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Tapered conductivity material for radar cross section reduction

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Radar sensors are used for more and more applications from space surveillance, navigation, speed measurement, landmine detection, anticolision systems, space investigations etc. All these applications depend on the signal reflected from an object. The processing algorithms as well as the hardware available had been constantly improved such as today radars are able to detect objects with very small radar cross section (RCS). Nevertheless in the field of surveillance radar there is a lot of work in progress to improve the shape and to use low reflecting materials to decrease the power backscattered by the radar and, in this way, to diminish the probability of detection. The work presented in this paper regards tapered conductivity materials which can be used to decrease the radar cross section. The incident energy, or part of it, has to be transformed in heat within the material layer. To this end it has to have some lossy mechanisms. One possibility is to increase the conductivity. In the paper, the behavior of a material layer with certain electromagnetic parameters as a function of thickness, frequency, gradient conductivity and incidence angle is studied by computer simulations. This type of materials may be used for radar cross section reduction and to built anechoic chambers for measurements.

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Keywords: Tapered conductivity, Reflection coefficient, Radar cross section

1. Introduction

One of the most appealing applications of the electromagnetic waves is to discover different kinds of objects on the ground, in the ground or above the ground. The device used for these applications is called radar and operates based on the basis of the reflected signal from that object. In other words the radar transmits a pulse or a continuous wave signal which travels through the propagation media to the object and part of it is back scattered and received by the radar. If the received signal exceed a certain level than the object is detected. Moreover, the distance can be measured based on the delay between the transmitted and received signals (for pulse radar) or the phase difference between them (for continuous wave radar). In time, more advanced signal processing techniques as well as better high frequencies devices improved the detection properties of radar systems but specialists started to look for procedures to decrease the power back scattered by an object irradiated with electromagnetic energy. Among other procedures one was to cover it with some materials having absorbing properties [1]. The amount of electromagnetic energy scattered by an object depend on the reflection properties of the material it is made of, the shape of the object, the ratio between the object dimension and the wavelength and the incidence angle. In most cases, the object radar is used to detect has a metallic surface or at least a metallic structure. As a result the absorbing properties of material layer have to be searched against a metallic surface. In this paper the possibility to decrease the radar cross section [2] of a metallic surface by covering it with a tapered conductivity material is studied.

2. Theory

Consider an electromagnetic wave incident on a tapered conductivity material at any incidence angle. The incident wave has the electrical component of electromagnetic field parallel with the separation plane between the free space and the material. Let us suppose that the material layer starts at z=0 and finishes at z=l. The electromagnetic phenomenons are complete characterized by Maxwell's equations which, for a lossy medium, linear, immobile and without charges, are:

$$\begin{cases} rot\overline{\mathrm{E}} = -j\cdot\omega\cdot\mu_0\cdot\overline{\mathrm{H}} \\ rot\overline{H} = \left(\sigma(z) + j\cdot\omega\cdot\varepsilon_0\right)\cdot\overline{E} \\ div\overline{D} = 0 \\ div\overline{B} = 0 \\ \overline{B} = \mu\cdot\overline{H} \\ \overline{D} = \varepsilon\cdot\overline{E} \end{cases} ,\qquad (1)$$

where:- μ_0-is the magnetic permeability of free space;
ε_0-is dielectric permitivity of free space;
σ-is the conductivity of the material and it depends of the z;
E, H- are the electrical and magnetic field.

From (1) the following equation is derived:

$$\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}+\beta_0^{\,2}\left(1-j\frac{\sigma(z)}{\omega\cdot\varepsilon_0}\right)\cdot E_x=0\,. \tag{2}$$

The incident, reflected and refracted electromagnetic wave can be written as [3]:

$$\begin{cases} E_x^{\,i}(y,z)=E_i\cdot e^{-j\beta_0\cdot(y\cdot\sin\theta_i+z\cdot\cos\theta_i)} \\ E_x^{\,r}(y,z)=E_r\cdot e^{-j\beta_0\cdot(y\cdot\sin\theta_r-z\cdot\cos\theta_r)},z<0\,, \\ E_x^{\,t}(y,z)=E_t\cdot e^{-j\beta_t\cdot(y\cdot\sin\theta_t+z\cdot\cos\theta_t)},z>l \end{cases} \tag{3}$$

where:

$\beta_0^{\,2}=\omega^2\cdot\mu_0\cdot\varepsilon_0$;

$\beta_t^{\,2}=\omega^2\cdot\mu_0\cdot\varepsilon_0\cdot\left(1-j\cdot\frac{\sigma(z)}{\omega\cdot\varepsilon_0}\right)$;

E_i, E_r, E_t – are the magnitude of incident, reflected and transmitted wave;

θ_i, θ_r, θ_t – are the angles between the line perpendicularly on the separation surface and the direction of incident, reflected and transmitted wave. Supposing that there are no losses along Oy axis and that the variation of the electric field is given by $e^{-j\cdot\beta_0\cdot y\cdot\sin\theta}$, (2) becomes:

$$\frac{\partial^2 E_x}{\partial z^2}+\beta_0^{\,2}\left(\cos^2\theta_i-j\frac{\sigma(z)}{\omega\cdot\varepsilon_0}\right)\cdot E_x=0\,. \tag{3}$$

