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The influence of dimensional and structural shifts of the elastic constant values in cylinder fiber composites

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The paper analyses the main dimensional and structural shifts that appear in fiber-reinforced composites. Long, aligned fibers are considered to be made up of upright circular cylinders in a regular network. In fact, this arrangement is purely theoretical and impossible to achieve in practice. Dimensional and structural inconsistencies can appear due to the fact that fibers cannot be placed at equal distances, cannot be parallel and layers cannot be placed at equal distances. The paper presents the determination of shifts from theoretical formulae in the structure of composites.

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1. Introduction

During manufacture and afterwards, in the use of composites, a series of imperfections can appear, as well as chemical and diffusion phenomena that will lead to the existence of a significant difference between the composite and its theoretical model. In this case it can be seen, e.g. that the models of hexagonal or rectangular networks are far from the actual reinforced composites with hexagonal fibers.

The following differences can be noted between theoretical models and real materials:

- Differences between the theoretical size of mechanical features of the matrix and reinforcement material and their real values. Due to manufacturing processes, these values in the technical notes can shift from nominal values, which can be influenced by the manufacturer as well. It is necessary to make an analysis in order to establish whether these possible shifts can significantly influence the results obtained with calculations in formulae;
- Differences between the theoretical geometric shape of the reinforcement materials and the geometric shape of the materials used. The precise shapes cannot be obtained in fabrication. For instance, it is difficult to obtain cylinder fibers, due to the shift from circular shape in the manufacturing procedure. It is also difficult to obtain spherical reinforcement materials, as their deviation is smaller or larger than the theoretical shape. Studies are needed to determine the extent to which the deviation can influence the calculations, considering the theoretical geometric shape;
- Dimensional differences of the reinforcement materials. These differences will naturally appear in the fabrication process and it is compulsory to know the extent to which dimensional deviations can influence the behavior of the composite material;
- Differences in the geometric layout of the reinforcement materials. In the studied models, the fibers are considered to be in perfectly hexagonal model or rectangular model, which in many cases is far from being true. In this case, studies must be made to determine how a particular layout with high dispersion, as compared to the theoretical model, will influence the behavior of the composite material.

All these shifts will alter the results obtained on theoretical models (presented, e.g. in [1] and it is necessary to make an analysis to determine which of the formulae used by various authors are the most adequate to analyze a certain type of composite.

2. The influence of dimensional and structural deviations on the value of elastic constants

Dimensional and structural differences of fiber composites can lead to mechanical properties that are different in the real material as compared to the theoretical considerations in ideal conditions of manufacture and use. In [2] an analysis was made of the factors that cause deviations from the theoretical aspects and it was noted that in certain cases these differences could be very big. Measurements to confirm this have been carried out in [4-7]. Fig.1 and fig.2 present, after [3], the distribution of diameters in two types of SMC composites. A normal distribution of diameters is seen. In this case, dispersion becomes important for the considered distribution. In the following, an analysis will be made of the influence of dimensional variations on the values of certain mechanical constants. Let us note dispersion σ and mean diameter of the fiber d, for a hexagonal network:

$$v_f = \frac{\pi d^2}{4S}$$

where v_f is the fibers volume fraction and S represents the section of the hexagonal cell [2].

