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The influence of quantum effects on spatial distribution of carriers in surrounding-gate cylindrical MOSFETs

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The previously developed model describing quantum correction of carriers' concentration spatial distribution in ultra small MOS/MOD devices has been applied to Surrounding-Gate Cylindrical MOSFETs. The inevitable role of quantum effects over almost the entire sample has been confirmed and supported by numerical calculations for a specific set of geometrical parameters.

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1. Introduction

The evolution of modern device physics has led to the realization of devices with different promising geometries. Among them, the cylindrical geometry is most frequently used in order to achieve device size reduction and better packaging without the appearance of short-channel effects [1, 2]. Surrounding-gate devices are the best candidate because of their unbroken explicit cylindrical symmetry. They can appear as Si MOSFETs, as well as thin-film transistors (TFT) produced by means of organic semiconductors [3]. If first ones are considered, the inevitable step is to determine the spatial distribution of carriers over the whole semiconducting structure. So far the usual procedure was to solve the Poisson's equation for an undoped sample assuming that the carriers obeyed Boltzmann statistics. On the other hand, the sample size is usually even smaller than a thermal de Broglie wavelength λ_D (at room temperature approximately 15nm in silicon) making quantum-mechanical effects become prominent [1]. It might be possible to try to evaluate concentration of carriers by means of wave function obtained as a solution of 1D (one-dimensional) radial Schrodinger equation, assuming that they are subject to equilibrium Fermi-Dirac distribution. Apart from the shortcomings described in previous papers [4], this procedure would cause more serious problems than in the case of flat geometry (due to the more complex character of wave function) and could hardly lead to a cylindrical symmetry of concentration of carriers as expected.

Therefore we will directly solve the transport equation described in our previous papers, this time introducing cylindrical symmetry by hand [4, 5]. This equation contains quantum correction term and is especially convenient near strong barriers, where the curvature of $n(r)$ becomes significant [1].

2. Device structure

Let us consider the structure shown in Fig. 1. In the absence of lateral transport (voltage V_{DS}=0) the concentration of carriers can be described by a set of following equations [1]:

$$\Phi_t grad\left(n\right) - grad\left(V+q\right) = 0 \tag{1a}$$

$$q\left(\vec{r}\right) = \frac{\hbar^2}{2me}\frac{1}{\sqrt{n}}\Delta_{\vec{r}}\left(\sqrt{n}\right) \tag{1b}$$

where Φt=kT/e, V is the electrostatic potential, q is the quantum correction term, n is the concentration.

First of them is the transport equation in equilibrium and leads to a straight-forward Boltzmann-like solution:

$$n\left(\vec{r}\right) = n_0 \cdot \exp\left(\frac{V+q}{\Phi_t}\right) \tag{2}$$

with quantum-correction term defined by means of relation (1b) and n_0 being an arbitrary parameter imposed by boundary conditions.

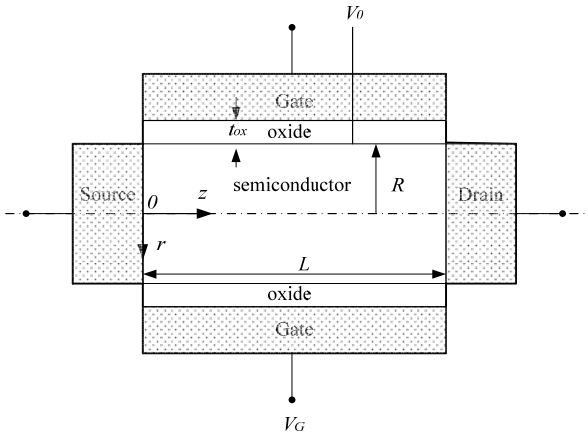

Fig. 1. Cross-sectional view of the cylindrical SG MOSFET.

