

The fitting of magnetic hysteresis curves using the Jiles -Atherton model(II)

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This paper presents the way of designing a computer programme made by using Visual Basic, a software which fits ferromagnet experimental hysteresis curves. The application makes use of the equations from Jiles-Atherton model in the differential form. The software itself succeeds in bringing nearer the theoretical and the experimental curves with an average relative deviation which for some materials is under 1%. The parameters which intervene in the model equations are determined with an under 2% precision. By determining them we can draw certain conclusions referring to the magnetic properties of magnetostriction phenomena at a micro level. The programme might be developed and used both to anticipate the magnetic properties of some materials which have not yet been created and to study magnetostriction phenomena.

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1. Introduction

In this paper the equations of the Jiles Atherton[1] model are used in order to represent the magnetic hysteresis curve and to fit experimentally-obtained curves for certain materials. The designed computer programme uses these equations even for the fitting of some magnetization curves which were obtained under stress and, as we shall see, its accuracy is very precise. The “c” and “a” coefficients which are part of this model’s equations are determined by using other equations than the ones used by Jiles[2,3]. The very good results obtained in the fitting process show that the inferred equations which are used in calculating these coefficients are more easily-used in the computer programme in relation with those inferred by Jiles.

There is not a unique theory to thoroughly describe the entire range of hysteresis curves’ shapes for ferromagnetic materials. The Jiles-Atherton Model describes the sigmoid shapes of these loops quite well. The aim of this model is to find the dependence of magnetization according to field H and which, when graphically represented, should overlap as well as possible the experimental hysteresis curves. This phenomenological model is a construction which is based on a series of hypotheses concerning the manifestation of magnetism at the micro level of matter and we are going to tackle these hypotheses further on. The first hypothesis considers the fact that each domains’ magnetic moment is placed in an effective magnetic field which increases linearly with the magnetization of the sample. $H_e = H + \alpha M$, where α is a coefficient which can be interpreted as a measure of the coupling between the adjoining magnetic domains. The second hypothesis refers to the fact that the material has a series of anisotropies caused by impurities which can be in most of the cases nonmagnetic, faults, inhomogeneities of mechanical tension, etc.- which are

called pinning sites. Another hypothesis is that in the material there is a homogeneous spatial distribution of the pinning sites. It is known that the magnetization of the substance takes place through two mechanisms: the motion of the domain walls and the mechanism of magnetic moments’ rotation. The “pinning site” coining comes from the fact that the domain walls, during their motion under the action of the exterior magnetic field, remain temporary impeded in these zones and when the field increases with a minimum quantity, they will irreversibly move beyond the above mentioned zone. The pinning of these zones is made by losing energy. The model suggested by Jiles and Atherton draws a balance of the energies involved in the magnetization process, their calculation on a certain basis, and, in the end, an equation involving the magnetization and the field is obtained. This is a differential equation which can be directly integrated only if certain approximations are made, or in specific conditions. However, the equation can be numerically integrated. The dependence of the magnetization on the applied field, i.e. the classical hysteresis loop, can be represented with the help of the above mentioned equation. The fact that the mentioned differential equation has four parameters which are determined within the fitting programme must be underlined. The fitting procedure is based on a series of successive steps in which the average relative deviation in absolute value is gradually minimized. The average relative deviation is the average of the relative errors in absolute value calculated point by point on the entire curve of the hysteresis and is given by:

$$\frac{\sum_{i=1}^N \left| \frac{M_i - M_{Exp\ i}}{M_{Exp\ i}} \right|}{N}, \quad (1)$$

where M_i represents the value of magnetization calculated by the computer programme for a H_i field very close to the experimental H_{expi} and M_{expi} is the value of magnetization experimentally determined for the H_{expi} magnetic field. The computer programme makes a gradual variation of the magnetic field so that the theoretically evaluated points should not exactly cut across the experimental points but could be "as close as possible". Mention must be made that the fitting accuracy is limited by the simplifying hypotheses of the model. A better fitting accuracy is noticed for soft magnetic materials than for hard magnetic materials. The fitting accuracy highly depends on the value of the saturation magnetization. In order to obtain an as good as possible fitting it is necessary that the hysteresis curve should get as close as possible to the saturation.

