The multimode photonic crystal resonator and its application to unidirectional optical energy transfer

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The current paper analyses the behavior of a photonic crystal cavity coupled with optical guides and able to transfer electromagnetic energy only in the forward direction. This simple photonic device with potential applications in photonic crystal microcircuits, especially intersection of guides, is studied, using "coupled mode theory" which is an approximate method that allows relatively simple derivations of optimal design parameters. Cavities coupled to guides appear everywhere in optical circuits. In many cases, they induce parasitic effects like important reflections back to the source. In other situations, if they are carefully tuned, optical devices consisting of micro resonators and guides can act as filters or unidirectional optical valves. The purpose of this article is to establish a procedure for designing an efficient intersection of guides, where the energy in one optical guide does not leak into the other, using a combination of analytical formula and numerical simulations.

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1. Introduction

Photonic crystals are periodic artificial dielectric nanostructures (Fig. 1) which affect the propagation of electromagnetic waves in much the same way as the periodic potential in a semiconductor influences the electrons motion, imposing allowed and forbidden electronic energy bands [5]. The existence of forbidden frequency bands inside photonic crystals leads to the possibility of constructing some micro optical devices like: highly efficient omnidirectional mirrors (Fig. 2), low loss waveguides able to direct light even if sharp corners appear along their path (Fig. 4, Fig. 5) [6], miniature optical filters or one directional energy couplers (Fig. 6).

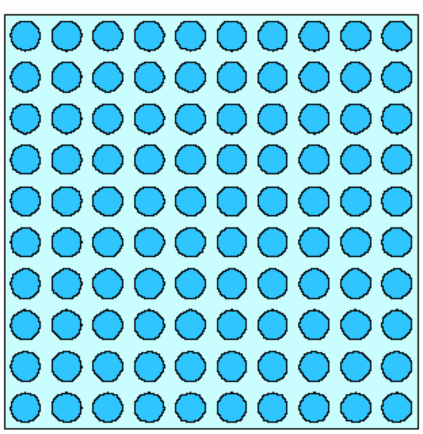

Fig. 1. Bidimensional photonic crystal.

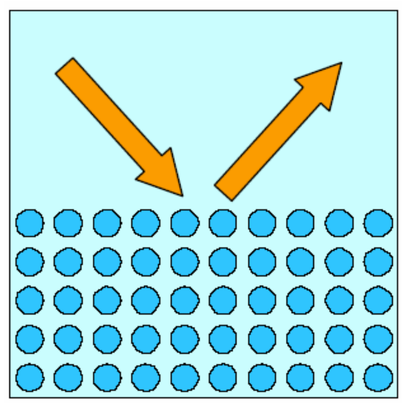

Fig. 2. Low loss omnidirectional reflector.

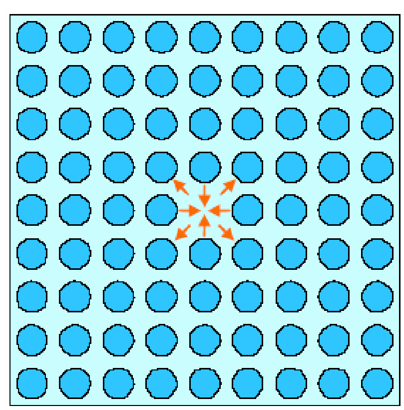

Fig. 3. 2D resonant cavity.

The current paper will deal with an extremely general and important aspect regarding the design of photonic crystal passive devices, namely “the coupling of electromagnetic energy between a multimode optical resonator and a number of optical guides”.

One way to directly analyze the behavior of waveguides, coupled with cavities, is to solve Maxwell Equations for a given photonic device (a spatial region consisting of a periodic dielectric pattern, possessing optical channels and cavities (Fig. 6)) [7]. A popular method that is regularly employed in studying the electromagnetic wave propagation is FDTD (Finite Difference Time Domain). However, this mathematical procedure does not give direct indications regarding the best configurations and optimal parameters. In most cases, it can be used just to verify some results obtained by other means. There are also situations when FDTD serves as a trial and error procedure, but changing the value of some parameters and running the algorithm again and again is a time consuming and laborious undertaking, in many circumstances, being hard to infer what values of input variables would lead to the desired behavior of the device under investigation.

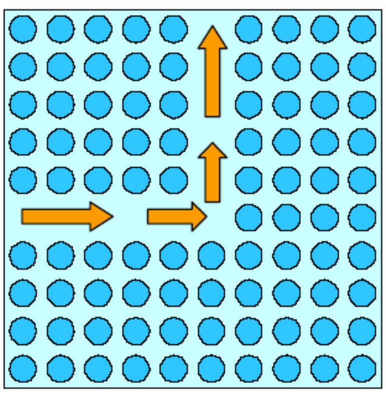

Fig. 4. Optical guide whose direction abruptly changes to 90^o. The forbidden band of the crystal denies any energy loss in the region of the corner.

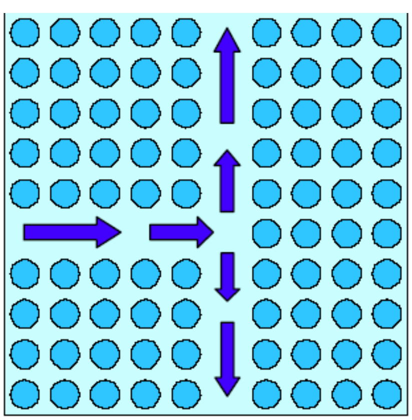

Fig. 5. Splitter.

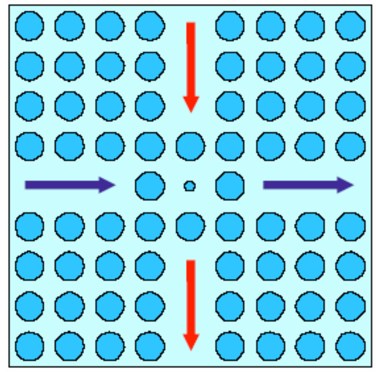

Fig. 6. Optical filter and, in the same time, intersection of guides. The cavity acts as both filter and unidirectional coupler.