Let us suppose a linear variation of the conductivity:

$$\sigma(z)=b\cdot z, 0\le z\le l\,, \tag{4}$$

where b is the gradient of the conductivity. If we denote:

$$\alpha=j\cdot\frac{b}{\omega\cdot\varepsilon_0}\,; \tag{5}$$

$$u=\cos^2\theta_i-\alpha\cdot z\,, \tag{6}$$

(3) becomes:

$$\frac{d^2E_x}{du^2}+\left(\frac{\beta_0}{\alpha}\right)^2\cdot u\cdot E_x=0 \tag{7}$$

which is a Bessel differential equation. It has the following solution [3]:

$$E_x(y,z)=(u)^{\frac{1}{2}}\cdot\left\{A\cdot H_{\frac{1}{3}}^{(1)}\left[\frac{2}{3}\cdot\left(\frac{\beta_0}{\alpha}\right)\cdot(u)^{\frac{3}{2}}\right]+ B\cdot H_{\frac{1}{3}}^{(2)}\left[\frac{2}{3}\cdot\left(\frac{\beta_0}{\alpha}\right)\cdot(u)^{\frac{3}{2}}\right]\right\}\cdot \cdot e^{-j\cdot\beta_0\cdot y\cdot\sin\theta_i}\,, \tag{8}$$

where:

$H_{\frac{1}{3}}^{(1)}, H_{\frac{1}{3}}^{(2)}$ - are third order Hankel's function;

A, B – are arbitrary constant.

If x is low one can use asymptotic representations of Hankel's functions:

$$\begin{cases} H_\nu^{(1)}(x)\cong\left(\frac{2}{\pi\cdot x}\right)^{\frac{1}{2}}\cdot e^{j\cdot\left(x-\frac{2\cdot\nu+1}{4}\cdot\pi\right)} \cdot\left(1-\frac{4\cdot\nu^2-1}{2\cdot j\cdot x}\right) \\ H_\nu^{(2)}(x)\cong\left(\frac{2}{\pi\cdot x}\right)^{\frac{1}{2}}\cdot e^{-j\cdot\left(x-\frac{2\cdot\nu+1}{4}\cdot\pi\right)}\cdot \left(1+\frac{4\cdot\nu^2-1}{2\cdot j\cdot x}\right) \end{cases}. \tag{9}$$

By substituting Hankel's functions with their asymptotic representations, the expression of electrical field becomes:

$$E_x(y,z)\cong\sqrt{\cos^2\theta_i-\alpha\cdot z}\cdot\left\{A\cdot\left(\frac{2}{\pi\cdot\frac{2}{3}\cdot\left(\frac{\beta_0}{\alpha}\right)\cdot u^{\frac{3}{2}}}\right)\cdot \cdot e^{j\left(\frac{2\cdot\beta_0}{3\cdot\alpha}\cdot u^{\frac{3}{2}}-\frac{5\cdot\pi}{12}\right)}\cdot\left(1-j\cdot\frac{5}{18\cdot\left(\frac{2\cdot\beta_0}{3\cdot\alpha}\right)\cdot u^{\frac{3}{2}}}\right)+ +B\cdot\left(\frac{2}{\pi\cdot\frac{2}{3}\cdot\left(\frac{\beta_0}{\alpha}\right)\cdot u^{\frac{3}{2}}}\right)\cdot e^{-j\left(\frac{2\cdot\beta_0}{3\cdot\alpha}\cdot u^{\frac{3}{2}}-\frac{5\cdot\pi}{12}\right)}\cdot \cdot\left(1+j\cdot\frac{5}{18\cdot\left(\frac{2\cdot\beta_0}{3\cdot\alpha}\right)\cdot u^{\frac{3}{2}}}\right)\right. \tag{10}$$

From simultaneous equations system (1) the expression of the magnetic field is:

$$H_y=-\frac{1}{j\cdot\omega\cdot\mu}\cdot\frac{\partial E_x}{\partial z}\cong\left[\left(-\frac{\alpha}{2\cdot u^{\frac{5}{4}}}+\frac{3\cdot\alpha}{4\cdot u^{\frac{5}{4}}}-j\cdot\beta_0\cdot u^{\frac{1}{4}}\right)\cdot \left(1-j\cdot\frac{5\cdot\alpha}{12\cdot\beta_0\cdot u^{\frac{3}{2}}}\right)+j\cdot\frac{15\cdot\alpha}{24\cdot\beta_0\cdot u^{\frac{11}{4}}}\right]\cdot e^{j\cdot\left(\frac{2\cdot\beta_0}{3\cdot\alpha}\cdot u^{\frac{3}{2}}-\frac{5\cdot\pi}{12}\right)}\cdot A\cdot\sqrt{\frac{3\cdot\alpha}{\pi\cdot\beta_0}}+\left[\left(-\frac{\alpha}{2\cdot u^{\frac{5}{4}}}+\frac{3\cdot\alpha}{4\cdot u^{\frac{5}{4}}}- -j\cdot\beta_0\cdot u^{\frac{1}{4}}\right)\cdot\left(1+j\cdot\frac{5\cdot\alpha}{12\cdot\beta_0\cdot u^{\frac{3}{2}}}\right)-j\cdot\frac{15\cdot\alpha}{24\cdot\beta_0\cdot u^{\frac{11}{4}}}\right]\cdot e^{-j\cdot\left(\frac{2\cdot\beta_0}{3\cdot\alpha}\cdot u^{\frac{3}{2}}-\frac{5\cdot\pi}{12}\right)}\cdot B\cdot\sqrt{\frac{3\cdot\alpha}{\pi\cdot\beta_0}} \tag{11}$$