2. The Jiles-Atherton model

The Langevin [4] equation which gives the magnetization according to the field applied to a paramagnetic substance is:

$$M = M_s \left(\coth \frac{\mu_0 m H}{k_B T} - \frac{k_B T}{\mu_0 m H} \right), \quad (2)$$

where m is the atomic magnetic moment. If we applied the same equation to a ferromagnetic material modifying the H field with its effective value within the material $H_e = H + \alpha M$, then the following equation would be obtained:

$$M_{an} = M_s \left(\coth \frac{H_e}{a} - \frac{a}{H_e} \right), \quad (3)$$

where $a = K_B T / \mu_0 m$ will be considered a coefficient which will be determined as a result of the fitting process. However, this equation does not describe the ferromagnetic hysteresis correctly. The magnetization calculated with the help of this equation is called anhysteretic and could describe the phenomenon of magnetization of an ideal ferromagnetic media in which there are not couplings between the magnetic moments or between adjoining magnetic domains, impurities, dislocations, inhomogeneities of mechanical tension, etc. In the case of the magnetization of an ideal ferromagnet (it does not have pinning sites), the magnetic energy spent during this process will be converted into potential magnetic energy in the volume of the magnetized material and its density is:

$$w = \int H dB_e = \frac{1}{\mu_0} \left(\int B_e dB_e - \int M_{an} dB_e \right), \quad (4)$$

where B_e is the effective magnetic induction within the material. In a cycle of hysteresis the term: $\int B_e dB_e = 0$. Therefore, the value of the energy density required for the magnetization of an ideal ferromagnet considered in

absolute value will be:

$$w = \frac{1}{\mu_0} \int M_{an} dB_e, \quad (5)$$

where $B_e = \mu_0(H + \alpha M)$. In the case of the magnetization of a real ferromagnet, a part of the magnetic energy used is wasted when the domain walls cut across the pinning sites. For this situation the potential energy density stored within the sample, in absolute value, will be:

$$w_R = \frac{1}{\mu_0} \int M dB_e \quad (6)$$

We shall note the energy density wasted when the domain walls cut across the pinning sites with E_{pin} . In the above mentioned hypotheses, Jiles shows that this energy density is:

$$E_{pin} = K \int_0^M dM. \quad (7)$$

The difference between the densities of potential magnetic energy in the case of ideal and real magnetization situations is the real density of dissipated energy E_{pin} . Therefore:

$$\frac{1}{\mu_0} \int_0^M M dB_e = \frac{1}{\mu_0} \int_0^M M_{an} dB_e - K \int_0^M dM. \quad (8)$$

This equation can be adjusted to:

$$dM = \frac{M_{an} - M}{\frac{K\delta}{\mu_0} - \alpha(M_{an} - M)} dH \quad (9)$$

where $\delta = 1$ if $dH/dt > 0$ and $\delta = -1$ if $dH/dt < 0$.

This is one of the basic equations of the model. There are a few phenomenological aspects which involve the readjusting of equation (9) and which implicitly lead to a better theoretical fitting of the experimental hysteresis loop. Suppose the sample has reached the saturation magnetization and starting from here the magnetic field starts to decrease. Until the field changes its direction it does not force any motion of the domain walls. Thus, as the walls are impeded precisely by the pinning sites they last cut across when field H increased, now when H starts to decrease, the change of magnetization cannot be made by the motion of the domain walls. Implicitly, the magnetization cannot be calculated by using equation (9). We call the magnetization calculated using equation (9) irreversible and we note it with M_{irr} . Therefore:

$$dM_{irr} = \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} dH \quad (10)$$

where $k = K/\mu_0$.

From (7) we infer that the value of k is proportional to the average energy lost as heat when a domain wall

irreversibly cuts across a pinning site.

Thus, the variation of magnetization in the saturation zone when the magnetic field gradually decreases is not due to the motion of the walls and, therefore, it has a reversible character (it is due both to the rotation of the domain moments, rotations which are caused by the anisotropies specific to the sample, and to the thermal agitation). Therefore, the total magnetization of the sample has both a reversible and an irreversible component: $M = M_{rev} + M_{irr}$. The irreversible component is given by equation (10) and the reversible one represents a certain percentage "c" from the difference of irreversible and anhysteretic magnetizations.

$$M_{rev} = c(M_{an} - M_{irr}) \quad (11)$$

Thus

$$dM_{rev} = c(dM_{an} - dM_{irr}), \quad (12)$$

and in the saturation zone $dM_{irr}=0$, this meaning that $dM_{rev}=c \cdot dM_{an}$.

Finally, the following equation yields:

$$M = cM_{an} + (1 - c)M_{irr} \quad (13)$$

3. The Calculation of the "a, α, k, c" parameters

Jiles and Thoeke [2,3] calculate the "a, α, k, c" coefficients which appear in section II by using some of the equations from the mentioned section.