In conclusion, an analytical formula, from witch an optimal set of parameters could be easily deduced, is needed. One relatively simple approach [2], [8] is to consider an idealized configuration, like the one in Fig. 7, described by the system (1) - (2) where a number, varying from one to *n*, optical guides converge toward a resonant cavity that receives and, in the same time, leaks energy from and into these optical branches. Such a model, (1) - (2), is based on “coupled mode theory” and was developed having in mind ordinary guides without any relation to photonic crystals [1]. So, the association between this theoretical model and photonic guides coupled to resonant monomod cavities is a bit forced and any theoretical prediction obtained using (1) - (2) has to be validated with the help of FDTD simulations.

2. Simplified theoretical model for a multimode cavity coupled with *N* guides

In general, the behavior of an optical monomod resonator, coupled to *n* input-output ports, can be approximately described [1], [2] by the system of equations (1) - (2):

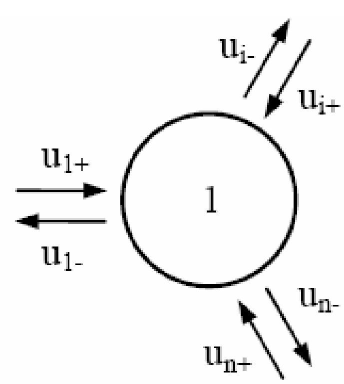

Fig. 7. Monomod cavity coupled with n input-output ports.

$$\frac{da(t)}{dt}=\left(j\omega_0-\frac{1}{\tau}\right)a(t)+\underbrace{(C_1 \quad \cdots \quad C_n)}_{\mathbf{C}^T}\underbrace{\begin{pmatrix}u_{1+}(t)\\ \vdots \\ u_{n+}(t)\end{pmatrix}}_{\mathbf{U}_+(t)}, \qquad (1)$$

$$\underbrace{\begin{pmatrix}u_{1-}(t)\\ \vdots \\ u_{n-}(t)\end{pmatrix}}_{\mathbf{U}_-(t)}=\underbrace{\begin{pmatrix}m_{11} & \cdots & m_{1n}\\ \vdots & \ddots & \vdots\\ m_{n1} & \cdots & m_{nn}\end{pmatrix}}_{\mathbf{M}}\underbrace{\begin{pmatrix}u_{1+}(t)\\ \vdots \\ u_{n+}(t)\end{pmatrix}}_{\mathbf{U}_+(t)}+\underbrace{\begin{pmatrix}p_1\\ \vdots \\ p_n\end{pmatrix}}_{\mathbf{p}}a(t), \qquad (2)$$

where a is the amplitude of the resonant mode with the frequency ω_0 and lifetime τ; u_{i+}, u_{i-}, amplitudes of input, output signals respectively and C_i, p_i, m_i some complex constants.

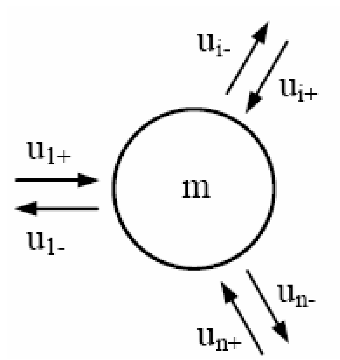

Fig. 8. Multimode cavity supporting m modes coupled to n input-output ports.

The simplified theory of the monomod resonator coupled with n waveguides (see Fig. 7 and equations (1) - (2)) is useful for cases when a number of guides, not coupled to each other, are coupled with a monomod resonator. However, situation exists, like “the intersection of two optical guides” when a bimode cavity have to be utilized.

Therefore, the system (1) - (2) has to be generalized for the case of a multimode cavity [2], [9], hoping to find some condition and criteria which permit designing more complex photonic devices than the ones whose behavior is described by (1) - (2).

The first step in the process of generalizing (1) - (2) resides in writing expression (1) for m modes, as follows:

$$\begin{cases}\dfrac{da_1(t)}{dt}=\left(j\omega_{01}-\dfrac{1}{\tau_1}\right)a_1(t)+C_{11}u_{1+}(t)+C_{12}u_{2+}(t)+\cdots+C_{1n}u_{n+}(t)\\ \dfrac{da_2(t)}{dt}=\left(j\omega_{02}-\dfrac{1}{\tau_2}\right)a_2(t)+C_{21}u_{1+}(t)+C_{22}u_{2+}(t)+\cdots+C_{2n}u_{n+}(t)\\ \vdots\\ \dfrac{da_m(t)}{dt}=\left(j\omega_{03}-\dfrac{1}{\tau_m}\right)a_m(t)+C_{m1}u_{1+}(t)+C_{m2}u_{2+}(t)+\cdots+C_{mn}u_{n+}(t)\end{cases}, \qquad (3)$$

which can be put in the equivalent matrix form (4):

$$\frac{d}{dt}\underbrace{\begin{pmatrix}a_1(t)\\ a_2(t)\\ \vdots\\ a_m(t)\end{pmatrix}}_{\mathbf{a}(t)}=\left[j\underbrace{\begin{pmatrix}\omega_{01} & 0 & \cdots & 0\\ 0 & \omega_{02} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \omega_{0m}\end{pmatrix}}_{\mathbf{\Omega}}-\underbrace{\begin{pmatrix}\frac{1}{\tau_1} & 0 & \cdots & 0\\ 0 & \frac{1}{\tau_2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \frac{1}{\tau_m}\end{pmatrix}}_{\mathbf{T}}\right]\cdot\underbrace{\begin{pmatrix}a_1(t)\\ a_2(t)\\ \vdots\\ a_m(t)\end{pmatrix}}_{\mathbf{a}(t)}+\underbrace{\begin{pmatrix}C_{11} & C_{12} & \cdots & C_{1n}\\ C_{21} & C_{22} & \cdots & C_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ C_{m1} & C_{m2} & \cdots & C_{mn}\end{pmatrix}}_{\mathbf{C}^T}\cdot\underbrace{\begin{pmatrix}u_{1+}(t)\\ u_{2+}(t)\\ \vdots\\ u_{n+}(t)\end{pmatrix}}_{\mathbf{U}_+} \qquad (4)$$