A and B constant are computed using boundary conditions for z=l. If the material covers a metallic surface, then, for z=l, E_x=0. If we denote:

$$q_l = \frac{2\cdot\beta_0}{3\cdot\alpha}\cdot u_l^{\frac{3}{2}} ; \tag{12}$$

$$q_0 = \frac{2\cdot\beta_0}{3\cdot\alpha}\cdot\cos^3\theta_i ; \tag{13}$$

$$u_l = \cos^2\theta_i - \alpha\cdot l ; \tag{14}$$

$$u_0 = \cos^2\theta_i . \tag{15}$$

then from:

$$E_x\Big|_{z=l} = 0 , \tag{16}$$

one can obtain:

$$A = -B\cdot\frac{H_{\frac{1}{3}}^{(2)}(q_l)}{H_{\frac{1}{3}}^{(1)}(q_l)} = \frac{e^{-j\cdot\left(q_l-\frac{5\cdot\pi}{12}\right)}\cdot\left(1+j\cdot\frac{5}{18\cdot q_l}\right)}{e^{+j\cdot\left(q_l-\frac{5\cdot\pi}{12}\right)}\cdot\left(1-j\cdot\frac{5}{18\cdot q_l}\right)} . \tag{17}$$

The input impedance, for z=0, is:

$$Z = \frac{E_x}{H_y}\Bigg|_{z=0} . \tag{18}$$

Denote:

$$L = -j\cdot\omega\cdot\mu_0\cdot\left[\left(1-j\cdot\frac{5}{18\cdot q_l}\right)\cdot\left(1+j\cdot\frac{5}{18\cdot q_0}\right)\cdot\sqrt{\frac{3\cdot\alpha}{\beta_0\cdot\pi}}\cdot\frac{1}{(u_0)^{\frac{1}{4}}} - \right. ;$$
$$\left. -\left[\left(1+j\cdot\frac{5}{18\cdot q_l}\right)\cdot\left(1-j\cdot\frac{5}{18\cdot q_0}\right)\cdot\sqrt{\frac{3\cdot\alpha}{\beta_0\cdot\pi}}\cdot\frac{1}{(u_0)^{\frac{1}{4}}}\cdot e^{-2\cdot j\cdot(q_l-q_0)}\right]\right. \tag{19}$$

$$M = \sqrt{\frac{3\cdot\alpha}{\pi\cdot\beta_0}}\cdot\left(1-j\cdot\frac{5}{18\cdot q_l}\right)\cdot\left[\left(1+j\cdot\frac{5}{18\cdot q_0}\right)\cdot\left(\frac{-\alpha}{2\cdot u_0^{\frac{5}{4}}}+\frac{3\cdot\alpha}{4\cdot u_0^{\frac{5}{4}}}+ +j\cdot\beta_0\cdot u_0^{\frac{1}{4}}\right)-j\cdot\frac{15\cdot\alpha}{24\cdot\beta_0}\cdot\frac{1}{u_0^{\frac{13}{4}}}\right] ; \tag{20}$$

$$N = \sqrt{\frac{3\cdot\alpha}{\pi\cdot\beta_0}}\cdot\left(1+j\cdot\frac{5}{18\cdot q_l}\right)\cdot\left[\left(1-j\cdot\frac{5}{18\cdot q_0}\right)\cdot\left(\frac{-\alpha}{2\cdot u_0^{\frac{5}{4}}}+\frac{3\cdot\alpha}{4\cdot u_0^{\frac{5}{4}}}- -j\cdot\beta_0\cdot u_0^{\frac{1}{4}}\right)+j\cdot\frac{15\cdot\alpha}{24\cdot\beta_0}\cdot\frac{1}{u_0^{\frac{13}{4}}}\right]\cdot e^{-2\cdot j\cdot(q_l-q_0)} ; \tag{21}$$

then:

$$Z_i = Z\Big|_{z=0} = \frac{L}{M+N} . \tag{22}$$

The reflection coefficient is given by:

$$\rho = \frac{Z_i - Z_0}{Z_i + Z_0} . \tag{24}$$

3. Computed results

3.1 Introduction

In order to study the variation of the material properties as a function of different parameters several computer simulations had been made based on the algorithm presented above. Taking into account the purpose of this work the simulations were made supposing that a material layer having certain electromagnetic properties given by μ, ε and σ is applied on a metallic surface. The electromagnetic wave is incident under different angles on this structure, so part of the energy will be back scattered at the interface between the free space and the material and part of it will go through. The electromagnetic wave which propagates trough the material layer will be attenuated proportional with the conductivity of the material. If the conductivity increases linearly from the free space interface towards the metallic surface than the impedance will decrease in the same direction matching the free space impedance (376,7 Ω). The matching depends on the dielectric permitivity of the support material and the gradient of the conductivity. It is measured by the value of the reflection coefficient. This is way it is computed by simulations against different parameters of the materials.

