The polarization-spectral structure of the modulated light by means of modulator with two ADP crystals and a half-wave plate

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An analysis of the polarization dynamics of the light modulated by means of the transverse electrooptic effect in a modulator with two ADP crystals and a half-wave plate placed between them is performed in the Jones matrix formalism and in terms of coherence matrix , for an arbitrary bias voltage applied to the ADP crystals. Two important cases for applications, no bias voltage and quarter-wave plate bias voltage applied to the ADP crystals, are particularized.

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1. Introduction

The light can be electrooptic modulated by Pockels or Kerr effects, the first being most used. The Pockels effect is used both in longitudinal and transverse mode. The advantage of using the transverse electrooptic effect is that the half-wave voltage may be much lower [1, 2]. The applications of the transverse modulators include broadband optical communication, display and printing systems, fast image and signal recorders [3].

For the transverse Pockels electrooptic effect the light does not propagate along the optical axis of the crystal, thus, there is a natural birefringence which depend on temperature. The elimination of the natural birefringence may be realized by mounting of the two crystals, between them being placed a half-wave plate [2, 4], as in the figure 1. The ordinary ray in the first crystal becomes extraordinary ray in the second crystal, and vice versa.

The description of the state of polarizition of the light and the interaction between the polarized light and the time-varying polarization devices can be realized in the Jones or Stokes matrix formalisms [5-10] or in a pure operatorial one [11].

In this work we performed an analysis, in Jones and coherence matrix formalisms, of the polarization-spectral structure of the modulated light by means of modulator with two ADP crystals and a half-wave plate placed between them. It also presents spectral structure of the modulated light by the three-dimension vectorial diagrams.

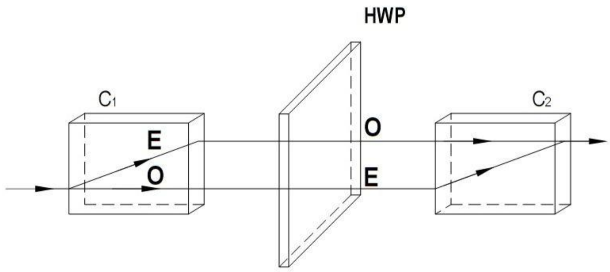

Fig. 1. The half -wave plate compensates the natural birefrigence of the ADP crystals

2. The matrix of the modulator

The modulator consists of two ADP crystals that exhibit transverse electrooptic Pockels effect and a half-wave plate placed between the crystals**.** The ADP crystals are identical, with parallel electrodes, the modulating voltage is applied perpendicular on the direction of propagation of the light. The scheme of the modulator is presented in Fig. 2.

In order to determine the Jones matrix of the modulator we use the *OXYZ* coordinate system. The neutral lines of the half-wave plate are parallel with the axes of the *OXYZ* coordinate system (fast axis of the half-wave plate is parallel with *OX* axis). The light propagates in the *Z* direction, polarized at 45^o to the *X* axis.

If it is applied a voltage to ADP crystals, with a d.c. component U_0, as well as a harmonically-varying one, $U_m \sin\Omega t$, as in Fig. 2, the phase shift between linearly polarized components along the principal axes X and Y is [12-14]:

$$\Phi = 2(\Phi_0 + \Gamma \sin\Omega t) \quad (1)$$

$$\Phi_0 = \pi \frac{U_0}{U_{\lambda/2}}, \quad \Gamma = \pi \frac{U_m}{U_{\lambda/2}},$$

where $U_{\lambda/2}$ is the half-wave voltage of the crystal.

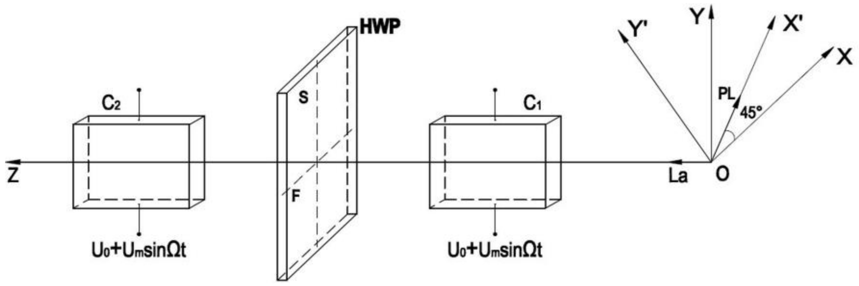

Fig. 2. The setup of two ADP crystals in tandem with a half-wave plate between them.