In these equations a series of differential susceptibilities (dM/dH sau dM_{an}/dH) arises, i. e. the derivatives of magnetization related to the H field at different points of the anhysteresis curve. We shall mark with χ'_{an} , χ'_{anmax} , χ'_{anc} , χ'_{anm} - the initial, remanent, coercive and saturation corresponding anhysteresis differential susceptibilities. The differential susceptibilities on the major magnetization curve corresponding to the already mentioned zones- χ'_{in} , χ'_{max} , χ'_c , χ'_m - are also included in these equations.

Therefore in the vicinity of $H=0$, $M=0$, as the result of the serial development, we have:

$$M_{an} \cong M_s \frac{H_e}{3a} = \frac{M_s}{3a} (H + \alpha M) \quad (14)$$

By using 13, we have:

$$M_{an} = \frac{M_s}{3a} [H + \alpha \cdot c \cdot M_{an} + \alpha \cdot (1 - c)M_{irr}] \quad (15)$$

For $H=0$ we will have $M_{an}=0$. We have in view the fact that in this zone the irreversible component of the magnetization is null (for very small fields the domain

walls perform reversible motions.). $M_{irr}=0$. What results from equation (10) is:

$$\left. \frac{dM_{irr}}{dH} \right|_{H=0} \cong 0. \quad (16)$$

Taking into consideration equation (13) and (16) we may write:

$$\chi'_{in} = \left. \frac{dM}{dH} \right|_{H=0} \cong c \left. \frac{dM_{an}}{dH} \right|_{H=0} \quad (17)$$

As a matter of fact, equation (17) presents "c" as the relation between the differential magnetic susceptibilities calculated in the beginning zone of the curve of first magnetization:

$$c \cong \frac{\chi'_{in}}{\chi'_{an}}. \quad (18)$$

The following two equations are presented as they were calculated by Jiles and Thoeke [3]:

$$k = \frac{M_{an}(H_c)}{1 - c} \left\{ \alpha + \frac{1}{\chi'_{max} - \frac{c}{1 - c} \cdot \frac{dM_{an}(H_c)}{dH}} \right\}, \quad (19)$$

$$M_R = M_{an}(M_R) + \frac{k}{\frac{\alpha}{1 - c} + \frac{1}{\chi'_{rem} - c \frac{dM_{an}(M_R)}{dH}}}, \quad (20)$$

M_R is the remanence magnetization.

Knowing the "c" coefficient from 18 we can find „k” and respectively "α" from 19 and 20.

By deriving equation 15 in $H=0$ and taking 16 into account, we have:

$$\begin{aligned} \left. \frac{dM_{an}}{dH} \right|_{H=0} &\cong \frac{M_s}{3a} \left(1 + \alpha \cdot \left. \frac{dM}{dH} \right|_{H=0} \right) = \\ &= \frac{M_s}{3a} \left(1 + \alpha \cdot c \cdot \left. \frac{dM_{an}}{dH} \right|_{H=0} \right). \end{aligned} \quad (21)$$

From here the following results:

$$\chi'_{an} \cong \frac{M_s}{3a} (1 + \alpha \cdot \chi'_{in}) \quad (22)$$

or,

$$\chi'_{an} \cong \frac{M_s}{3a - \alpha \cdot c \cdot M_s}. \quad (23)$$

From 23 we have:

$$a \cong \frac{M_s}{3\chi'_{an}} + \frac{\alpha \cdot c \cdot M_s}{3} \quad (24)$$

In conclusion, if the graph of a magnetization curve, including the anhysteresis curve, is known, we can calculate the "a, α , k, c" coefficients by using the equations 18, 19, 20, 24 and calculating the susceptibilities involved in these equations first.

In the calculations that Jiles made, the following approximation is used near $H=0$:

$$M_{an} \cong M_s \frac{H + \alpha M_{an}}{3 \cdot a} \quad (25)$$

(we use equation 14). If we use equation 25, then 24 becomes:

$$a \cong \frac{M_s}{3\chi'_{an}} + \frac{\alpha \cdot M_s}{3} \quad (26)$$

Real situations show that the second term on the right side of 24 or 26 is negligible in relation with the first, therefore in the process of calculating the "a" coefficient, the differences between the two equations do not have essentially different solutions. These differences are under 0,3%.