However, the diagonal form of $\mathbf{\Omega}$ and $\mathbf{T}$ is not satisfactory, for all possible situations. For instance, in case of a bimode resonator coupled with only one guide, matrix $\mathbf{T}$ can not have a diagonal form because the two fundamental conditions, namely, time inversion and energy conservation, that (1) - (2) has to satisfies (according to the theory of monomod resonators coupled with n guides) would not be met. In consequences $\mathbf{\Omega}$ and $\mathbf{T}$ must be extended at their generalized form:

$$\mathbf{\Omega}=\begin{pmatrix}\omega_{11} & \omega_{12} & \cdots & \omega_{1m}\\ \omega_{21} & \omega_{22} & \cdots & \omega_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ \omega_{m1} & \omega_{m2} & \cdots & \omega_{mm}\end{pmatrix},$$

$$\mathbf{T}=\begin{pmatrix}\frac{1}{\tau_{11}} & \frac{1}{\tau_{12}} & \cdots & \frac{1}{\tau_{1m}}\\ \frac{1}{\tau_{21}} & \frac{1}{\tau_{22}} & \cdots & \frac{1}{\tau_{2m}}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{1}{\tau_{m1}} & \frac{1}{\tau_{m2}} & \cdots & \frac{1}{\tau_{mm}}\end{pmatrix}. \qquad (5)$$

In the same way (1) was transformed in (4), where $\mathbf{\Omega}$ and $\mathbf{T}$ are given by (5), an analog generalized expression for (2) can be written:

$$\underbrace{\begin{pmatrix} u_{1-}(t) \\ u_{2-}(t) \\ \vdots \\ u_{n-}(t) \end{pmatrix}}_{\mathbf{U}_-} = \underbrace{\begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{m1} & m_{m2} & \cdots & m_{mn} \end{pmatrix}}_{\mathbf{M}} \cdot \underbrace{\begin{pmatrix} u_{1+}(t) \\ u_{2+}(t) \\ \vdots \\ u_{n+}(t) \end{pmatrix}}_{\mathbf{U}_+} + \underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nm} \end{pmatrix}}_{\mathbf{p}} \cdot \underbrace{\begin{pmatrix} a_1(t) \\ a_2(t) \\ \vdots \\ a_m(t) \end{pmatrix}}_{\mathbf{a}(t)} . \tag{6}$$

Therefore, it can be said that the behavior of a multimode resonator, supporting *m* modes and coupled with *n* guides, is given by the following two matrix equations which form a system:

$$\frac{d\mathbf{a}(t)}{dt} = (j\mathbf{\Omega} - \mathbf{T})\mathbf{a}(t) + \mathbf{C}^T \mathbf{U}_+(t), \tag{7}$$

$$\mathbf{U}_-(t) = \mathbf{M}\mathbf{U}_+(t) + \mathbf{p}\mathbf{a}(t). \tag{8}$$

In the same way as in the case of (1) - (2), it is expected that **Ω**, **C**, **T**, **M**, **p** are not independent but some connections exists between them which follow from the conditions of time inversion and energy conservation as already discussed above.

Thus, if the cavity in Fig. 8 is supposed to have the initial energy $E(0)=\mathbf{a}^+(0)\mathbf{a}(0)$, and all the input signals are zero, $\mathbf{U}_+(t)=\mathbf{0}$, from the principle of energy conservation it follows that:

$$\frac{d\left(\mathbf{a}^+(t)\mathbf{a}(t)\right)}{dt} = -\mathbf{U}_-^+(t)\mathbf{U}_-(t) = -[\mathbf{p}\mathbf{a}(t)]^+\mathbf{p}\mathbf{a}(t) = -\mathbf{a}^+(t)\mathbf{p}^+\mathbf{p}\mathbf{a}(t) \tag{9}$$

where *"+"* at the exponent of a matrix signifies an operator which transform the matrix in its adjunct (ex: $a^+=(a^T)^*$, *"T" means transpose and "*" conjugate*). As a remark, (9) made use of the matrix identity: $(\mathbf{AB})^+=\mathbf{B}^+\mathbf{A}^+$. Expression (9) means that the decrease of energy inside the resonator must equal the radiated power through the *n* output ports.

On the other side, according to (7):

$$\frac{d\left(\mathbf{a}^+(t)\mathbf{a}(t)\right)}{dt} = \frac{d\mathbf{a}^+(t)}{dt}\mathbf{a}(t) + \mathbf{a}^+(t)\frac{d\mathbf{a}(t)}{dt} = [(j\mathbf{\Omega}-\mathbf{T})\mathbf{a}(t)]^+\mathbf{a}(t) + \mathbf{a}^+(t)[(j\mathbf{\Omega}-\mathbf{T})\mathbf{a}(t)] =$$
$$= \mathbf{a}^+(t)[j\mathbf{\Omega}]^+\mathbf{a}(t) - \mathbf{a}^+(t)\mathbf{T}^+\mathbf{a}(t) + \mathbf{a}^+(t)j\mathbf{\Omega}\mathbf{a}(t) - \mathbf{a}^+(t)\mathbf{T}\mathbf{a}(t) = -2\mathbf{a}^+(t)\mathbf{T}\mathbf{a}(t) \quad ,(10)$$

where it was considered that: $\mathbf{\Omega}^+ = \mathbf{\Omega}$ and $\mathbf{T}^+ = \mathbf{T}$ as a supplemental condition.