The Jones matrices of the ADP crystals are identical, they are diagonal matrices [15]:

$$M_1 = M_2 = \begin{pmatrix} e^{i\Phi} & 0 \\ 0 & e^{-i\Phi} \end{pmatrix} \quad (2)$$

The half-wave plate have Jones matrix [16]:

$$M_{\lambda/2} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3)$$

The matrix of the modulator is [17]:

$$M = M_2 M_{\lambda/2} M_1 \quad (4)$$

By replacing the corresponding matrices M_1, M_2 and $M_{\lambda/2}$, we obtain the modulator matrix

$$M = \begin{pmatrix} e^{2i\Phi} & 0 \\ 0 & -e^{-2i\Phi} \end{pmatrix} = \begin{pmatrix} e^{i(2\Phi_0 + 2\Gamma\sin\Omega t)} & 0 \\ 0 & -e^{-i(2\Phi_0 + 2\Gamma\sin\Omega t)} \end{pmatrix} \quad (5)$$

Thus, the electrooptic modulator behaves as a time varying phase shift plate.

The Jones matrix of the modulator, M', in the $OX'Y'Z'$ coordinate system is determinate in accordance with the Jones matrix, M, in the $OXYZ$ coordinate system, by the formula [18]:

$$M' = R(-\theta) M R(\theta) \quad (6)$$

where, $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, is Jones matrix of rotation, and θ the angle of rotation of the $OX'Y'Z'$ coordinate system relative to the $OXYZ$ coordinate system.

In our case, $\theta = 45^\circ$, we obtain for M' Jones matrix the following expression:

$$M' = \begin{pmatrix} i\sin 2\Phi & \cos 2\Phi \\ \cos 2\Phi & i\sin 2\Phi \end{pmatrix} = \begin{pmatrix} i\sin 2(\Phi_0 + \Gamma\sin\Omega t) & \cos 2(\Phi_0 + \Gamma\sin\Omega t) \\ \cos 2(\Phi_0 + \Gamma\sin\Omega t) & i\sin 2(\Phi_0 + \Gamma\sin\Omega t) \end{pmatrix} \quad (7)$$

3. The electric field intensity spectrum of the modulated light

The incident light on modulator is characterized, in the $OX'Y'Z'$ coordinate system, by J'_{in} Jones vector. It has the formula [7, 17]:

$$J'_{in} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i\omega_0 t} \quad (8)$$

Thus, the J'_M Jones vector **of** the modulated light**,** in the $OX'Y'Z'$ coordinate system, is [17]:

$$J'_M = M' J'_{in} = \begin{pmatrix} i\sin 2\Phi \\ \cos 2\Phi \end{pmatrix} = \begin{pmatrix} i\sin 2(\Phi_0 + \Gamma \sin \Omega t) \\ \cos 2(\Phi_0 + \Gamma \sin \Omega t) \end{pmatrix} = \begin{pmatrix} E_{X'}(t) \\ E_{Y'}(t) \end{pmatrix} \quad (9)$$

The J'_M vector is a periodically time-varying Jones vector, it represents a periodically time-varying state of polarization. Thus, the state of polarization of the light emerging from modulator is temporally modulated. Jones vector components, J'_M, according to $OX'Y'Z'$ reference system , are the intensities of the electric field of the modulated light along the OX' and OY' axes, $E_{X'}(t)$ and $E_{Y'}(t)$.

In order to determinate the electric field intensity vector spectra of the modulated light given above, are used decomposed formulas [19]:

$$\cos\left(2\Gamma \sin \Omega t\right) = J_0(2\Gamma) + 2\sum_{n=1}^{\infty} J_{2n}\left(2\Gamma\right)\cos\left(2n\Omega t\right)$$

$$\sin\left(2\Gamma \sin \Omega t\right) = 2\sum_{n=1}^{\infty} J_{2n-1}\left(2\Gamma\right)\sin\left[\left(2n-1\right)\Omega t\right] \quad (10)$$

in relation (9). Hence, we obtained the following expression for intensity vector of the electric field:

$$\begin{pmatrix} E_{X'}(t) \\ E_{Y'}(t) \end{pmatrix} = \begin{pmatrix} i\sin 2\Phi_0 \sum_{n=-\infty}^{\infty} J_{2n}\left(2\Gamma\right) e^{i(\omega_0 + 2n\Omega)t} + \cos 2\Phi_0 \sum_{n=-\infty}^{\infty} J_{2n-1}\left(2\Gamma\right) e^{i[\omega_0 + (2n-1)\Omega]t} \\ \cos 2\Phi_0 \sum_{n=-\infty}^{\infty} J_{2n}\left(2\Gamma\right) e^{i(\omega_0 + 2n\Omega)t} + i\sin 2\Phi_0 \sum_{n=-\infty}^{\infty} J_{2n-1}\left(2\Gamma\right) e^{i[\omega_0 + (2n-1)\Omega]t} \end{pmatrix} \quad (11)$$

From spectral development (11) we see that both components of the electric field intensity are linearly polarized along the OX' and OY'axes, containing the optical carrier, ω_0, and all harmonics of even and odd order, situated on the both sides of the optical carrier, at whole multiples of the modulation frequency.

An intuitive graphical representation of the spectral development (11) is given by the three-dimension vectorial diagrams, one for each component of the light electric field [12,15]. Those diagrams describe the amplitude, frequency, phase and the state of polarization of every harmonics.

The modification of the harmonics phase linearly polarized is given by the vector rotation corresponding around the ω axis. The vectorial diagram it is a graphical representation of the optical signal structure at a given moment, so it indicate the momentary phase of every harmonics. In fig. 3 it is represented the harmonic structure of the components $E_{X'}$ and $E_{Y'}$ at the particular moment of t = 0.

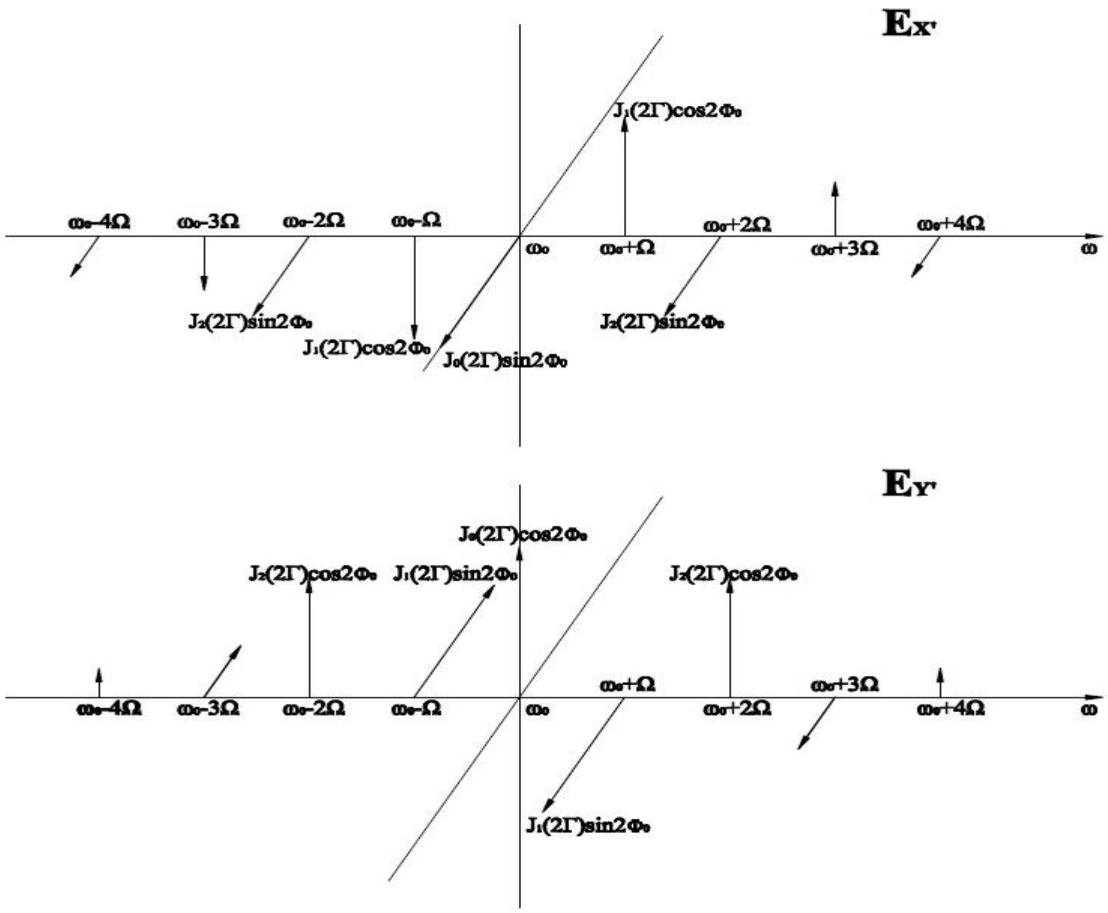

Fig. 3 Three-dimensional vectorial diagram of modulated light for an arbitrary bias voltage.