4. The method of fitting the hysteresis curves

In order to fit the experimental hysteresis curves a mathematical technique which will be described further on was used. In the computer programme H takes values between 0 and $+H_{max}$, then between $+H_{max}$ and $-H_{max}$ and finally between $-H_{max}$ and $+H_{max}$. The passing between the above mentioned extremes is made through the gradual modification of the value of H with a ΔH quantity much smaller than the difference between two consecutive experimental values. The value of ΔH is established in the reading subroutine of the initial data. Each modification of the field with the ΔH value involves a modification of the irreversible magnetization with the value ΔM_{irr} as given by equation (10), thus M_{irr} becomes $M'_{irr} = M_{irr} + \Delta M_{irr}$. For the calculation of the anhysteretic magnetization equation (3) is used and then it is replaced in equation (13) together with M'_{irr} . Starting from these aspects, the computer programme is going to graphically represent the points (H, M) by using the above mentioned equations so that these points might be as close as possible to the experimental ones. The programme has four main parts. Zero procedure refers to the experimental data uploading and the initializing of the variables used in the programme. Here, we also calculate the saturation magnetization by extrapolating the experimental data. Procedure I consists in the calculation of the value of magnetization for a value of the variable magnetic field applied to the sample, as we have already described, then the calculation of the sum of the relative deviations of the theoretical values of

magnetization related to the experimental ones for a cycle of hysteresis (the magnetic field H has values between $-H_{max}$ and $+H_{max}$ and vice versa). This deviation is given by equation (1). The same procedure also graphically represents the points corresponding to the experimental hysteresis loop and to the loop calculated in the Jiles Atherton model. In the procedure III a certain condition corresponding to a minimum difference between the two plots is fulfilled. In fact, the execution of the programme is finalized then. Procedure II: the modification of the "a, α ,k,c" parameters plus or minus the quantities " $\pm\Delta a$, $\pm\Delta\alpha$, $\pm\Delta k$, $\pm\Delta c$ " so that the deviation calculated in procedure I is minimised. Therefore, procedure II appeals, in turns, to procedure I for each of the four parameters. As a result of the appeal, the value S_j of the deviation given by I is calculated each time and, in this way, by comparing it with its previous value, S_{j-1} , the choice whether to modify the referred parameter plus or minus one of the mentioned quantities is made. Procedure III: resumes the process of minimising the average relative deviation calculated by using equation (1). This procedure appeals to procedures I and II until the difference between two consecutive average relative deviations is below the fixed value (usually less than 10^{-12}) from the value of the previous deviation. This procedure has a subroutine through which, if the minimising cannot be made any longer, then the quantities " $\pm\Delta a$, $\pm\Delta\alpha$, $\pm\Delta k$, $\pm\Delta c$ " are lowered to the values: " $\pm\Delta a/2$, $\pm\Delta\alpha/2$, $\pm\Delta k/2$, $\pm\Delta c/2$ ". Then the appellation of the mentioned procedures is next. A simplified logical scheme of the programme is shown in Fig. 1.

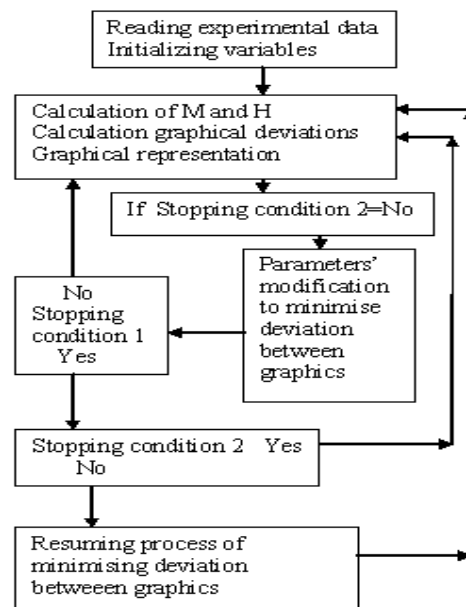


Fig. 1 The simplified logical scheme of the programme

In order to fit the hysteresis loops for the samples subjected to mechanical stress we can use the same equations that are used by the programme for the fitting of the curves in which the sample is not under stress and we

expect that as a result of the fitting process by the computer all the coefficients of the model should modify more or less. However, the question is whether the values of these coefficients are the real ones or not.