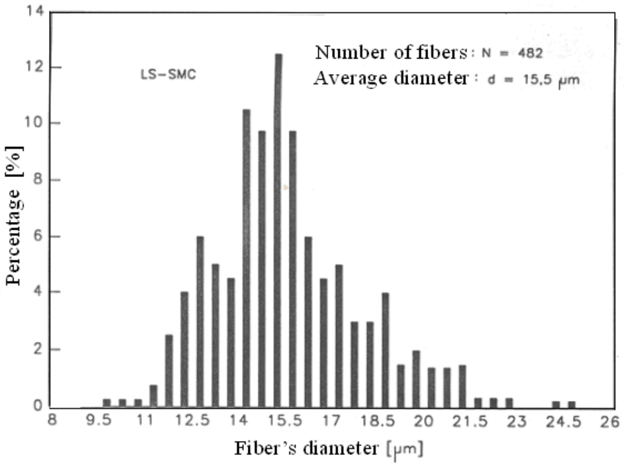

Fig. 1. Distribution of fibers for a SMC, charge 164

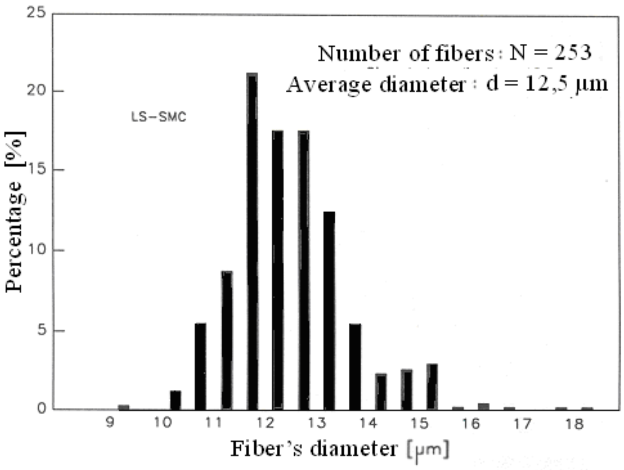

Fig. 2. Distribution of fibers for a SMC, charge 150

In this case, if the diameter of the fiber varies with $\pm\,\Delta d$, the fiber percentage varies within the limits:

$$v_{f(\min-\max)} = \frac{\pi(d \pm \Delta d)^2}{4S} = \frac{\pi d^2 \pm 2\pi d\,\Delta d + \pi(\Delta d)^2}{4S} =$$

$$= \frac{\pi d^2}{4S} \pm \frac{2\pi d\,\Delta d}{4S} + \frac{\pi(\Delta d)^2}{4S} \cong$$

$$\cong \frac{\pi d^2}{4S} \pm \frac{\pi d^2\, 2\Delta d}{4S d} = v_f \pm v_f \frac{2\Delta d}{d} =$$

$$= v_f (1 \pm \frac{2\Delta d}{d}) = v_f (1 \pm 2\varepsilon)$$

Size $(\Delta d)^2$ is considered very small and thus could be neglected and it was noted:

$$\varepsilon = \frac{\Delta d}{d} .$$

It follows from here that for a 10 % variation in the fiber diameter, a variation of 20 % in the procentual volume is obtained. Let us analyze the influence of this variation on the behavior of the composite.

Let us consider the formulae that give the volume module, longitudinal elasticity module and the Poisson coefficient:

$$\frac{\hat{v}_f k_f (k_m + m_m) + \hat{v}_m k_m (k_f + m_m)}{\hat{v}_f (k_m + m_m) + \hat{v}_m (k_f + m_m)} \le k \le$$

$$\le \frac{\hat{v}_f k_f (k_m + m_f) + \hat{v}_m k_m (k_f + m_f)}{\hat{v}_f (k_m + m_f) + \hat{v}_m (k_f + m_f)}$$

$$\hat{v}_f E_f + \hat{v}_m E_m + \frac{4\hat{v}_f \hat{v}_m (\nu_f - \nu_m)^2}{(\frac{\hat{v}_f}{k_m} + \frac{\hat{v}_m}{k_f} + \frac{1}{m_m})} \le E \le$$

$$\le \hat{v}_f E_f + \hat{v}_m E_m + \frac{4\hat{v}_f \hat{v}_m (\nu_f - \nu_m)^2}{(\frac{\hat{v}_f}{k_m} + \frac{\hat{v}_m}{k_f} + \frac{1}{m_f})}$$

$$\hat{v}_f \nu_f + \hat{v}_m \nu_m + \frac{(\nu_f - \nu_m)\hat{v}_f \hat{v}_m \left(\frac{1}{k_m} - \frac{1}{k_f}\right)}{\left(\frac{\hat{v}_f}{k_m} + \frac{\hat{v}_m}{k_f} + \frac{1}{m_m}\right)} \le \nu \le$$

$$\le \hat{v}_f \nu_f + \hat{v}_m \nu_m + \frac{(\nu_f - \nu_m)\hat{v}_f \hat{v}_m \left(\frac{1}{k_m} - \frac{1}{k_f}\right)}{\left(\frac{\hat{v}_f}{k_m} + \frac{\hat{v}_m}{k_f} + \frac{1}{m_f}\right)}$$