The modulator emergent light is, generally, elliptically polarized, because in this structure the components $E_{X'}$ and $E_{Y'}$ coexist. From (11) we observe that every harmonics of even order from the spectral development of the component $E_{X'}$ it is advanced with $\frac{\pi}{2}$ in phase toward to same order harmonics from the spectral development of the component $E_{Y'}$, for odd harmonics, the corresponding phase difference it is $-\frac{\pi}{2}$. Thus, the spectral components of even order of the modulator emergent light are elliptically polarized and the odd order ones are, also, elliptically polarized, but in reverse way.

The intensity vector spectral structure of the electric field of modulated light depends of the bias voltage applied to the modulating crystals, thenceforth we analyze this dependence in two particularly cases:

a) Without bias voltage applied to ADP crystals: $U_0 = 0$, $\Phi_0 = 0^\circ$

From (11) we obtain:

$$\begin{pmatrix} E_{X'}(t) \\ E_{Y'}(t) \end{pmatrix} = \begin{pmatrix} \sum_{n=-\infty}^{\infty} J_{2n-1}(2\Gamma) e^{i[\omega_0 + (2n-1)\Omega]t} \\ \sum_{n=-\infty}^{\infty} J_{2n}(2\Gamma) e^{i(\omega_0 + 2n\Omega)t} \end{pmatrix} \quad (12)$$

All the spectral components of the modulated light are linear polarized. The even order components are perpendicularly polarized with respect to the polarization direction of incident light, and odd order components are parallel polarized toward the incident light polarization direction. Through the linear polarizer placed after modulator, of whom transmittance axis is parallel with one of those directions, would be selected optical carrier and the odd, respectively, even harmonics. Three-dimensional vectorial diagram, which correspond to the relation (12), it is represented in Fig. 4.

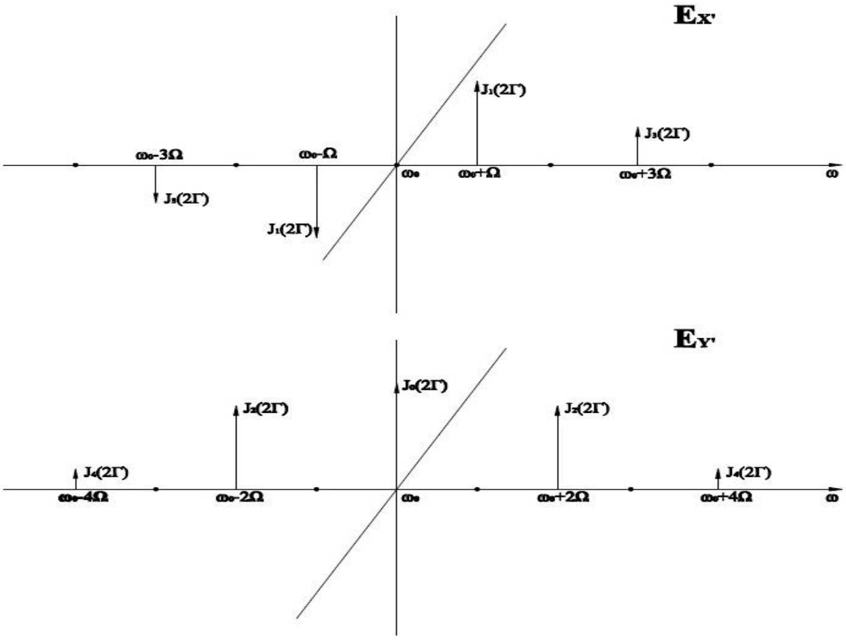

Fig. 4 Three-dimensional vectorial diagram of modulated light, bias voltage U_0 = 0

b) Bias voltage of quarterwave plate applied to the modulator: $2U_0 = U_{\lambda/4}$, $2\Phi_0 = \frac{\pi}{2}$. From the (11) we obtain:

$$\begin{pmatrix} E_{X'}(t) \\ E_{Y'}(t) \end{pmatrix} = \begin{pmatrix} i\sum_{n=-\infty}^{\infty} J_{2n}\left(2\Gamma\right) e^{i(\omega_0+2n\Omega)t} \\ i\sum_{n=-\infty}^{\infty} J_{2n-1}\left(2\Gamma\right) e^{i[\omega_0+(2n-1)\Omega]t} \end{pmatrix} \quad (13)$$

Three-dimension vectorial diagram corresponding to the relation (13) is represented in the Fig. 5.