For a given deviation between the experimental and theoretical plots, there could be more solutions for the mentioned coefficients. The real values of the coefficients could be calculated using the equations established in section III (eq.18,19,20,24). A programme subroutine can be appealed after the fitting has finished and this one will calculate of the “a, α , k, c” coefficients. Then the loop corresponding to the model is plotted. The practice shows that there are minor differences between the loops and that through both methods close values are obtained. As a matter of fact, this is the verification of the method. The conclusion is that the application fits the experimental loop through two methods. The first has in view the modification of the “a, α , k, c” parameters until the theoretical and experimental graphs are optimately close. The second method uses the graph from the first method and determines the “a, α , k, c” coefficients with the help of the equations from section III. In fact, this method gives the real values for the mentioned coefficients from a physical point of view. We are going to further name these methods 1 and 2.

5. About the interface of the programme

In order for the application to be functional it is necessary that the experimental data should be read from a text file with the numbers that represent the values of the field and of the magnetization written on two columns with space between them (more than 3 characters) or separated by commas. This text file must have the name “his” and must be in the same directory as the executable file corresponding to our fitting programme. We specify that the values of the magnetization and of the field must be given in fundamental measuring units, A/m, otherwise the computer programme will have inherent errors of calculation. The interface is easy to use; it contains a series of buttons which, when clicked on, they will produce the specified effects. The button which starts the rolling of the programme has “Start” on it. By pressing it, the rolling of the fitting programme will start without taking into consideration the α coefficient. Then you may press the button labelled “ α ” and the programme will be resumed including this coefficient. There is a button with the help of which the fitting will be done more and more precisely; it is labelled “Error reducing”. It will be appealed the moment we want to get the theoretical curve closer to the experimental one.

The interface has also buttons which can have as effect vertical or horizontal translations of the experimental graphic affected by measuring errors which lead to such deviations. There are buttons with functions representing the error bars (up to 5%) for each experimental point and displaying the graphic of the hysteresis of the $\lambda(H)$ magnetostriction coefficient.

Next figure (fig. 2) presents the interface of the application before the fitting programme rolling:

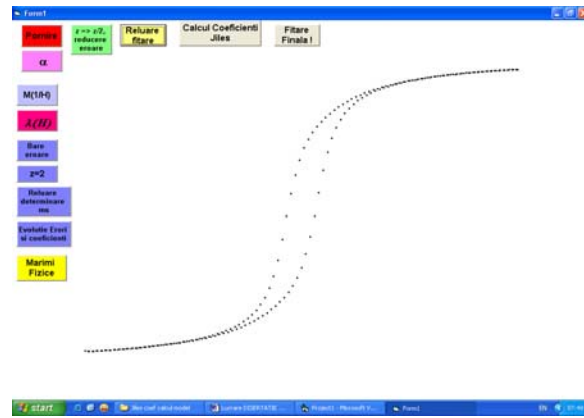


Fig. 2

The experimental points that form the experimental loop of the magnetic hysteresis can be noticed in relative coordinates (H/H_{max} , M/M_S).

The buttons that must not be pressed appear in the interface only at the right moment. An optimum use is made after several attempts. The values of the four “a, α , k, c” coefficients and other values of the necessary parameters are displayed in the left down corner of the screen at the end of the programme rolling, such as: the saturation magnetization (which the programme determines from the very beginning when the data are being uploaded through a fitting method), the coercive field, the remanent magnetization, the average relative error expressed in percentage (calculated by using equation 1), the number of experimental data, etc.

After the ending of a fitting process, the process can be resumed with a more precise fitting so that the average relative error might drop more. We can stop resuming the fitting when the error starts to modify somewhere at the sixth decimal. After these stages the interface corresponding button that appeals to the procedure for the calculation of the coefficients using the equations deduced by Jiles and Thoenke can be pressed. For the last phase we appeal to the procedure of graphical representation with the help of these coefficients.

6. Results

The experimental data used by the application are determined by a vibrating samples magnetometer (VSM). The samples which were the focus of the fitting process are: the first one, a cobalt ferrite and, the second one a 60%CoFe₂O₄ + 40%BaTiO₃ composite. The values of the coefficients and of the other specific magnetic measures can be noticed in table 1. It can easily be shown that the value of “a” is directly proportional to the density of the magnetic domains and the value of “k” is directly

proportional to the density of the pinning sites. Following the dynamic of these coefficients certain conclusions concerning the state and magnetic behaviour of the two materials can be drawn. This fitting example argues, through the obtained results, that the fitting programme proves to be useful and practical in studying magnetic materials.