If (9) is compared to (10) the following relation between **T** and **p** follows immediately:

$$\mathbf{p}^+\mathbf{p} = 2\mathbf{T}. \tag{11}$$

Coming back to (7) and supposing again that the input signals are zero, the solution of (7) is:

$$\mathbf{a}(t) = e^{(j\mathbf{\Omega}-\mathbf{T})t}\mathbf{a}(0), \tag{12}$$

As a note, relation (12) was obtained using the mathematical theory which tells that the solution of the matrix equation $\mathbf{f}'(t) = \mathbf{A}\mathbf{f}(t)$ is $\mathbf{f}(t) = e^{\mathbf{A}t}\mathbf{f}(0)$, where **A** is a square matrix of *k x k* elements and **f** a vector of length *k*.

Using (12) and (8) the expression of the output signals can be written as:

$$\mathbf{U}_-(t) = \mathbf{p}e^{(j\mathbf{\Omega}-\mathbf{T})t}\mathbf{a}(0). \tag{13}$$

The radiation of the energy inside the cavity through all output ports has to be reversible in time (as already discussed). Therefore, if the system in Fig. 8 is fed with the signal:

$$\mathbf{U}_-(t) = \mathbf{p}^* e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0), \tag{14}$$

which is the time reverse of (13), the energy should accumulate inside the multimode resonator till it reaches $E(0)=\mathbf{a}^+(0)\mathbf{a}(0)$. Therefore, if (14) is fed as input signal, $\mathbf{U}_+$, in (7), it follows that:

$$\frac{d\mathbf{a}(t)}{dt} = (j\mathbf{\Omega}-\mathbf{T})\mathbf{a}(t) + \mathbf{C}^T\mathbf{p}^* e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0). \tag{15}$$

The energy absorbed by the cavity can be calculated using a procedure similar to (10). Thus, the absorbed power has the expression:

$$\frac{d\left(\mathbf{a}^+(t)\mathbf{a}(t)\right)}{dt} = \frac{d\mathbf{a}^+(t)}{dt}\mathbf{a}(t) + \mathbf{a}^+(t)\frac{d\mathbf{a}(t)}{dt}, \tag{16}$$

Where

$$\frac{d\mathbf{a}^+(t)}{dt}\mathbf{a}(t) = \left((j\mathbf{\Omega}-\mathbf{T})\mathbf{a}(t) + \mathbf{C}^T\mathbf{p}^* e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0)\right)^+\mathbf{a}(t) =$$
$$= \mathbf{a}^+(t)(j\mathbf{\Omega}-\mathbf{T})^+\mathbf{a}(t) + \mathbf{a}^+(0)\left(e^{(j\mathbf{\Omega}+\mathbf{T})t}\right)^+\mathbf{p}^T\mathbf{C}^*\mathbf{a}(t) \quad ,(17)$$

$$\mathbf{a}^+(t)\frac{d\mathbf{a}(t)}{dt} = \mathbf{a}^+(t)(j\mathbf{\Omega}-\mathbf{T})\mathbf{a}(t) + \mathbf{a}^+(t)\mathbf{C}^T\mathbf{p}^* e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0) \tag{18}$$

In conclusion:

$$\left.\frac{d\left(\mathbf{a}^+(t)\mathbf{a}(t)\right)}{dt}\right|_{t=0} = \mathbf{a}^+(0)(-2\mathbf{T})\mathbf{a}(0) + \mathbf{a}^+(0)\mathbf{I}\mathbf{p}^T\mathbf{C}^*\mathbf{a}(0) + \mathbf{a}^+(0)\mathbf{C}^T\mathbf{p}^*\mathbf{I}\mathbf{a}(0) = \tag{19}$$
$$= \mathbf{a}^+(0)\left(-2\mathbf{T} + \mathbf{p}^T\mathbf{C}^* + \mathbf{C}^T\mathbf{p}^*\right)\mathbf{a}(0)$$

On the other side, the rate of energy increase must equal the power absorbed by the cavity through its n ports. Therefore, using (8), the following equality can be written:

$$\left.\frac{d\left(\mathbf{a}^{+}(t)\mathbf{a}(t)\right)}{dt}\right|_{t=0} = \mathbf{U}_{-}^{+}(t)\mathbf{U}_{-}(t)\Big|_{t=0} = \mathbf{a}^{+}(0)\mathbf{I}\mathbf{p}^{T}\mathbf{p}^{*}\mathbf{I}\mathbf{a}(0) . \qquad (20)$$

From (19) and (20) it follows that:

$$\mathbf{a}^{+}(0)\left(-2\mathbf{T}-\mathbf{p}^{T}\mathbf{p}^{*}+\mathbf{p}^{T}\mathbf{C}^{*}+\mathbf{C}^{T}\mathbf{p}^{*}\right)\mathbf{a}(0)=0 . \qquad (21)$$

On the other side, $2\mathbf{T}=\mathbf{p}^{+}\mathbf{p}=2\mathbf{T}^{*}=\mathbf{p}^{T}\mathbf{p}^{*}$ (see (11)). Thus,

$$-\mathbf{p}^{T}\mathbf{p}^{*}-\mathbf{p}^{T}\mathbf{p}^{*}+\mathbf{p}^{T}\mathbf{C}^{*}+\mathbf{C}^{T}\mathbf{p}^{*}=0 . \qquad (22)$$