3.2 Reflection coefficient variation against the thickness of the material

In this case an electromagnetic wave is incident on different angles on a linearly variation conductivity material. The study is made for different values of thickness and frequencies.

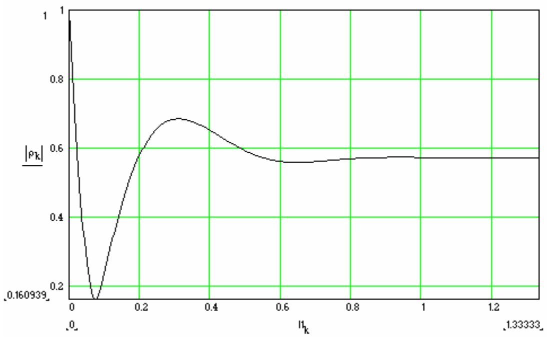

Fig.1 Reflection coefficient modulus as a function of l/λ, for f=2 GHz, b=1 S/m^2, θ=60°.

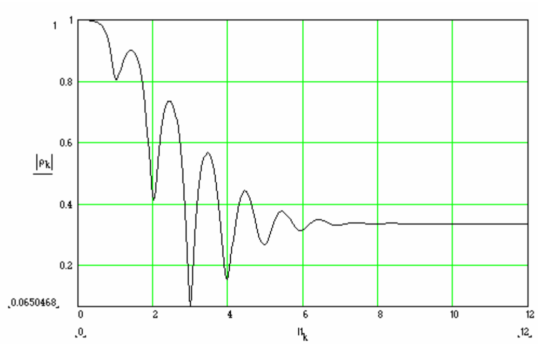

Fig.2 Reflection coefficient modulus as a function of l/λ, for f=18 GHz, b=1 S/m^2, θ=60°

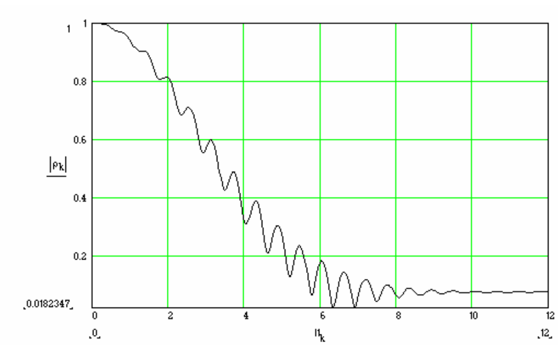

Fig.3 Reflection coefficient modulus as a function l/λ, for f=18 GHz, b=1 S/m^2, θ=30°

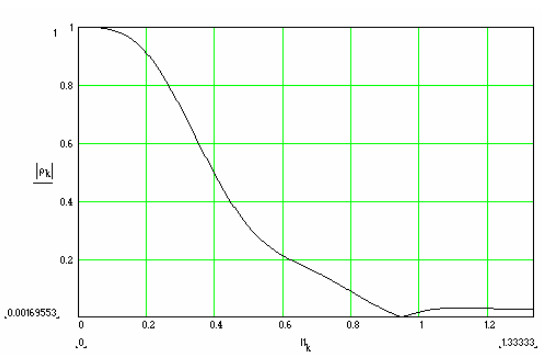

Fig. 4 Reflection coefficient modulus as a function of l/λ, for f=2 GHz, b=1 S/m^2, θ=0°.

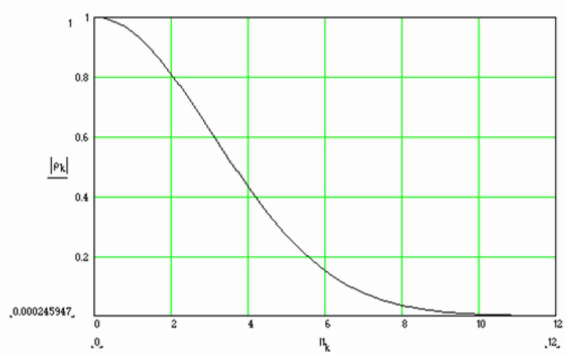

Fig.5 Reflection coefficient modulus as a function of l/λ, for f=18 GHz, b=1 S/m^2, θ=0°.

3.3 Reflection coefficient variation against the incidence angle

The incidence angle is very important for radar cross section (RCS). It has to be taken into account for RCS prediction because it will produce a random variation of the received signal. On the same time, when designing a radar system, the expected minimum value of RCS has to be known in order to establish the sensitivity of the receiver or the range of the radar. Several simulations had been made for different values of incidence angle and some of them are presented below.