Consider the relative variation of diameter ε that result in the relations:

$$k^- = \frac{\hat{v}_f (1 \pm 2\varepsilon) k_f (k_m + m_m) + [1 - \hat{v}_f (1 \pm 2\varepsilon)] k_m (k_f + m_m)}{\hat{v}_f (1 \pm 2\varepsilon)(k_m + m_m) + [1 - \hat{v}_f (1 \pm 2\varepsilon)](k_f + m_m)}$$

$$k^+ = \frac{\hat{v}_f (1 \pm 2\varepsilon) k_f (k_m + m_f) + [1 - \hat{v}_f (1 \pm 2\varepsilon)] k_m (k_f + m_f)}{\hat{v}_f (1 \pm 2\varepsilon)(k_m + m_f) + [1 - \hat{v}_f (1 \pm 2\varepsilon)](k_f + m_f)}$$

where, further, the exponent is noted – the lower limit of the measurement considered and + the upper margin. For the longitudinal elasticity module and the Poisson coefficient, the following relations are obtained:

$$E^{-} = \hat{v}_f(1 \pm 2\varepsilon)E_f + [1-\hat{v}_f(1 \pm 2\varepsilon)]E_m + \frac{4\hat{v}_f(1 \pm 2\varepsilon)[1-\hat{v}_f(1 \pm 2\varepsilon)](v_f - v_m)^2}{(\frac{\hat{v}_f(1 \pm 2\varepsilon)}{k_m} + \frac{1-\hat{v}_f(1 \pm 2\varepsilon)}{k_f} + \frac{1}{m_m})}$$

$$E^{+} = \hat{v}_f(1 \pm 2\varepsilon)E_f + [1-\hat{v}_f(1 \pm 2\varepsilon)]E_m + \frac{4\hat{v}_f(1 \pm 2\varepsilon)[1-\hat{v}_f(1 \pm 2\varepsilon)](v_f - v_m)^2}{(\frac{\hat{v}_f(1 \pm 2\varepsilon)}{k_m} + \frac{1-\hat{v}_f(1 \pm 2\varepsilon)}{k_f} + \frac{1}{m_f})}$$

$$\hat{v}^{-} = \hat{v}_f(1 \pm 2\varepsilon)v_f + [1-\hat{v}_f(1 \pm 2\varepsilon)]v_m + \frac{(v_f - v_m)\hat{v}_f(1 \pm 2\varepsilon)[1-\hat{v}_f(1 \pm 2\varepsilon)]\left(\frac{1}{k_m} - \frac{1}{k_f}\right)}{\left(\frac{\hat{v}_f(1 \pm 2\varepsilon)}{k_m} + \frac{1-\hat{v}_f(1 \pm 2\varepsilon)}{k_f} + \frac{1}{m_m}\right)}$$

$$\hat{v}^{+} = \hat{v}_f(1 \pm 2\varepsilon)v_f + [1-\hat{v}_f(1 \pm 2\varepsilon)]v_m + \frac{(v_f - v_m)\hat{v}_f(1 \pm 2\varepsilon)[1-\hat{v}_f(1 \pm 2\varepsilon)]\left(\frac{1}{k_m} - \frac{1}{k_f}\right)}{\left(\frac{\hat{v}_f(1 \pm 2\varepsilon)}{k_m} + \frac{1-\hat{v}_f(1 \pm 2\varepsilon)}{k_f} + \frac{1}{m_f}\right)}$$

3. Examples

In the following, a numerical analysis is made. This is needed because the analytical representation of function charts for the main mechanical properties and deviations is very difficult to draw up, due to the form of the relations used. A numerical analysis is handier, especially thanks to the performance of existing calculation methods.

Calculations will be made for three situations. In the first case (case study no.1) a composite made up of a matrix of longitudinal module with elasticity 0.4 MPa and Poisson coefficient 0.35 is considered. The fiber's longitudinal elasticity module is 10.5 MPa and Poisson coefficient is 0.22. Graphic representation of the volume module margin is made considering the rapport between the radius of the fiber and the radius of the composite cylinder, then considering the procentual fiber volume to the total volume of the material.

In the second case (case study no 2) we consider the composite made of epoxy resin with longitudinal elasticity module equal to 2.7 MPa and Poisson coefficient equal to 0.35 while the fiber has longitudinal elasticity module of 72.4 and Poisson coefficient 0.22.

In the third case (case study no. 3), the same values for the fiber as in the previous example were taken and the matrix was considered to have a longitudinal elasticity module ten times smaller, having the same Poisson coefficient.