Tabel 1. A comparison between the coefficients of the Jiles model for the used ferrimagnetic samples

	CoFe ₂ O ₄ Cobalt Ferrite	60%CoFe ₂ O ₄ +40%BaTiO ₃ Composite
ρ (g/cm ³)	4,89	3,81
Ms (kA/m)	415	156
Mrem (kA/m)	175	62,4
Hc (kA/m)	20	51,6
α -	0,1560	0,85
k(A/m)	21180	53964
c -	0,000189	0,000074
a(A/m)	31966	74656
Err	0,96%	3,13%

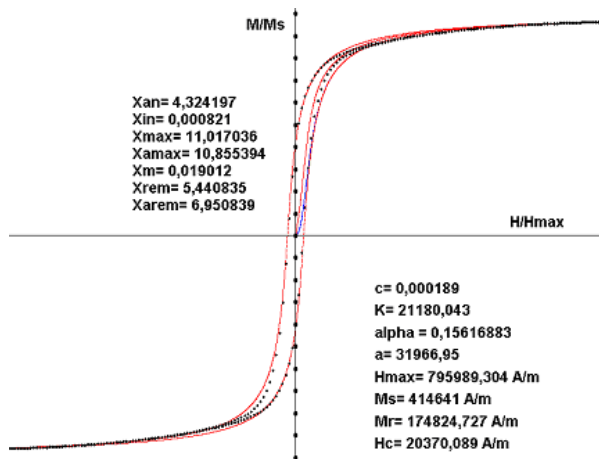


Fig. 3. Fitting graph- CoFe₂O₄.

A few conclusions regarding the dynamic of the coefficients:

- the increase of k implies a higher density of the pinning sites for the composite material;
- the value of a implies an almost double density of magnetic domains for the composite;
- the value of α must be understood as a measure of the interdomain coupling, otherwise the composite has more closely-related domains than the normal ferrite.

Concerning the values obtained by the fitting

programme it may also be added that in the case of the composite the decrease of the saturation magnetization and the increase of the coercive field take place, which was expected because the barium titanite leads to the increase of the number of the pinning sites within the structure of the magnetic material.

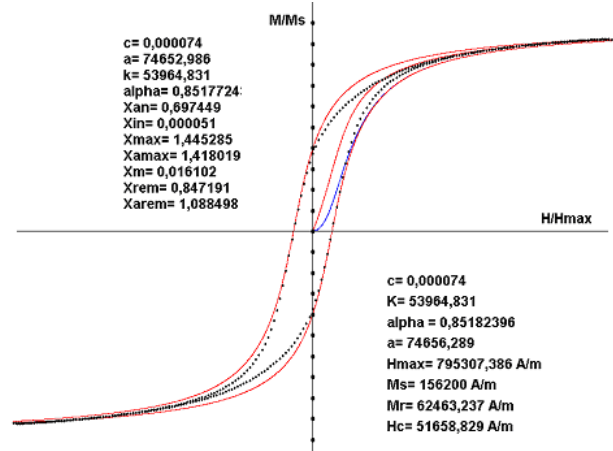


Fig. 4. Fitting graph-60%CoFe₂O₄ + 40% BaTiO₃

7. Conclusions

The designed software based on Jiles model can be used in fitting the experimental hysteresis loops of magnetic materials with high accuracy. The purpose of the fitting programme was the well-precise determination of the values of the model's coefficients for a certain material. This purpose was achieved. Moreover, this calculating programme can also be used for fitting the magnetization curves of samples under mechanical stress or under other interaction conditions and, in this way, the dynamic of the model's coefficients in the given situations can be observed. These coefficients, through their physical meanings, can provide useful information regarding the microstructure of magnetic materials and of the interactions at this level. Besides these aspects, the application also calculates the values for: the saturation magnetization, the remanent magnetization, the coercive field and the susceptibilities presented in section III. It may be noticed that in the case of the composite the average relative deviation between the graphs is about three times bigger than in the case of the cobalt ferrite. This thing shows that the more we deviate from a homogenous and isotropic structure of the magnetic material, the bigger the fitting errors are just because the model does not take into consideration heterogeneous structures.

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References

- [1] D. C. Jiles, D. L. Atherton, J. Appl.Phys. **55**, 2115 (1984).
- [2] D. C. Jiles, J. B. Thielke, IEEE Trans. Magn. **25**(5), 3928 (1989).
- [3] D. C. Jiles, J. B. Thielke, M. K. Devine, IEEE Trans. Magn. **28**(1), 27 (1992).
- [4] M. Langevin, Ann. Chem. Phys. **5**, 70 (1905).

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