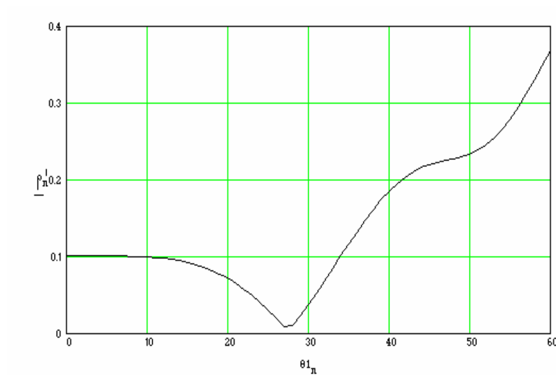

Fig. 6. Reflection coefficient modulus as a function of incidence angle ($\theta 1_n$ in degrees) for a structure with following parameters: l=0.05m, f=10 GHz, b=5 S/m^2.

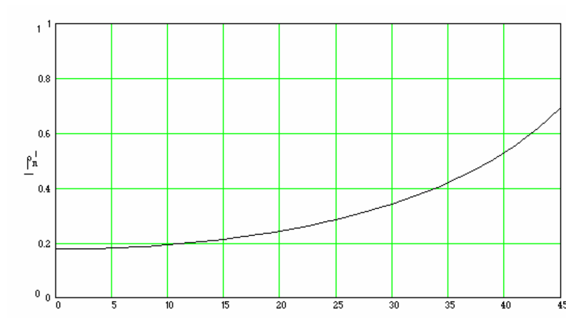

Fig.7 Reflection coefficient modulus as a function of incidence angle ($\theta 1_n$ in degrees) for a structure with following parameters: l=0.05 m, f=2 GHz, b=5 S/m^2.

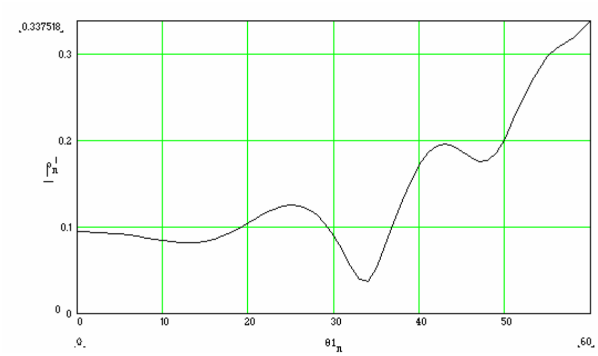

Fig.8 Reflection coefficient modulus as a function of incidence angle ($\theta 1_n$ in degrees) for a structure with following parameters: l = 0.05 m, f = 18 GHz, b = 5 S/m^2.

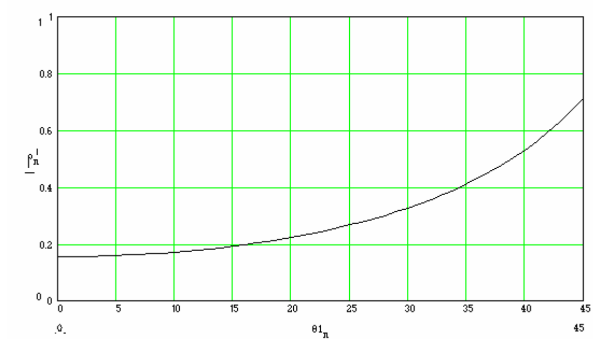

Fig. 9. Reflection coefficient modulus as a function of incidence angle ($\theta 1_n$ in degrees) for a structure with following parameters: l = 0.1 m, f = 2 GHz, b = 5 S/m^2.

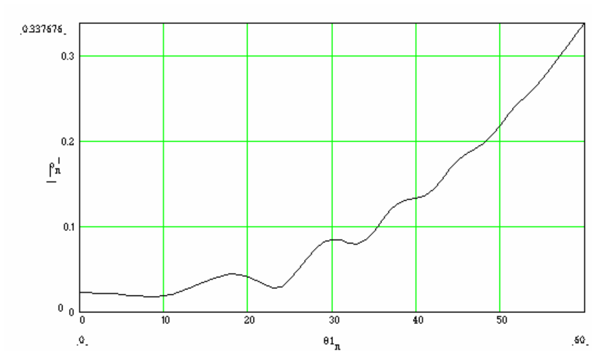

Fig. 10. Reflection coefficient modulus as a function of incidence angle ($\theta 1_n$ in degrees) for a structure with following parameters: l = 0.1 m, f = 18 GHz, b = 2 S/m^2.

3.4 Reflection coefficient variation against conductivity

In order to decrease the reflection coefficient, a part of the incident energy has to be transformed in heat within the material layer.

This process depends on the conductivity of the material. The higher the conductivity is the higher the losses are. High values of the conductivity, at the interface between free space and material, will increase the reflection at the interface, so a tapered conductivity variation is needed. The results obtain for different values of the conductivity gradient are presented below.

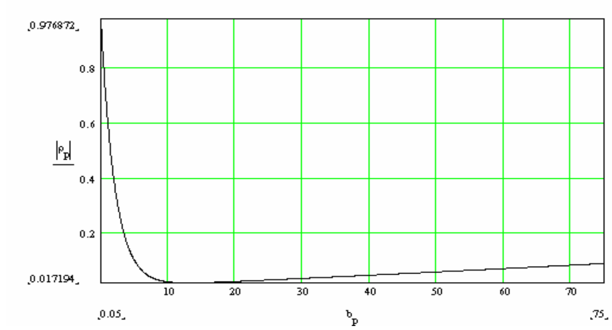

Fig.11 Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) for a structure with following parameters =10 GHz, l=.05m, θ=0°.