The equality (22) is satisfied if:

$$\mathbf{C}=\mathbf{p} . \qquad (23)$$

In conclusion, two important restrictions have been established till now: (1) $\mathbf{p}^{+}\mathbf{p}=2\mathbf{T}$ and (2) $\mathbf{C}=\mathbf{p}$ which connects matrices **C**, **p**, **T.** In the next lines, we will try to find a link between these three matrices and **M**. A first step to achieve this goal is to find the explicit solution, **a**(t), of the equation (15). Therefore, if both members of (15) are multiplied by $e^{-(j\mathbf{\Omega}+\mathbf{T})t}$ the following relation is obtained:

$$e^{-(j\mathbf{\Omega}+\mathbf{T})t}\frac{d\mathbf{a}(t)}{dt}-e^{-(j\mathbf{\Omega}+\mathbf{T})t}\left(j\mathbf{\Omega}-\mathbf{T}\right)\mathbf{a}(t)=e^{-(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{C}^{T}\mathbf{p}^{*}e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0) , \qquad (24)$$

which means that:

$$\frac{d\left(e^{-(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(t)\right)}{dt}=e^{-(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{C}^{T}\mathbf{p}^{*}e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0) . \qquad (25)$$

By integrating (25), a formula for **a**(t) is found:

$$\mathbf{a}(t)=e^{(j\mathbf{\Omega}+\mathbf{T})t}\int_{0}^{t}e^{-(j\mathbf{\Omega}+\mathbf{T})(v)}\mathbf{C}^{T}\mathbf{p}^{*}e^{(j\mathbf{\Omega}+\mathbf{T})(v)}\mathbf{a}(0)dv+e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{Ct}$$

where

$$\mathbf{Ct}=\mathbf{a}(0) \qquad (26)$$

Once **a**(t) has been calculated, the condition of zero radiation at $t=0$ can be imposed. Therefore, the equation (8), where the excitation is the signal in (14) and **a**(t) is given by (26), must have the left member null at $t=0$. Thus,

$$0=\mathbf{M}\mathbf{p}^{*}e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0)+\mathbf{p}e^{(j\mathbf{\Omega}+\mathbf{T})t}\int_{0}^{t}e^{-(j\mathbf{\Omega}+\mathbf{T})(v)}\mathbf{C}^{T}\mathbf{p}^{*}e^{(j\mathbf{\Omega}+\mathbf{T})(v)}\mathbf{a}(0)dv+\mathbf{p}e^{(j\mathbf{\Omega}+\mathbf{T})t}\mathbf{a}(0)\Big|_{t=0} , \qquad (27)$$

which means that:

$$\mathbf{M}\mathbf{p}^{*}=-\mathbf{p} . \qquad (28)$$

In conclusion, the restrictions, in which (7) - (8) exist, are:

$$\mathbf{C}^{+}\mathbf{C}=2\mathbf{T} ,\ \mathbf{M}\mathbf{C}^{*}=-\mathbf{C} \text{ and } \mathbf{C}=\mathbf{p} \qquad (29)$$

The conditions (29) are intrinsic restrictions that must be satisfied by any multimode cavity coupled to a number of guides. Only after these conditions are imposed other supplemental restrictions that vary from case to case, like zero unitary transmission between two guides coupled to a resonator, have to be imposed.

3. The intersection of two photonic guides

The theory in the precedent paragraph will be used in order to design an efficient intersection of guides with minimal cross talk between its two optical paths and physical dimensions as little as possible, which represents a particular case of passive photonic devices that can have potential applications for optical microcircuits [10], [11].

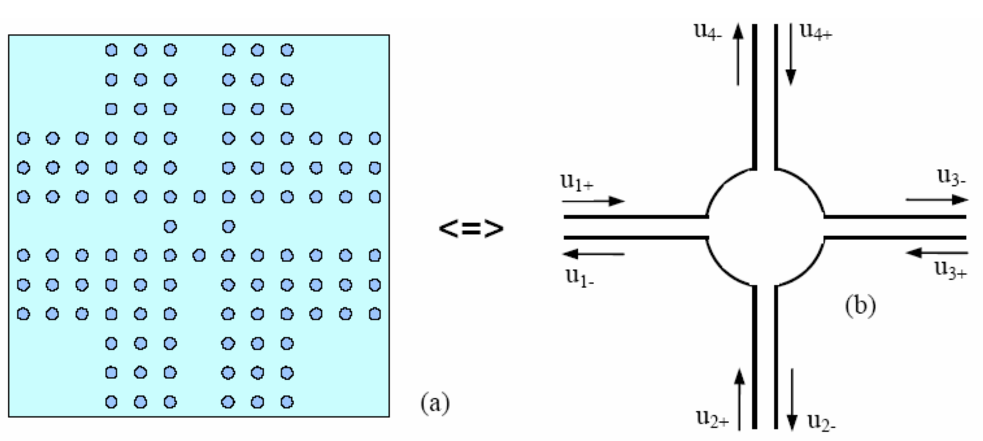

Fig. 9. (a) Intersection of photonic guides. (b) Simplified representation of (a).

If the cavity, from the intersection of the two guides in Fig. 9, is considered monomod, it can be proven quickly, using the theory in the precedent paragraph, that the resonator radiates its energy in all the four channels no matter which guide brings energy into the cavity.

Since the monomode resonator was found not to be satisfactory, the next step consists in trying the case of a bimode resonator. The equations that describe the device in Fig. 9 (b), where the cavity is considered as supporting two modes, have the form (30) - (31), according to the theory in paragraph 2 (see (7) - (8) and conditions (29)).