Let us start by considering the variations of mechanical properties in the two phases. The formulae used were presented previously. Let us consider the variation of the elasticity module of the fiber, because in reinforced materials with long fibers they decisively influence the behavior of the composite. Fig. 3 shows Poisson coefficient for case study no. 1, considering that the fiber's elasticity module varies by ± 10%. In this case a variation seen in this parameter and influences Poisson's coefficient, which abides by the law of mixtures and is not influenced by the variation of the other elastic parameters. Fig. 4 shows the same parameter in case study no. 2.

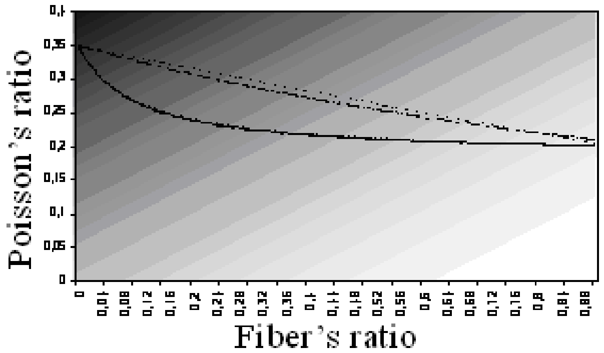

Fig. 3. Poisson coefficient for ±10% variations in the elasticity module for case study no. 1

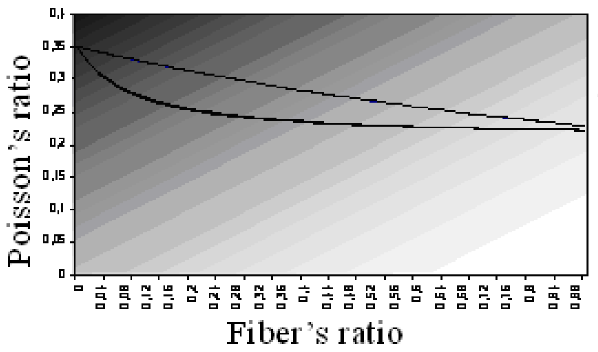

Fig. 4. Poisson coefficient for ±10% variations in the elasticity module for case study no. 2

For case studies no. 2 and 3, it is noted there is no difference regarding the variation of Poisson coefficient even if the longitudinal elasticity module varies greatly. This is not the case with the other properties of the material.

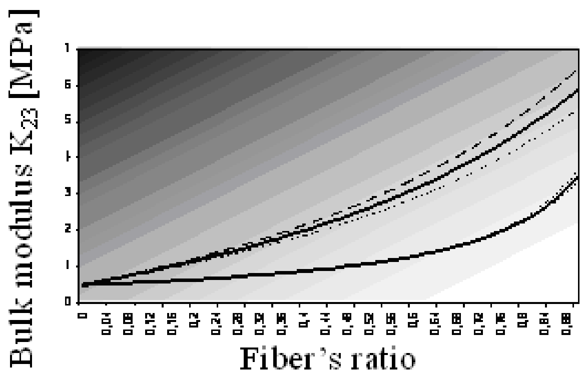

Fig. 5. Upper and lower margins for the volume module for ± 10% variations of the elasticity module in case study no. 1

Studying the variation of the volume coefficient depending on the concentration of the phases and considering that the properties of the fiber material can vary within a range of ± 10%, it is noted that the values obtained are situated pretty close to exact values. Thus, in this case, only for high procentual values of the fiber deviations start to appear from exact theoretical values. For small fiber concentrations, even a great deviation in the transversal elasticity module will lead to slight deviations from theoretical values in the margins of the volume module (fig.5).

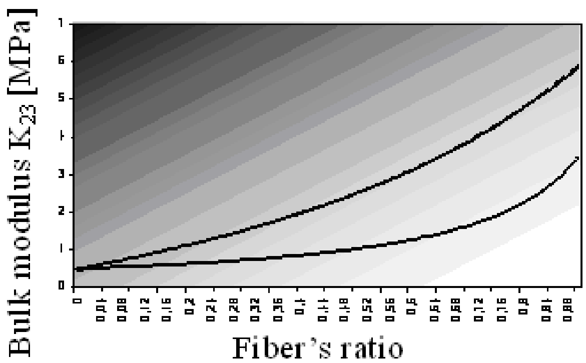

Fig. 6. Upper and lower margins for the volume module for ± 1% variations in the elasticity module in case study no.1

If we consider for the same case to have a deviation of only ± 1% it is noted that values obtained for this case for the volume modules coincide (fig.6). It follows that for small variations around the mean values of the fiber's mechanical properties the results obtained with this formula and the calculation presented in [2] differs insignificantly. Disregard of geometry, shape or material in these cases in the vicinity of theoretical values will influence the final result to a very small extent. The formulae under conditions in [2] prove to be very stable.