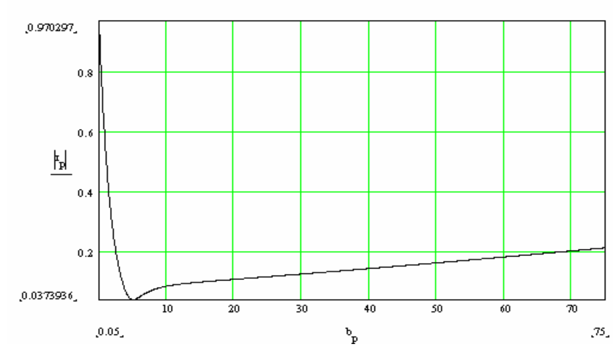

Fig. 12. Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) for a structure with following parameters: f = 10 GHz, l =. 05m, θ = 30°.

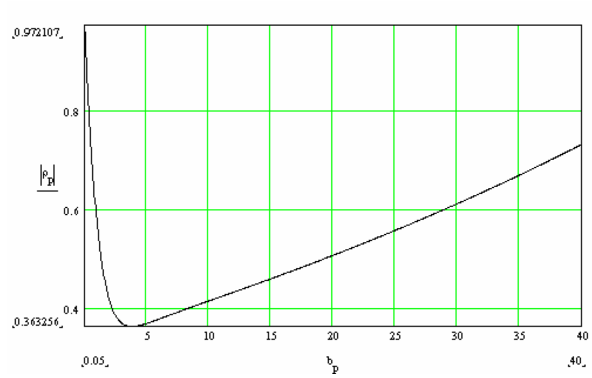

Fig.13 Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) for a structure with following parameters: f = 10 GHz, l =. 05m, θ = 60°.

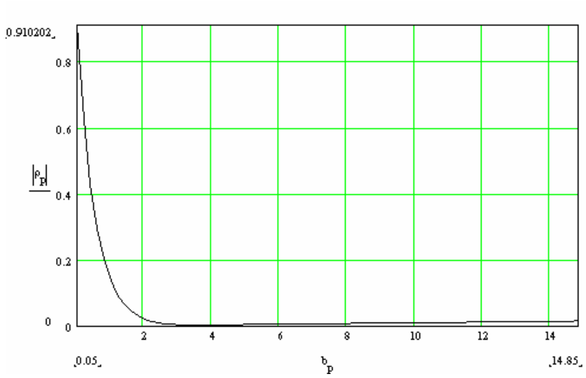

Fig.14 Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) for a structure with following parameters: f=10 GHz, l=.1m, θ=0°.

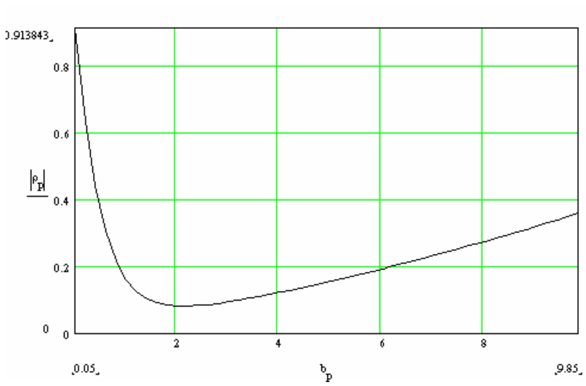

Fig.15 Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) f or a structure with following parameters: f=2 GHz, l=.1m, θ=0°.

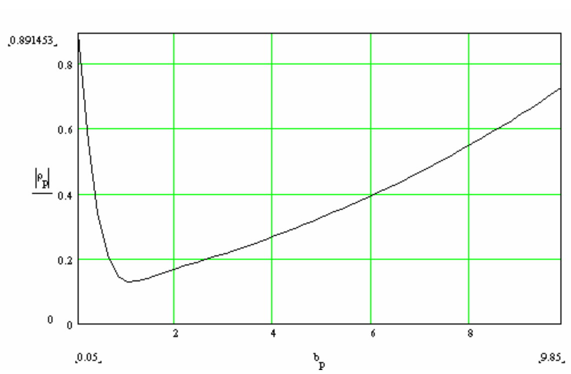

Fig.16 Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) for a structure with following parameters: f=2 GHz, l=.1m, θ=30°.

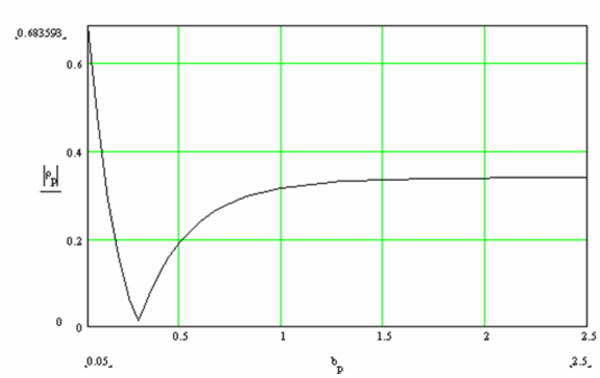

Fig.17 Reflection coefficient modulus as a function of conductivity gradient (b_p in S/m^2) for a structure with following parameters: f=18 GHz, l=.1m, θ=60°.