If the channel length L is much greater than the radius of the sample, all the equations become 1D, depending only on radial coordinate r:

$$n(r) = n_0 \cdot \exp\left(\frac{V+q}{\Phi_t}\right) \tag{3a}$$

$$q(r) = \frac{\hbar^2}{2me} \cdot \frac{1}{\sqrt{n}} \cdot \left(\frac{d^2}{dr^2} + \frac{1}{r}\cdot\frac{d}{dr}\right)\cdot\sqrt{n} \tag{3b}$$

Eliminating $q(r)$ from the above set of relations, one derives a new equation that describes the variation of carrier's concentration in the presence of the applied external voltage $V(r)$:

$$\frac{d^2n}{dr^2} - \frac{1}{2n}\cdot\left(\frac{dn}{dr}\right)^2 + \frac{1}{r}\cdot\frac{dn}{dr} + \frac{4me}{\hbar^2}\cdot\left(V(r) - \Phi_t\cdot\ln\frac{n}{n_0}\right)\cdot n = 0 \tag{4}$$

This voltage is certainly subject to Poisson's equation, but at the moment our goal is rather to investigate the consequences of equation (4) than to construct a closed self-consistent procedure. At first it must be observed that the equation (4) can be rewritten in a dimensionless form:

$$\frac{d^2\breve{n}}{dr^2} - \frac{1}{2\breve{n}}\cdot\left(\frac{d\breve{n}}{dr}\right)^2 + \frac{1}{r}\cdot\frac{d\breve{n}}{dr} + \frac{4me\Phi_t}{\hbar^2}\cdot\left(\frac{V(r)}{\Phi_t} - \ln\breve{n}\right)\cdot\breve{n} = 0 \tag{5}$$

where: $n(r) = n_0\cdot\breve{n}(r)$ is valid.

Equation (5) turns out to be very important due to its universal character and therefore deserves to be investigated in details, while the influence of particular sample features is accounted for by taking different values of n_0. The choice $n_0 = N_A\cdot\exp\left(\frac{-2\Phi_F}{\Phi_t}\right)$ describes p-doped sample, while $n_0 = n_i\cdot\exp\left(\frac{-\Phi_F}{\Phi_t}\right)$ is valid for an undoped semiconductor, etc. Both cases assume that near the axis of the sample bulk is formed, in spite of the fact that small size of the sample (a~5nm-10nm) implies this statement to be contemplated much more carefully [4].

The universal character of equation (5) provides the possibility of imposing universal boundary conditions:

$$\breve{n}(a) = 0 \tag{6a}$$

$$\breve{n}(0) = 1 \tag{6b}$$

First of them is a consequence of quantum-mechanical requirement that the carriers are not able to leave semiconductor substrate and penetrate into oxide, while the second one is enabled by the arbitrarity of parameter n_0 [4].

Although equation (5) does not seem difficult for numerical solving at first sight, its nonlinear term can cause some problems. Fortunately, there exists a straightforward procedure to linearize it. If a new quantity $z(r)$ according to:

$$\sqrt{\breve{n}(r)} = z(r) \tag{7}$$

is introduced, equation (5) appears in a much more convenient form:

$$\frac{d^2z}{dr^2} + \frac{1}{r}\cdot\frac{dz}{dr} + \frac{4me\Phi_t}{\hbar^2}\cdot\left(\frac{V(r)}{2\Phi_t} - \ln z\right)\cdot z = 0 \tag{8a}$$

together with derived boundary conditions:

$$z(a) = 0 \tag{8b}$$

$$z(0) = 1 \tag{8c}$$

Similar to the flat geometry case, it is very useful to introduce a new parameter with the dimension of length [4]:

$$\lambda^2 = \frac{\hbar^2}{8me\Phi_t}, \quad \text{i.e.} \quad \lambda = \frac{\hbar}{\sqrt{8me\Phi_t}} \tag{9}$$