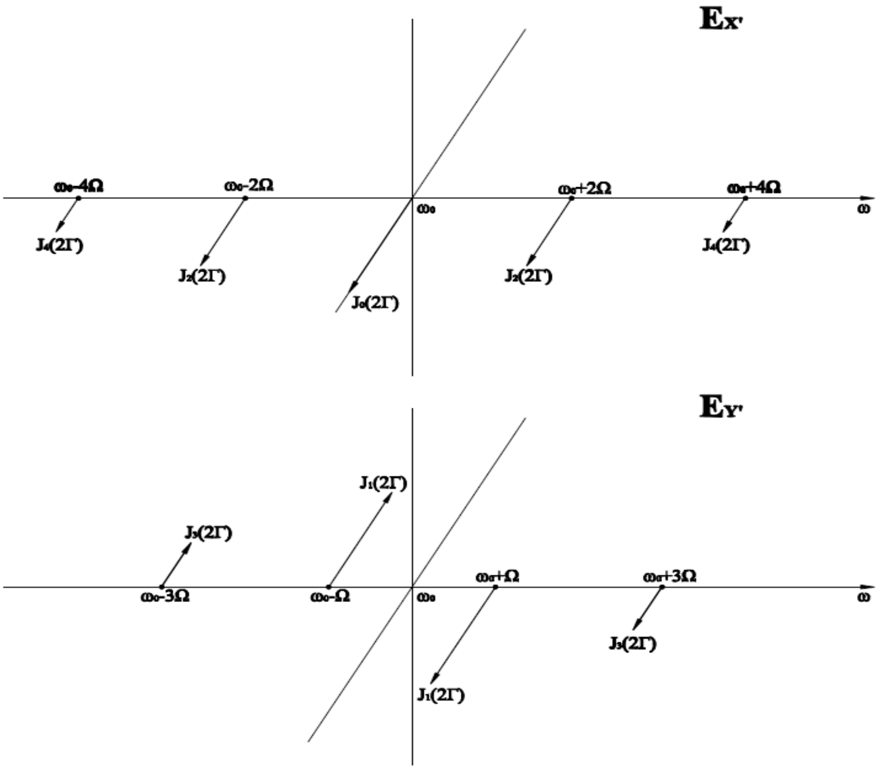

Fig. 5 Three-dimension vectorial diagram of modulated light, bias voltage

In this case, for even ordered components, the modulator doesn't change the light state of polarization, those components being parallel polarized towards the polarization direction of the incident light, and for the odd order components, the modulator act like a half-wave plate which rotate the direction of the polarized incident light with 90º, hence the even order components are perpendicularly polarized in respect with the polarization direction of the incident light

4. Spectral expansion of the coherence matrix

The Jones vector describe the totally polarized light using simple algebraically notions. This vector isn't a measurable entity. The algebraically structures which contains measurable entities are Stokes vector and coherence matrix [10]. Further, we will determinate and develop in Fourier series the coherence matrix, M_c, of the emergent light from the modulator, in the $OX'Y'Z'$ coordinate system.

The coherence matrix is found according to the Jones vector components J', $E_{X'}(t)$ and $\mathbf{E_{Y'}(t)}$, by formula [7]:

$$M_C = \begin{pmatrix} E_{X'}(t)E^*_{X'}(t) & E_{X'}(t)E^*_{Y'}(t) \\ E^*_{X'}(t)E_{Y'}(t) & E_{Y'}(t)E^*_{Y'}(t) \end{pmatrix} \quad (14)$$

By substituting the electric field intensities expressions, $E_{X'}(t)$ and $\mathbf{E_{Y'}(t)}$, from the expression (9) to equation (14), we obtain for the coherence matrix M_C the following expression:

$$M_C = \begin{pmatrix} \sin^2 2\Phi(t) & i\sin 2\Phi(t)\cos 2\Phi(t) \\ -i\sin 2\Phi(t)\cos 2\Phi(t) & \cos^2 2\Phi(t) \end{pmatrix} \quad (15)$$

So, the coherence matrix M_C is a matrix with variation in time, thereby give the temporal modulation of the light polarization state.