3.5 Reflection coefficient variation against frequency

The frequency range of a radar absorbing material is very important. The radars used for surveillance work in different frequencies bands from hundreds of MHz to tens of GHz. The higher the frequency is the higher the resolution of the radar. Although there are some materials used to protect different targets against electromagnetic radiation in a narrowband, usually the designers are looking for materials working for a frequency range as large as possible. The investigations made in this paper start from 2 GHz and go to 40 GHz. This frequency range has been chosen because most of the operational radars used today work in this frequency range.

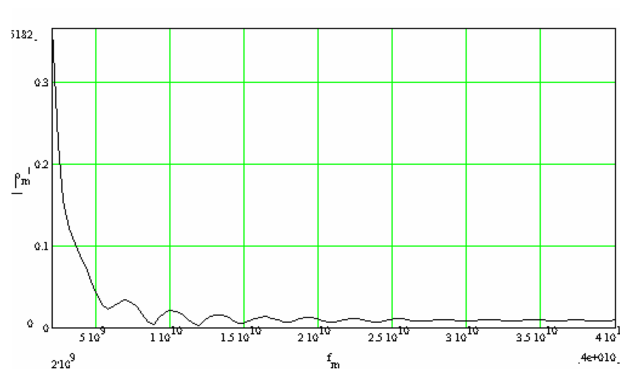

Fig.18 Reflection coefficient modulus as a function of frequency (f_m in Hz) for a structure with following parameters: l=0.05m, b=10 S/m^2, θ=0°.

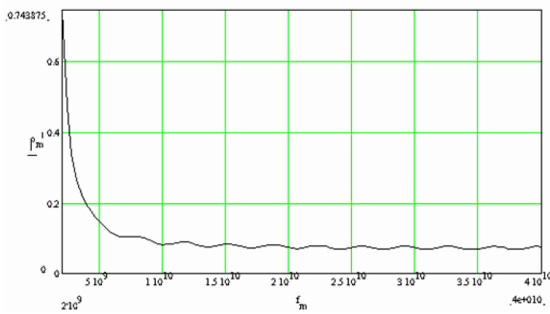

Fig.19 Reflection coefficient modulus as a function of frequency (f_m in Hz) for a structure with following parameters: l=0.05 m, b=10 S/m^2, θ=30°.

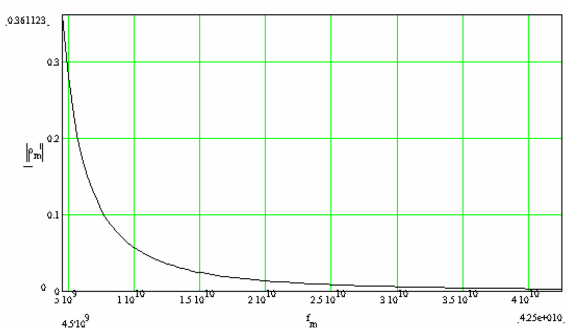

Fig.20 Reflection coefficient modulus as a function of frequency (f_m in Hz) for a structure with following parameters: l=0.05 m, b=50 S/m^2, θ=0°.

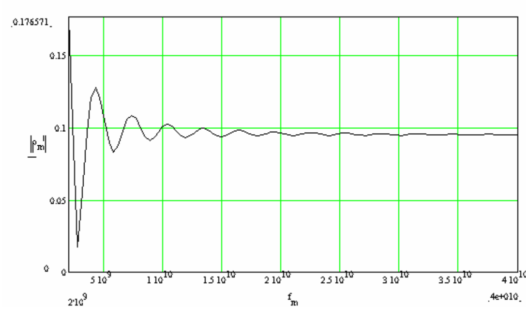

Fig.21 Reflection coefficient modulus as a function of frequency(f_m in Hz) for a structure with following parameters: l=0.05 m, b=5 S/m^2, θ=0°.

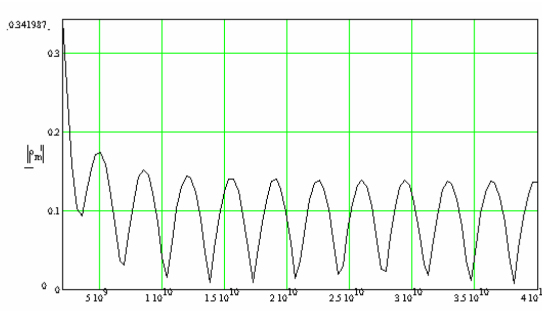

Fig.22 Reflection coefficient modulus as a function of frequency (f_m in Hz) for a structure with following parameters: l=0.05 m, b=5 S/m^2, θ=30°.