With the effective mass considered, at the room temperatures the estimated value for silicon becomes $\lambda\approx 1.4$nm. This length plays a significant role in investigating how deep quantum effects penetrate into the sample. Being comparable with the radius of the sample in

the order of magnitude, it gives a reliable hint that quantum effects become really prominent in the sense they can affect the device operation. Finally, the equation to be treated numerically becomes:

$$\frac{d^2z}{dr^2}+\frac{1}{r}\cdot\frac{dz}{dr}+\frac{1}{2\lambda^2}\cdot\left(\frac{V(r)}{2\Phi_t}-\ln z\right)\cdot z=0 \qquad (10)$$

with boundary conditions mentioned above (8b, 8c). This equation is going to be solved in two different cases of special importance. The first one will investigate carriers' spatial distribution in the absence of external field ($V(r)$=0, $\forall r\in Dr$), while the second one will exploit the following form of the applied voltage (Fig. 2):

$$V(r)=V_0\cdot\left(\frac{r}{a}\right)^{\alpha} \qquad (11)$$

This step is supposed to be sufficient due to the limited range with respect to radial coordinate r and the dimensionless parameter α being greater than one in order to achieve the expected convexity.

3. Numerical procedure

The first problem one had to face with was the existence of boundary conditions specified at two different points (6, 8b, 8c). This fact has prevented the straightforward use of previously developed Runge-Kutta method and made one to search for alternative ones [6]. As the best candidate a "shooting" method has emerged. The essence of this procedure is to start with the presumed boundary conditions at one side of the sample [6]:

$$z(a)=0 \qquad (12a)$$

$$\frac{dz}{dr}(a)=\theta \qquad (12b)$$

where θ is the free parameter finding itself in the range (0; +∞).

The suggested procedure was to solve equation (10) together with boundary conditions (12a, 12b) varying the parameter θ over the whole range until the solution satisfying both boundary conditions (8b, 8c) emerged. Only then we were able to go throw Runge-Kutta steps suggested in the literature [6]. To achieve this goal, it is convenient to renormalize the radial coordinate r in the following manner:

$$y=\ln\frac{a}{r},\quad 0\le r\le a,\quad 0\le y\le+\infty \qquad (13)$$

giving equation (10) even simpler form:

$$\frac{d^2z}{dy^2}+\frac{a^2}{2\lambda^2}\cdot\exp(-2y)\cdot\left(\frac{V(y)}{2\Phi_t}-\ln z\right)\cdot z=0 \qquad (14)$$

$$V(y)=V_0\cdot\exp(-\alpha\cdot y),\ z(0)=0,\quad \lim_{y\to+\infty}z(y)=1 \qquad (15)$$

The solution $z(y)$ of equation (14) is shown in Figs. 2 and 3 and deserves some general remarks:

a) due to the exponential term exp(-2y) the solution $z(y)$ strongly tends to its asymptotic (constant) value even for moderate values of the unknown y;

b) if $V(y)$=0 the solution $z(y)$ is convex upwards over the entire domain D_y: (0; +∞) as expected (Fig. 2), but introducing $V(y)$ according to (15) will change this conclusion (Fig. 3). The concentration of carriers has a sharp peak near the oxide/semiconductor interface surpassing the asymptotic value, as also expected. It is usually said that the carriers are attracted by the potential $V(y)$ and thus removed from the core of the sample to its border. At the very end, it is interesting to go back to initial quantities, i.e. to go through all transformations in the reverse direction; the result $\breve{n}(r)$ is shown in Fig. 4. with the mostly expected behavior.

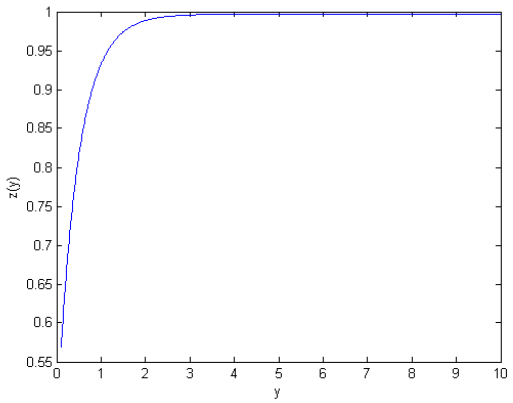

Fig. 2a. The results of calculation according to equation (14) z(y), V_0=0, a=5nm