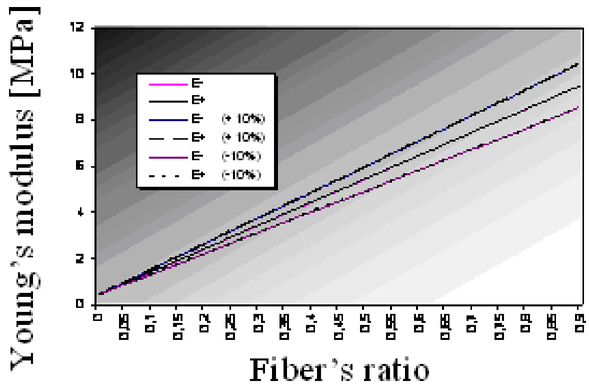

Fig. 7. Upper and lower margins for the longitudinal elasticity module for ± 10% variations of the fiber's elasticity module in case study no. 1

In the case of the longitudinal elasticity module, the law of mixtures being respected, a variation in the longitudinal elasticity module of the fiber generally higher than the matrix' will lead to a variation that is almost identical to the longitudinal elasticity module for the composite (fig.7). The fiber module is dominant and determines the result, thus its variation will trigger a nearly similar variation in the module of the composite. Fig. 7 presents the longitudinal elasticity module of the composite in case study no. 1 and fig. 8. shows the same measurement for case study no. 2. If the two figures are analyzed, it is seen that the variation of the fiber module is approximately the same as the variation of the composite module as compared to the theoretical value. It follows then that for determination in calculations of this measurement we have to be careful of the values we work with and their deviations from theoretical values, as we can induce meaningful errors. Due to dimensional variations, the formulae used can lead to erroneous estimates of the longitudinal elasticity module.

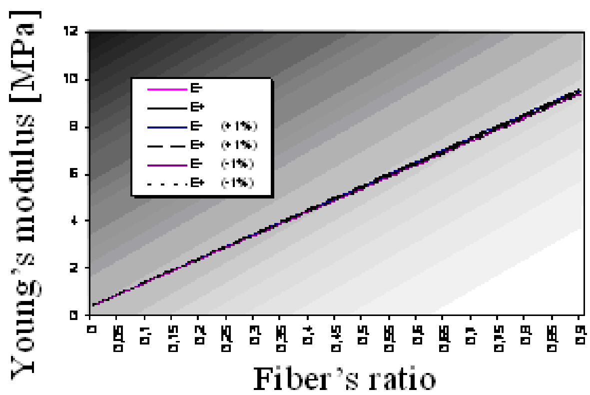

Fig. 8. Upper and lower margins for the longitudinal elasticity module for ± 1% variation of the fiber's elasticity module in case study no.1

Considering concentration variations of the two phases as compared to theoretical values, the chart in fig. 9 is obtained for the volume module, which indicates a high dispersion of the results. It follows then that for an accurate determination by calculus of the mechanical measurements the concentration of phases must generally be well estimated, as it intervenes in linear form in the majority of proposed formulae. Thus, deviations from theoretical values are directly proportional to the deviations in the theoretical values of the phase concentration. If the variation in the theoretical values is only 1% then the graph in fig. 10 is obtained, but even in this case, the differences are notable.