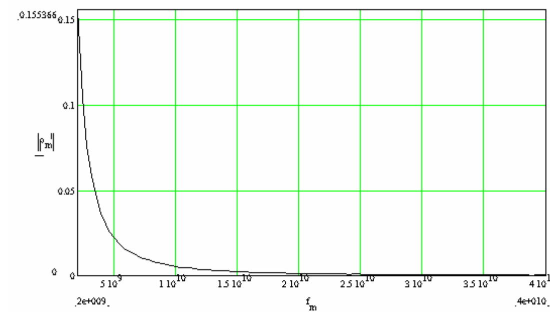

Fig.23 Reflection coefficient modulus as a function of frequency (f_m in Hz) for a structure with following parameters: l=0.1 m, b=5 S/m^2, θ=0°.

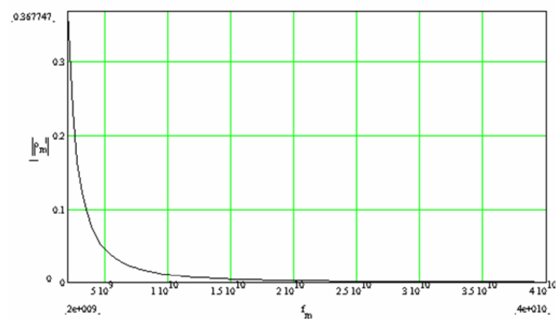

Fig.24 Reflection coefficient modulus as a function of frequency (f_m in Hz) for a structure with following parameters: l=0.1 m, b=10 S/m^2, θ=0°.

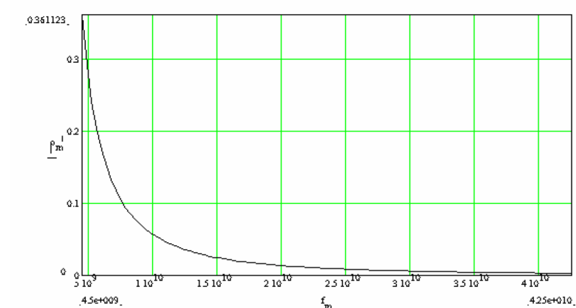

Fig.25 Reflection coefficient modulus as a function of frequency(f_m in Hz) for a structure with following parameters: l=0.1 m, b=50 S/m^2, θ=0°

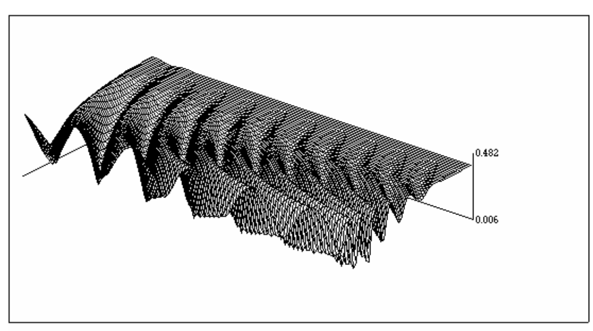

Fig.26 Reflection coefficient modulus for $\theta \in$ [0°,60°] and $f \in$[2,18] GHz, b=0.5 S/m^2, l=0.1 m

4. Conclusions

For given values of frequency, incidence angle and conductivity gradient there is an optimum thickness of the material layer which minimises the reflection coefficient.

The minimum value of reflection coefficient decreases as frequency increases, for instance, at normal incidence and for b=5 S/m^2, the following values had been obtain: ρ_{min}=0.1295, for f=2 GHz and a thickness of 6 cm; ρ_{min}=0.001238, for f=10 GHz and a thickness of 7.5 cm ; ρ_{min}=0.000303, for f=18 GHz and a thickness of 7.16 cm.

For l=0.05m and b=5 S/m^2, due to the interferences between direct and reflected waves, the reflection coefficient has a high ripple.

If the incidence angle is high, θ=60°, the behaviour of the material layer is less thickness dependent.

The modulus of reflection coefficient decreases approximately exponentially with frequency.

If the thickness, the frequency and the conductivity gradient are constant the lowest reflections are obtain at normal incidence.

The modulus of reflection coefficient increases almost after a squared function with the increase of the incidence angle and, if the incidence angle is kept constant, the minimum value of the reflection coefficient decreases as frequency increases.

When the working frequency is high and the layer is thin there is an incidence angle for which the reflections are lower than for normal incidence. This result may be used to choose the positions of the antennas in an anechoic chamber.

If all the other parameters are constant there is a value of the conductivity gradient which minimises the reflections.

The simulations made as a function of the conductivity showed that the reflection coefficient modulus decreases as the gradient increases until a certain value is reached. Than it increases with a steep rate direct proportional with the incidence angle and inverse proportionally with frequency, for the same thickness of material layer.

As frequency increases the value of the conductivity gradient, which minimises the reflections, decreases with the increase of incidence angle.

The dependency of the reflections on conductivity gradient is stronger for higher incidence angle.

The thickness of the layer, which minimises the reflections, can be chosen form the pictures presented if the frequency, the conductivity and the incidence angle are known. The best value obtained in the simulations is $1.88*10^{-4}$ (≅-74.516 dB) for b=5 S/m^2, θ=0°, f=10 GHz and 16.5cm thickness.

The tapered conductivity material can be manufacture by mixing a low permitivity dielectric material as polyethylene, polyvinyl chloride or rubber with a high conductivity carbon black.

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