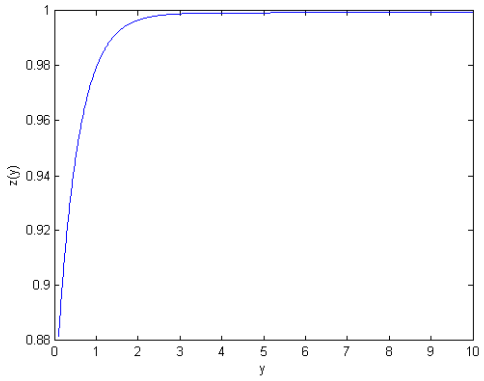

Fig. 2b. The results of calculation according to equation (14) z(y), V_0=0, a=2.5nm

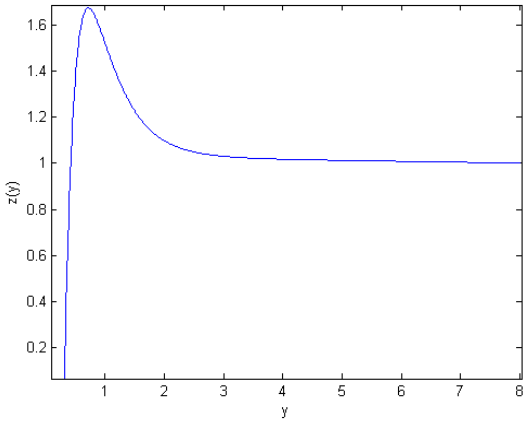

Fig. 3. The results of calculation according to equation (14) z(y), V_0=1.2V, α=3, a=5nm

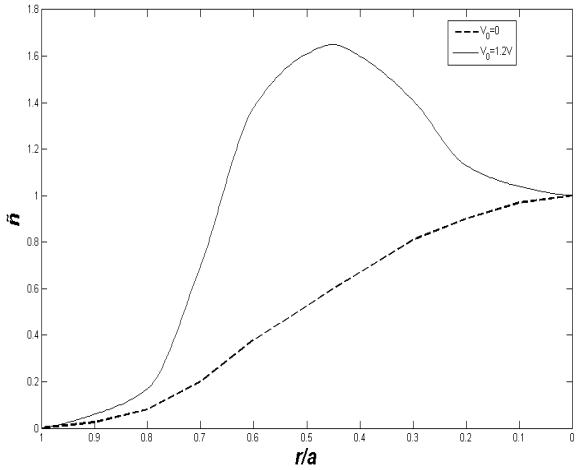

Fig. 4. The calculated profile of carriers' spatial distribution in SG MOSFET.

In addition to equation (14), it is useful to assume that the potential $V(y)$ is subject to Poisson's equation. This set of conjugated equations must be solved exploiting selfconsistent procedure, but its foundation in this case is far beyond the goal of this paper.

5. Conclusion

A method for introducing quantum effects on the carriers' concentration profile and their transport properties, previously developed bringing together the achievements of quantum mechanics and statistical physics, has been applied to devices with recently designed and promising cylindrical geometry. The 1D equation describing this concentration profile for rather long-channel devices has been derived, transformed and numerically solved in two cases of the greatest interest ($V(y)$=0 and $V(y)\neq 0$ as postulated). The invariance of the obtained equation under multiplication of $n(r)$ by an arbitrary quantity n_0 provides it some kind of universal character. That means it can be solved for any constant value n_0 (for example n_0=1) and only then the actual boundary conditions for r=0 are implemented.

The role of quantum effects is unquestionable. The lateral dimension of the sample implies that quantum effects (here appearing as edge effects) penetrate deep into semiconductor sample and spread over the prevailing part of it. Therefore, it can be reasonably assumed that these quantum effects have a great influence on a device operation.

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