$$\underbrace{\frac{d}{dt}\begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}}_{\mathbf{a}(t)} = \left[\underbrace{j\begin{pmatrix} \omega_{01} & 0 \\ 0 & \omega_{02} \end{pmatrix}}_{\mathbf{\Omega}} - \underbrace{\begin{pmatrix} \frac{1}{\tau_1} & 0 \\ 0 & \frac{1}{\tau_2} \end{pmatrix}}_{\mathbf{T}} \right] \cdot \underbrace{\begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}}_{\mathbf{a}(t)} + \underbrace{\begin{pmatrix} C_{11} & C_{21} & C_{31} & C_{41} \\ C_{12} & C_{22} & C_{32} & C_{42} \end{pmatrix}}_{\mathbf{C}^T} \cdot \underbrace{\begin{pmatrix} u_{1+}(t) \\ u_{2+}(t) \\ u_{3+}(t) \\ u_{4+}(t) \end{pmatrix}}_{\mathbf{U}_+} \tag{30}$$

$$\underbrace{\begin{pmatrix} u_{1-}(t) \\ u_{2-}(t) \\ u_{3-}(t) \\ u_{4-}(t) \end{pmatrix}}_{\mathbf{U}_-} = \underbrace{\begin{pmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{13} & 0 \\ 0 & 0 & 0 & m_{14} \end{pmatrix}}_{\mathbf{M}} \cdot \underbrace{\begin{pmatrix} u_{1+}(t) \\ u_{2+}(t) \\ u_{3+}(t) \\ u_{4+}(t) \end{pmatrix}}_{\mathbf{U}_+} + \underbrace{\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{pmatrix}}_{\mathbf{C=p}} \cdot \underbrace{\begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}}_{\mathbf{a}(t)} . \tag{31}$$

Matrix **M** has the form in (31) for matching the conditions of zero coupling when (see Fig. 9) $u_{i-}(t)$ depends only of $u_{i+}(t)$.

The system (30) - (31) can also be put in the form:

$$\frac{da_1(t)}{dt} = \left(j\omega_{01} - \frac{1}{\tau_1} \right) a_1(t) + C_{11}u_{1+}(t) + C_{21}u_{2+}(t) + C_{31}u_{3+}(t) + C_{41}u_{4+}(t),$$
$$\frac{da_2(t)}{dt} = \left(j\omega_{02} - \frac{1}{\tau_2} \right) a_2(t) + C_{12}u_{1+}(t) + C_{22}u_{2+}(t) + C_{32}u_{3+}(t) + C_{42}u_{4+}(t), \tag{32}$$

$$\begin{aligned} u_{1-}(t) &= m_{11}u_{1+}(t) + C_{11}a_1(t) + C_{12}a_2(t), \\ u_{2-}(t) &= m_{12}u_{2+}(t) + C_{21}a_1(t) + C_{22}a_2(t), \\ u_{3-}(t) &= m_{13}u_{3+}(t) + C_{31}a_1(t) + C_{32}a_2(t), \\ u_{4-}(t) &= m_{14}u_{4+}(t) + C_{41}a_1(t) + C_{42}a_2(t). \end{aligned} \tag{33}$$

In order to calculate transmission coefficients T_{12}, T_{14}, T_{13}, (not to be confused with the elements of matrix **T**) and the reflection coefficient R_{11}, the following particular case; $u_{1+}(t) = u_{1+}e^{j\omega t}$; $u_{2+} = u_{3+} = u_{4+} = 0$; is considered. If (32) is solved with these values as input signals then the following two relations can be written:

$$\lim_{t\to\infty} a_1(t) = \frac{u_{1+}C_{11}e^{j\omega t}}{j(\omega - \omega_{01}) + \dfrac{1}{\tau_1}},$$
$$\lim_{t\to\infty} a_2(t) = \frac{u_{1+}C_{12}e^{j\omega t}}{j(\omega - \omega_{02}) + \dfrac{1}{\tau_2}}. \tag{34}$$

Using expressions (34) and the system (33), the complex transmission and reflection coefficients, corresponding to the port 1, results. Thus,

$$t_{12} = \frac{u_{2-}}{u_{1+}} = \frac{C_{21}C_{11}}{j(\omega - \omega_{01}) + \dfrac{1}{\tau_1}} + \frac{C_{22}C_{12}}{j(\omega - \omega_{02}) + \dfrac{1}{\tau_2}}, \tag{35}$$

$$t_{13} = \frac{u_{3-}}{u_{1+}} = \frac{C_{31}C_{11}}{j(\omega - \omega_{01}) + \dfrac{1}{\tau_1}} + \frac{C_{32}C_{12}}{j(\omega - \omega_{02}) + \dfrac{1}{\tau_2}}, \tag{36}$$

$$t_{14} = \frac{u_{4-}}{u_{1+}} = \frac{C_{41}C_{11}}{j(\omega - \omega_{01}) + \dfrac{1}{\tau_1}} + \frac{C_{42}C_{12}}{j(\omega - \omega_{02}) + \dfrac{1}{\tau_2}}, \tag{37}$$

$$r_{11} = \frac{u_{1-}}{u_{1+}} = m_{11} + \frac{C_{11}^2}{j(\omega - \omega_{01}) + \dfrac{1}{\tau_1}} + \frac{C_{12}^2}{j(\omega - \omega_{02}) + \dfrac{1}{\tau_2}}, \tag{38}$$

where $T_{12} = |t_{12}|^2$, $T_{14} = |t_{14}|^2$, $T_{13} = |t_{13}|^2$, $R_{11} = |r_{11}|^2$.

Once the four coefficients calculated, conditions (29) have to be imposed for finding the connections that exist between coefficients C_{ij}. Therefore, forcing the condition: $\mathbf{MC}^* = -\mathbf{C}$, some relation between the phases, φ_{ij}, of C_{ij} appears:

$$\begin{pmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{12} & 0 & 0 \\ 0 & 0 & m_{13} & 0 \\ 0 & 0 & 0 & m_{14} \end{pmatrix} \begin{pmatrix} C_{11}^* & C_{12}^* \\ C_{21}^* & C_{22}^* \\ C_{31}^* & C_{32}^* \\ C_{41}^* & C_{42}^* \end{pmatrix} = -\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{pmatrix} \Rightarrow$$
$$\begin{cases} \varphi_{12} = \varphi_{11} + k_1\pi \\ \varphi_{22} = \varphi_{21} + k_2\pi \\ \varphi_{32} = \varphi_{31} + k_3\pi \\ \varphi_{42} = \varphi_{41} + k_4\pi \end{cases} . \tag{39}$$