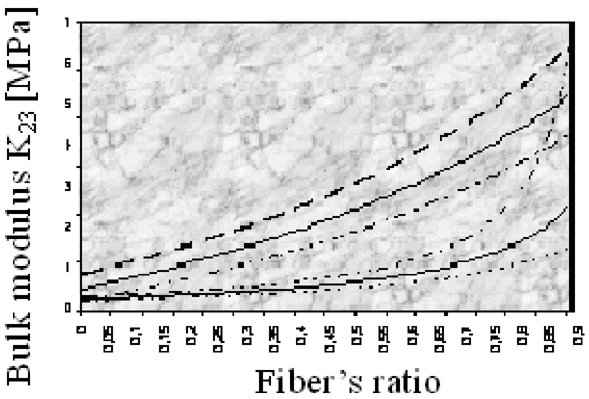

Fig. 9. Upper and lower margins for the volume module for ± 10% variation in the fiber concentration in case study no.1

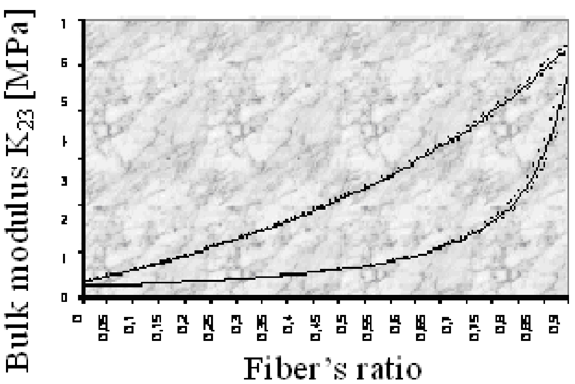

Fig. 10. Upper and lower margins for the volume module ± 1% variation in the fiber concentration in case Study no.1

Analyzing the formulae proposed for the transversal elasticity module, it is seen that the upper margin is sensitive to variations of the longitudinal elasticity module while the lower margin is less sensitive.

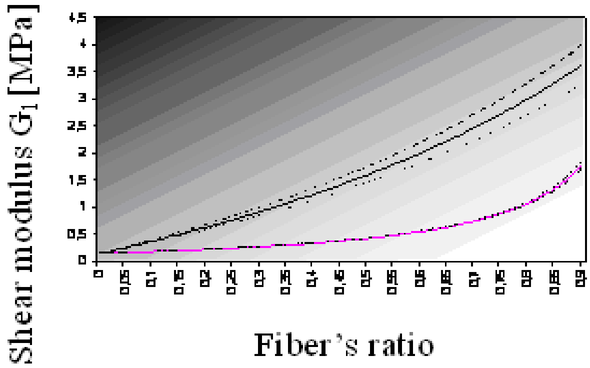

Fig. 11. Upper and lower margins of the transversal elasticity module for ± 10% variation of the longitudinal elasticity module in case study no.1

Fig. 11 shows the transversal elasticity module considering a variation of ± 10% of the longitudinal elasticity module and fig. 12 considers a ± 1% variation. If the variation is smaller, deviations from exact values are also very small.

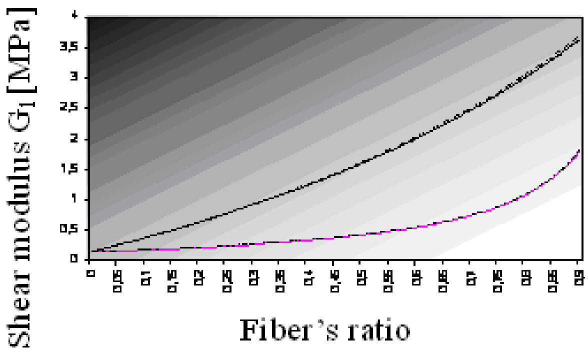

Fig. 12. Upper and lower margins of the transversal elasticity module for ± 1% variation of the longitudinal elasticity module in case study no.1

4. Discussion and conclusions

Following analysis for the formulae presented in [2] for the determination of mechanical properties of composite materials it can be concluded that variations in the properties of the fiber or the matrix can generally lead to variations, sometimes significant ones, in the properties of the composite material.

A conclusion based on the formulae used and the charts is that longitudinal properties that are generally described in formulae abiding by the law of mixtures are influenced to the same extent with the deviations from the theoretical values. The longitudinal elasticity module and the volume module are especially considered. The Poisson coefficient is affected to a small extent by the discrepancies between the theoretical and experimental values.

The dimensions that describe transversal features are significantly affected by deviations from theoretical values. This happens because the proposed formulae are based on less perfect models when calculating the dimensions corresponding to transversal deformations.

As a general conclusion, we must bear in mind the deviations from the ideal, theoretical model and the real, fabricated model, produced at the end of a specific technological model, when calculating mechanical dimensions of a composite. Dimensional deviations can lead to values that can influence – sometimes decisively – the behavior of the considered material.

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