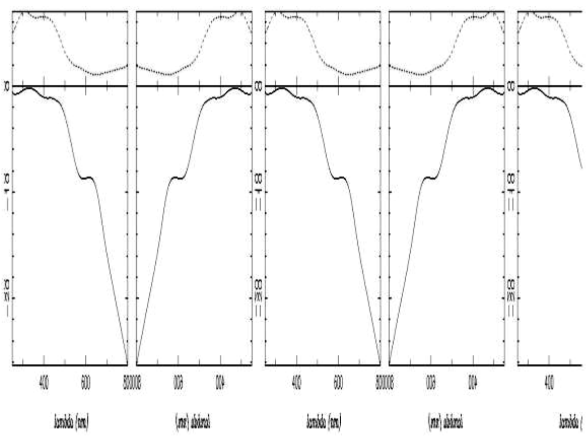

Fig. 1. Gold Dielectric constant as given by Drude Dots is for imaginary part and line for real part.

2.1 Typical Mode of a Simple Interface

Maxwell equations are used to determine the surface wave analytical characteristics. Hence it can be shown that the surface waves appear in ***p*** polarization (they may appear in ***s*** polarization in magnetic materials) and their dispersion relation can be computed for a planar interface [5]. As has been discussed earlier, surface waves are electromagnetic resonance of the planar interface and are evanescent in the direction perpendicular to the planar interface where the do propagate. Resonance of a structure can be studied using a matrix formalism linking the incoming and outgoing waves hence one gets:

$$\mathrm{k_r} = k_p = \sqrt{\frac{\varepsilon_r\varepsilon_d}{\varepsilon_r+\varepsilon_d}} \tag{6}$$

For a given impulse, the plasmon wave vector is greater than that of a phonon as shown in Fig. 2. Fig. 2 shows that excitation of a plasmon mode obeys $k>k_d$. This property ensures an imaginary k_z, i.e., an interface mode which, however, does not allow the light excitation.

The plasmon resonant frequency given by

$$\omega_p = \frac{4\pi Ne^2}{m}$$

where m is the electron effective mass, e the electron charge and N the metal electron density.

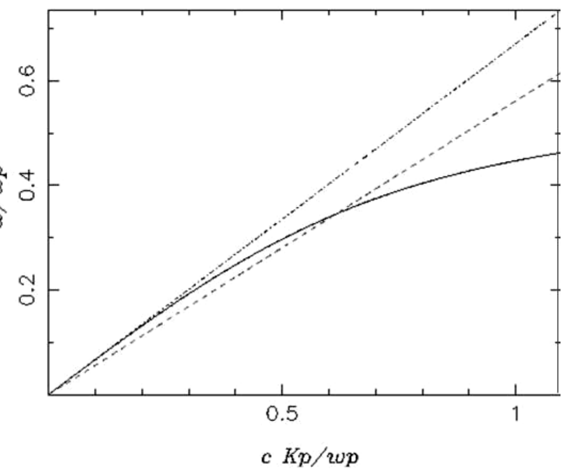

Fig. 2. Dispersion relation for a simple interface mode

2.2 Refection Coefficient and Wave length

The interpretation of the experimental results needs the reflection coefficient R_p of a dissymmetrical system of 3 layers (dielectric/metal/dielectric). The field continuity conditions at the interface allow the determination of this reflection coefficient as:

$$R_p = \frac{\left(rp_{md} + rp_{mp}exp\left(2\,ia_m e\right)\right)}{\left(1 + rp_{dm}rp_{pd}exp\left(2\,ia_m e\right)\right)} \tag{7}$$

with r_{pmd} r_{pmp} the reflection coefficients at the interfaces metal/dielectric and metal /prism, respectively, and *e* is the metallic layer thickness as shown in Fig.1.