If we develop sinus and cosine functions from the expression (15) according to the argument ($2\Phi=2\Phi_0+2\Gamma\sin\Omega t$), we obtain:

$$M_C = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -\cos 4\Phi_0 & i\sin 4\Phi_0 \\ -i\sin 4\Phi_0 & \cos 4\Phi_0 \end{pmatrix}\cos\left(4\Gamma\sin[\![\Omega t]\!]\right) + \frac{1}{2}\begin{pmatrix} \sin 4\Phi_0 & i\cos 4\Phi_0 \\ -i\cos 4\Phi_0 & -\sin 4\Phi_0 \end{pmatrix}\sin(4\Gamma\sin[\![\Omega t]\!]) \quad (16)$$

By using the formulae (10), we obtain the next spectral development for the coherence matrix M_C with variation in time of the modulated light:

$$M_C = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}J_0(2\Gamma)\begin{pmatrix} -\cos 4\Phi_0 & i\sin 4\Phi_0 \\ -i\sin 4\Phi_0 & \cos 4\Phi_0 \end{pmatrix} + \sum_{n=1}^{\infty} J_{2n}(2\Gamma)\begin{pmatrix} -\cos 4\Phi_0 & i\sin 4\Phi_0 \\ -i\sin 4\Phi_0 & \cos 4\Phi_0 \end{pmatrix}\cos(2n\Omega t) + \sum_{n=1}^{\infty} J_{2n-1}(2\Gamma)\begin{pmatrix} \sin 4\Phi_0 & i\cos 4\Phi_0 \\ -i\cos 4\Phi_0 & -\sin 4\Phi_0 \end{pmatrix}\sin\left[(2n-1)\Omega t\right] \quad (17)$$

Coherence matrix elements oscillate at even and odd multiples of the modulation frequency.

From (17) we identify two representative coherence matrices:

$$M_{C1} = \begin{pmatrix} -\cos 4\Phi_0 & i\sin 4\Phi_0 \\ -i\sin 4\Phi_0 & \cos 4\Phi_0 \end{pmatrix} \quad \text{and} \quad M_{C2} = \begin{pmatrix} \sin 4\Phi_0 & i\cos 4\Phi_0 \\ -i\cos 4\Phi_0 & -\sin 4\Phi_0 \end{pmatrix} \quad (18)$$

First matrix, M_{C1}, characterize the type of the state of polarization of the even components, respectively, M_{C2}, determine the type of the state of polarization of the odd components of modulated light. Representative coherence matrices depend of the bias voltage applied on the ADP crystals:

a) Without bias voltage on the ADP crystals: $U_0 = 0$, $\Phi_0 = 0º$. Coherence matrices have the following expressions:

$$M_{C1} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad M_{C2} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad (19)$$

b) Bias voltage of quarterwave plate applied to the modulator: $2U_0 = U_{\lambda/4}$, $2\Phi_0 = \frac{\pi}{2}$ Representative coherence matrices are:

$$M_{C1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad M_{C2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (20)$$

Modulated light can be analyzed by coherence matrix in temporal domain using (15) or frequency domain using (17).

5. Conclusions

In this paper we studied the temporal modulation of the state of polarization of the emergent light from a modulator with two ADP crystals and a half-wave plate placed between them, in Jones matrix and coherence matrix formalism. The emergent polarized light has the following features: the spectral structure of the intensity of the electric field of the modulating light depends of the bias voltage applied to the modulator crystals; both components of the electric field intensity are linearly polarized along the OX' and OY' axes, they contain the optical carrier, ω_0, and all even and odd order harmonics, situated on the both sides of the optical carrier, at whole multiplies of the modulation frequency; the three-dimension vectorial diagrams of the light electric field is an intuitive graphical representation of the spectral development of the light electric field; the coherence matrix is a matrix with variation in time, thereby give the temporal modulation of the state of polarization of the light; coherence matrix elements oscillate at even and odd multiplies of the modulation frequency; representative coherence matrices depend of the bias voltage applied on the ADP crystals; modulated light can be analyzed by coherence matrix in temporal domain or frequency domain.

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