Another condition that has to be satisfied is $\mathbf{C}^+\mathbf{C} = 2\mathbf{T}$ (see (29)). Thus,

$$\begin{pmatrix} C_{11}^* & C_{21}^* & C_{31}^* & C_{41}^* \\ C_{12}^* & C_{22}^* & C_{32}^* & C_{42}^* \end{pmatrix} \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{pmatrix} = 2\begin{pmatrix} \dfrac{1}{\tau_1} & 0 \\ 0 & \dfrac{1}{\tau_2} \end{pmatrix}. \tag{40}$$

With the help of (39), the equality (40) turns into:

$$\begin{aligned} &|C_{11}|^2 + |C_{21}|^2 + |C_{31}|^2 + |C_{41}|^2 = 2/\tau_1 \\ &n_1|C_{12}||C_{11}| + n_2|C_{22}||C_{21}| + n_3|C_{32}||C_{31}| + n_4|C_{42}||C_{41}| = 0, \\ &|C_{12}|^2 + |C_{22}|^2 + |C_{32}|^2 + |C_{42}|^2 = 2/\tau_2 \end{aligned} \tag{41}$$

where $n_i = 1$ if k_i is even and $n_i = -1$ if k_i is odd. As a remark, relations (39) between the phases of coefficients C_{ij} and (41) between their modules are intrinsic conditions that, according to the theory in the precedent paragraph, the device in Fig. 9 (b) has to satisfy automatically, but they are not enough to force no cross talk between the two propagation paths *1-3* and *2-4*. Therefore, supplemental non interferential conditions have to be added. For the input-output port *1* these restrictions are:

$$T_{12} = T_{14} = 0 \text{ and } T_{13} = 1 . \quad (42)$$

$T_{12}=|t_{12}|^2=0$ means that:

$$T_{12} = \left| \frac{C_{21}C_{11}D_2 + C_{22}C_{12}D_1}{D_1 D_2} \right|^2 = 0 , \quad (43)$$

where:

$$D_1 = j(\omega - \omega_{01}) + 1/\tau_1 , \ D_2 = j(\omega - \omega_{02}) + 1/\tau_2 . \quad (44)$$

Making the notation $C_{21}C_{11}D_2 = G_1$ and $C_{22}C_{12}D_1 = G_2$, the condition (43) becomes equivalent with:

$$|G_1 + G_2|^2 = 0 \ \Rightarrow$$

$$|G_1|^2 \cdot \left| (|G_2|/|G_1|) e^{j(\arg(G_2) - \arg(G_1))} + 1 \right|^2 = 0 \text{ with } |G_1| \neq 0 \quad (45)$$

$$\Rightarrow$$

$$|G_1|^2 \cdot \left[\left(\frac{|G_2|}{|G_1|} \cos(\arg(G_2) - \arg(G_1)) + 1 \right)^2 + \frac{|G_2|^2}{|G_1|^2} \sin^2(\arg(G_2) - \arg(G_1)) \right] = 0$$

with

$$|G_1| \neq 0 , \quad (46)$$

which leads to the solutions:

$$\text{(a) } |G_1| = |G_2| = 0 \text{ ,(b) } |G_2|/|G_1| = 1 \text{ and}$$
$$\arg(G_2) - \arg(G_1) = (2k_{12} + 1)\pi \text{ where } |G_1| \neq 0 . \quad (47)$$

In the case (47) (a), which is the only one that will be discussed,

$$T_{12} = 0 \Rightarrow C_{21}C_{11} = 0 , \ C_{22}C_{12} = 0 . \quad (48)$$

Analogously:

$$T_{23} = 0 \Rightarrow C_{31}C_{21} = 0 , \ C_{32}C_{22} = 0 ,$$
$$T_{34} = 0 \Rightarrow C_{41}C_{31} = 0 , \ C_{42}C_{32} = 0 , \quad (49)$$
$$T_{41} = 0 \Rightarrow C_{11}C_{41} = 0 , \ C_{12}C_{42} = 0 .$$

The system formed by equations (48) - (49) has a few sets of solutions from which (50) distinguishes as the set of practical interest:

$$C_{21} = 0 , \ C_{41} = 0 , \ C_{32} = 0 , \ C_{12} = 0 . \quad (50)$$

Forcing the condition $T_{13}=1$ and using (36), (41) and (50), the following results are obtained:

$$|C_{11}|^2 = |C_{31}|^2 = 1/\tau_1 \quad (51)$$

and, in an analogous way, $T_{24}=1$ leads to:

$$|C_{22}|^2 = |C_{42}|^2 = 1/\tau_2 . \quad (52)$$

In consequence, if the cavity in Fig. 9 (b) supports two modes, out of which, one couples just to the guides *1-3* with equal coupling coefficients and the other just to the ports *2-4*, also with equal coupling coefficients, then there is no cross talk between the two propagation paths, *1-3* and *2-4*. Also, the transmissions in the forward direction are unitary for the resonant frequencies ω_{01} and ω_{02} of each mode and have the general expressions:

$$T_{13} = \frac{1/\tau_1^2}{(\omega - \omega_{01})^2 + 1/\tau_1^2} , \ T_{24} = \frac{1/\tau_2^2}{(\omega - \omega_{02})^2 + 1/\tau_2^2} . \quad (53)$$