If $|\varepsilon_r|>>\varepsilon_i$, the reflectivity expression is a Lorentzien of the form:

$$R_P = 1 - 4\frac{\left(\Gamma_i\Gamma_{rad}\right)}{\left(\left[k-\left(k_0+\Delta k\right)\right]^2 + \left(\Gamma_i+\Gamma_{rad}\right)^2\right)} \tag{8}$$

Equation 8 gives $k = k_{m/d} + \Delta k$ as the excitation condition of a plasmons mode and its propagation length is given as:

$$L_{ps} = \frac{1}{\left(2\Gamma_i + \Gamma_{rad}\right)} \tag{9}$$

Γ_i : Plasmon attenuation due to the joule losses and it is given by the imaginary part of the wave vector of the plasmon $\Gamma_i = \mathrm{Im}(k_{ps})$.

Γ_{rad}: is the energy transfer impedance of the plasmon to photon.

The propagation length of surface wave is inversely proportional to the imaginary part of the wave vector and the field expression can be written as [7]:

$$\mathbf{H} = H_0 \exp[-i(\omega t - k_r x)]\, f(z) \exp(-k_i x) \qquad (10)$$

Equation 10 characterizes a mode related to the interface metal-dielectric and propagating in parallel to the interface with a wave vector k_r and an attenuation factor $\exp(-k_i x)$. This allows defining the propagation length as:

$$L_{ps} = \frac{1}{2} k_i \qquad (11)$$

For a system Gold/PMMA; at λ=770nm, we observe L_{ps}= 27μm.

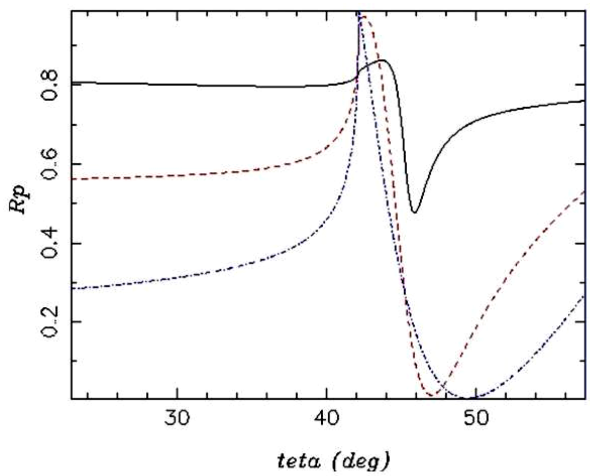

Fig. 3. Evolution of the reflectivity graph with respect to the incidence angle for different layer thicknesses at λ = 600nm, d=60nm,.... d=30nm,... d=25nm —

3. Brent method

The Brent method is defined as the combination of the dichotomy and the inverse quadratic interpolation methods. It is an iterative method using the square root setting and the inverse quadratic interpolation [8].

Assume a couple of points ***(a,b)*** and a function ***f*** continuous within the definition interval. If the product ***f (a)f (b) < 0***, the intermediate value theorem shows that the functions goes to zero at least once in its definition interval. This method is however, is lengthy for convergence. Using the Brent method, one needs to know the values of the function for three values

Assume (a, f(a)), (b, f(b)), and (c, f(c)) the interpretation expression given by

$$\begin{aligned} x = {} & \frac{(y - f(a))(y - f(b))c}{(f(c) - f(a))(f(c) - f(b))} + \\ & + \frac{(y - f(b))(y - f(c))a}{(f(a) - f(b))(f(a) - f(c))} \\ & + \frac{(y - f(c)(y - f(a))b}{(f(b) - f(c))(f(b) - f(a))} \end{aligned} \qquad (12)$$

For pour y= 0 Eq.12 can be written as:

$$x = b + \frac{P}{Q} \qquad (13)$$

Where P and Q are given by:

$$\begin{aligned} P &= S[T(R - T)(c - b) - (1 - R)(b - a)] \\ Q &= (T - 1)(R - 1)(S - 1) \end{aligned} \qquad (14)$$

R, S and T are expressed as:

$$R = \frac{f(b)}{f(c)}\; S = \frac{f(b)}{f(a)}\; T = \frac{f(a)}{f(c)} \qquad (15)$$

In practice, ***b*** is the first estimation of the root and ***P/Q*** is a correction factor.

4. Long-range surface plasmons

The magnetic field expression is given by [9]

$$\mathbf{H} = H_0 \exp[i(\omega t - k_r x)]\; f(z) = \mathbf{H}_y \qquad (16)$$

with H_0 is the normalizing constant, and f (z) expresses the dependency of **H** in terms of ***z*** such that ***f(z =0)=1.***

The decrease with respect to ***z*** is given by:

$$\begin{aligned} f(z) &= \exp.(-a_{d3} z) \qquad z > d \\ f(z) &= \exp(a d_1 z) \qquad z < 0 \end{aligned} \qquad (16')$$

d is the gold layer thickness with aj (j=d_1, d_3, m) are the propagation constants with respect to ***z*** within the different mediums.

$$\begin{aligned} &\mathbf{H} = \exp[i(\omega t - kx)][A\exp(a_m z) + B\exp(-a_m z)] \qquad 0 < z < d \\ &aj = (k^2 - (\omega/c)^2\, \varepsilon_j)^{1/2} \quad j = \{d_1, m, d_3\}\ \text{avec} R(a_{d1}) > 0\, \text{et}\, R(a_{d3}) > 0 \end{aligned} \qquad (17)$$

Solving Maxwell equations with the continuity applied for each surface, we get

$$\tanh(a_2 d) = \varepsilon_2 a_2 (\varepsilon_1 a_3 + \varepsilon_1 a_3)\; (a_2^2 \varepsilon_1 a_3 + a_1 \varepsilon_2^2 a_3) \qquad (18)$$

For a symmetrical system $d_1 = d_3$, Eq.18 splits into two parts where one is antisymetric given by:

$$\tanh(a_m d/2) = -\varepsilon_{d_1} a_m / \varepsilon_m a_{d1} \qquad (19)$$

and the other symetric

$$\tanh(a_m d/2) = -\varepsilon_m a_{d1} / \varepsilon_{d1} a_m$$

The symmetrical part corresponds to the long range plasmons. The Brent method has been used to solve the equation and the results are illustrated in Fig. 4.

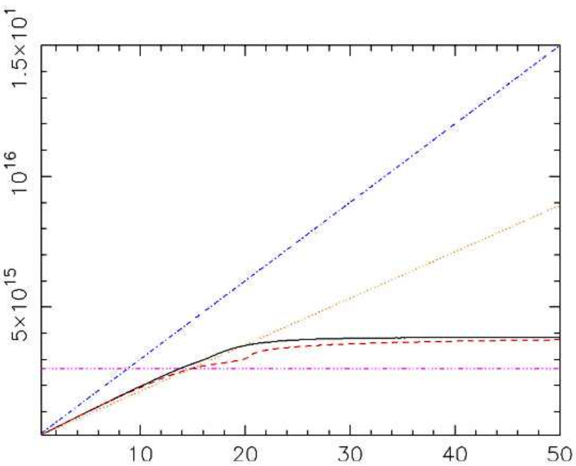

Fig. 4. Dispersion relationship for a three layer system d=60nm

For a thickness of roughly 60 nm the two curve parts are superposed, and suites the results of Fig.3

This method allows obtaining a propagation length of 140 µm order for a wave length of 770 nm and a thickness of 33 nm for the case of gold layers.

5. Results and discussions

The first part of this work focused on certain important parameter computation, such as the reflection coefficient. Figs. 5-7 show a comparison of the theoretical and experimental results in case of a symmetrical structure. It can be noticed that the plasmons range is greater than 100 µm. This corresponds to propagation length of surface plasmons-polariton given by: LPS $=1/[2(\Gamma_i + \Gamma_{rad})]$ $=1.09$µm.

As can be seen the experimental values are far from that predicted by theory, i.e., 140µm. This can be explained by the important radiative losses due to small layer thickness of PMMA that does not allow obtaining a plasmon polariton propagating over a long distance. However, this experiment allowed to characterize precisely the experimental parameters such as $\boldsymbol{\varepsilon_m}$, $\boldsymbol{d_1}$ the PMMA lateral thickness. Hence, the new parameters leading to the excitation of long range mode are: λ= 770nm, d_1 =2.6µm d_3 =33nm. The reflection coefficient computed from these values is LPS=1/2 $\Gamma_i \approx$180µm.