As a note, the entire theory developed till now is general and not specific for photonic guides coupled to photonic resonators. In consequence, conditions (50), (51), (52) are useful only if they can be physically implemented in an real intersection of photonic guides. Therefore, a numerical simulation that solves directly (using the FDTD method) Maxwell equations, for two guides, carved inside a photonic crystal, which intersect in one place, is needed for verifying the usefulness of the simplified theory we have discussed above. For this purpose, the structure in Fig. 10 is chosen. The building blocks of the crystal in Fig. 10 are characterized by $r=0.2a$ (the central “atom” in the middle of the resonant cavity is different having $r=0.3a$), $\varepsilon_{ra}=11.56$, $\varepsilon_{rb}=1$, where r is the ray of the circular dielectric “atom”, ε_{ra} represents the relative electric permittivity of the “atom” and ε_{rb} the same dielectric constant but corresponding to the material that is found around the “atom” in the remaining of the cell. The excitation source is a narrowband modulated pulse test signal, characterized by an uniform spectrum in the frequency range [*0.33(c/a), 0.4(c/a)*] (where *c* is the speed of light in vacuum and *a* is the length of the side of each elementary square cell) and a maximum amplitude *A=2 a.u.*. The source acts in the point marked with “+”, located at the left of Fig. 10, in the middle of the optical guide. As a remark, the frequency range [*0.33(c/a), 0.4(c/a)*] falls inside the forbidden band of the photonic crystal in Fig. 10 and also inside the pass band of the guides.

With the help of the excitation, whose characteristics were given above, it is found that the resonant frequencies of the cavity for its two modes are $f_{01}=f_{02}=0.368(c/a)$ or equivalent $\omega_{01}= \omega_{02}=2\pi f_{01}=2\pi f_{02}=2\pi\cdot 0.368(c/a)$.

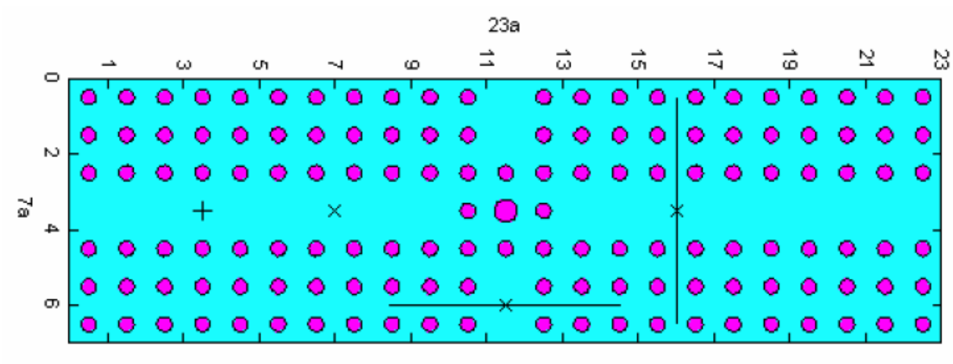

Fig. 10. Intersection of photonic guides.

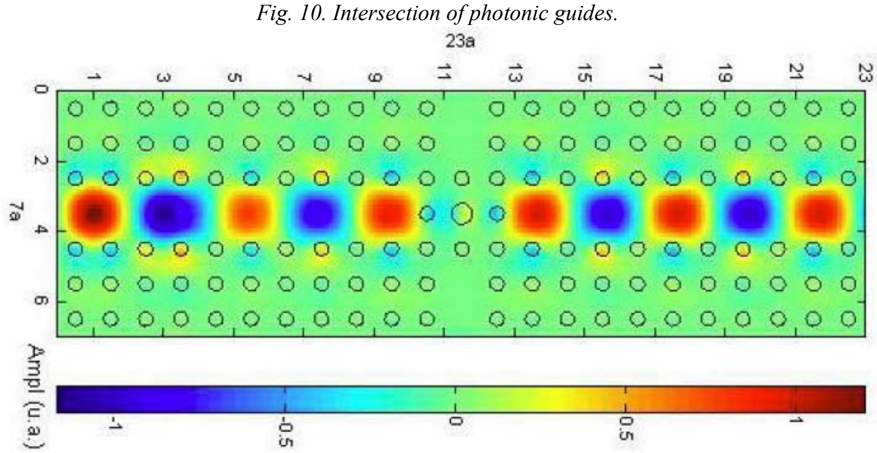

Fig. 11. The field map of the component E_z for the case when a sinusoidal excitation with the frequency f=0.368(c/a) is used to excite the structure shown in Fig. 10. The image is taken at t=282.84(a/c) which corresponds to a moment after the continuous regime has been established.

As can be seen in Fig. 11, there is no visible cross talk between the horizontal and the vertical guide, the light being channeled only in the forward direction and no visible power leaks at 90^o angle, a fact that is in concordance with the simplified theoretical model.

4. Conclusions

A photonic intersection of guides has been studied using a double approach. The first method, based on “coupled mode theory”, offers clear conditions for perfect transmission and zero cross talk between the two optical paths, being a powerful procedure for finding optimal parameters and explaining why the structure behaves like a band-pass filter. Despite the approximate nature of the method, the “coupled mode theory” proves to be a key tool that gives important information about the optimal configuration of photonic crystal devices. It is true, because of its generality, “coupled mode theory” does not tell what measures have to be taken in order to attain the right coupling coefficients. Their optimal values have to be found using FDTD in a trial and error process. Therefore, the analytical expression of *T* is only an approximation. Also, the values of $1/\tau$ need to be determined experimentally (numerically with the help of FDTD - Finite Difference Time Domain - procedure). However, the formula for *T* permits a quick identification of potential unsuitable values for $1/\tau$ (too small, big, etc.) and, in consequence, each τ can be adjusted accordingly, by inserting impurities of various diameters inside the cavity (a known procedure for modifying the emission absorption rate of resonators). After a few FDTD simulations the right τ is obtained.

As a remark, before proceeding to design an optical device using “coupled mode theory” and FDTD, the properties of the uniform photonic crystal (i.e. its frequency gaps) have to be determined using a procedure called Plane Wave Method (PWM) because, outside this forbidden ranges, electromagnetic energy will penetrate in the body of the crystal rendering it useless. Also, the properties of the particular optical guide and cavity utilized, more precisely their number of modes, the field maps of each mode, etc., have to be determined with the help of PWM.

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