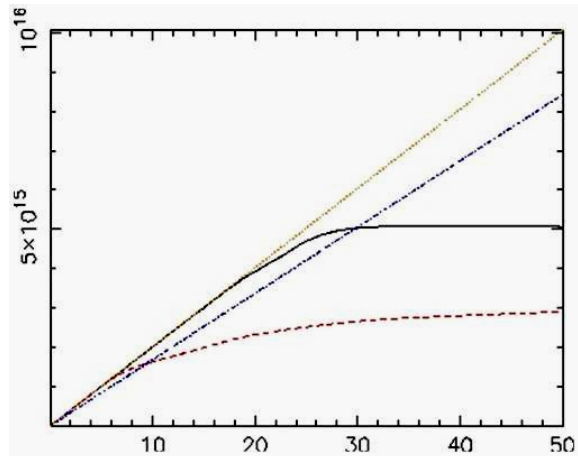

Fig 5. Dispersion relationship w= f(k) for a 3 layer system, d=33nm

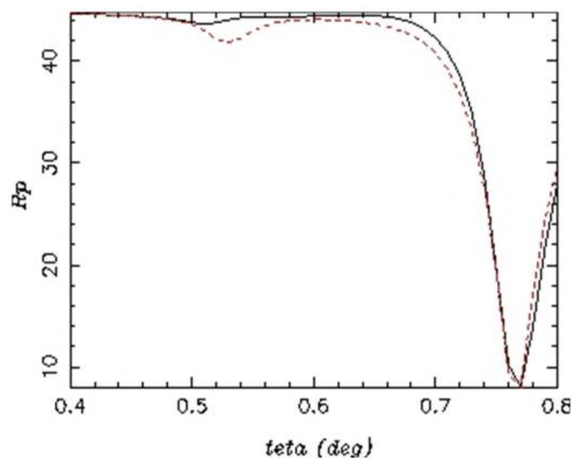

*Fig. 6. Experimental versus Theoretical Resu*lts

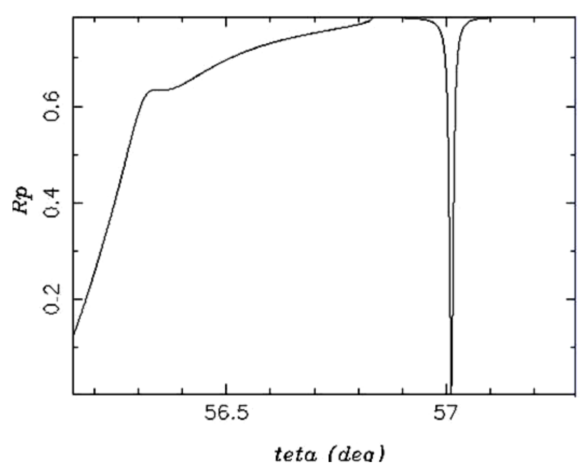

Fig. 7. Rp vs. Incidence angle for the System λ = 770nm, d1 =2.6µm d3 =33nm

6. Conclusion

The theoretical and experimental investigations on a symmetrical structure response under the plasmon-polariton mode of long range lead to a more precise characterization of the experimental parameters such as gold dielectric constant and the thickness of the PMMA, which are important in implementing the experiment.

In this work we have defined new theoretical parameters for exciting a long range mode, with parametric values λ= 770 nm, d1 =2.6 μm d=33 nm. For these parameters, we observed a propagation length of 180 μm.

Numerical simulations of certain parameter evolution were conducted which led to the study of the length that can be traveled by a wave. The proposed system supports a plasmon propagating over more than 140 μm distance.

Surface adherence problems did not allow validation of the last simulation results. Hence implementing a sample test PMMA structure of thickness 2.6 μm and that of gold of d=33 nm is one of the future possible venues of this investigation.

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