The research of high-speed target space location based on double-N multi-screen array

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In order to solve the measuring difficulties of position coordinate in target space of high-speed flight, the dissertation applies for the measurement theory of N-shaped multi-screen array. Through developing space plane equation of every detection plane and then combining this equation with unknown parameter trajectory space linear equation can acquire the solving formula of high speed target space location. Making use of the law of error propagation analyses the theoretical error of the high-speed target space location coordinates calculation formula and going on digital simulation can reach the distribution of errors of X, Y coordinates. While the error of Z -coordinate is double of the error of target distance. Applying for the experiment of live bullets shoot can manifest the correctness of coordinate measurement formula provided in the article. The errors of distribution and digital simulation are coherent.

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Keywords: High-speed target, Space location, Multi-light screens array, Detection light-screen

1. Introduction

The high-speed target (like high-speed flight bullet) space location technology is widely used in development of weapons and ammunition, vertical target density measurement during the production and military warnings. Nowadays, the high -speed flight target location devices which are based on optical principles mainly include CCD vertical target, optical grid target and light-screen array vertical target [1-3]. According to the measurement principle of CCD vertical target can conclude that it makes use of two line array CCD to collect the picture when the bullets fly through the reserved plane and then we can obtain the target coordinates on the reserved plane according to the geometrical relationship among line array CCD, bullet pictures and pellets, So its measurement accuracy is closely related to the quality of CCD linear array. The high-performance CCD machine is more expensive [4,5]. The optical grid target is made of frame, numerous transmitters and receivers, but it has the high chances to appear wrongness and it is difficult to protect it in advance as well as it could not form the quite bigger testation target plane [6]. Light-screen array vertical target get the trajectory equation through the time when the bullets cross the different detection light-screens, space relationship and the structure size of the array. Combining the trajectory equation and the reserved target plane can calculate the spatial coordinate when high-speed target flies over reserved position [7]. The Four-light photoelectric vertical target is the earliest device of multi-light screen array vertical target, which owns the simple calculation and requires the bullets drive into the slight-screen array vertically and has a high requirement to the installment and commission for detection light-screen [8-10]. In recent years, in order to satisfy the development needs of super high-speed ammunitions, the dense-array weapons and the detection requirements of high-air, high speed target. We have proposed a measurement model of multi-screen array [11] and measurement means so that we can not only obtain the coordinates when the bullet reach the point but can get the pellet's flight vector when the pellets cross the area of light-screen array. In this article, we will analyze the location theories relating to double-N multi-light screens array in order to provide the consultations for the engineering projects in the future.

2. The model of double-N multi-light screens array

Double-N multi-screen array is made of lots of photo electricity detectors with the background of sky, whose function is as the light source. This model can shape into the form similarly with the capital "N" in the space. Double-N multi-light screens array may be shown in Fig. 1.

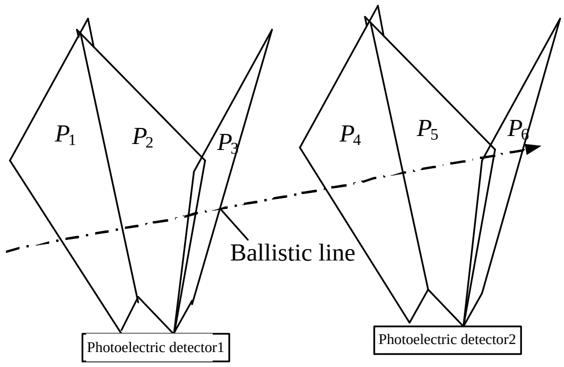


Fig. 1. Double-N multi-light screens array

In Fig. 1, the every detection direction of light-screen is the sky. What's more, there is no strict regulatory geometrical relationship such as parallel, vertical and etc. There is no prevention frame on detector only if the detector can satisfy the requirement of sensitivity so that can increase the detection area arbitrarily and accomplish the high-altitude and high-speed in the target space location orientation.

3. The arithmetic of high-speed target space location orientation

High-speed target can stab the multi-light screen array in arbitrary direction. In order to get the space location of high-speed target, we make a mathematic abstraction to the measurement model in Fig. 1. We can look on each detection light-screen as a space plane and regard the trajectory line as a space vertical one. Because the location of photo electricity detector as well as the location relationship among each formed detection plane are unchanged. Through measuring we can acquire the angle relationship among each detection light-screen so that we can offer the dot-method space plane equation in detection light-screen.

3.1. The foundation of double-N multi-light screen array plane equation

There is the foundation of space coordinate system shown in Fig. 2.

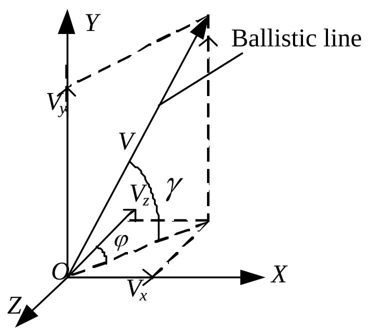


Fig. 2. Space coordinate system and the space parameter of trajectory line locus

In Fig. 2, the included angle between trajectory locus and horizontal plane XOZ is γ (pitching angle); the included angle between the projection line of trajectory locus on XOZ plane and OZ axle is ϕ .

If we project the double-N multi-light screen array according to Fig. 2, we can get the Figs. 3 and Fig. 4.

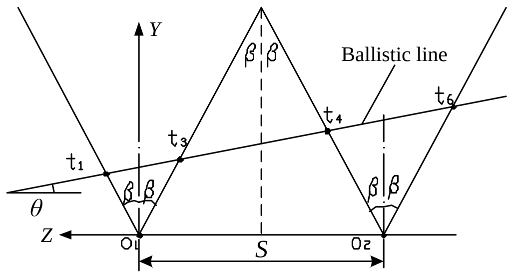


Fig. 3. The projection of double-N multi-light screen in plane YOZ

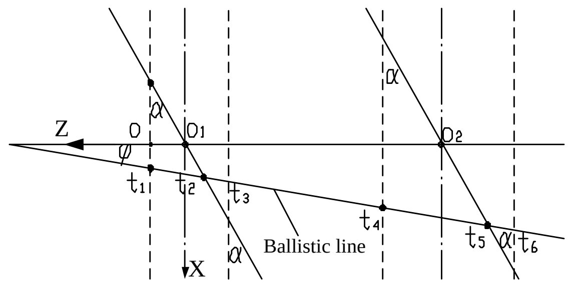


Fig. 4. The projection of the Double-N multi- screen in the plane XOZ

As we can see in the picture, α is the included angle between Light-screen P_2, P_5 and the plane of YOZ . β is the included angle between P_1, P_3, P_4, P_6 and plane YOZ . S is the distance between the two detectors. Establish the space plane equation on the light-screen with the dot-method. It may be shown by formula (1).

$$\begin{cases} P_1:[0 \quad -\sin\beta \quad \cos\beta]\cdot[x \quad y \quad z]' = 0 \\ P_2:[\sin\alpha \quad 0 \quad \cos\alpha]\cdot[x \quad y \quad z]' = 0 \\ P_3:[0 \quad \sin\beta \quad \cos\beta]\cdot[x \quad y \quad z]' = 0 \\ P_4:[0 \quad \sin\beta \quad -\cos\beta]\cdot[x \quad y \quad z+S]' = 0 \\ P_5:[\sin\alpha \quad 0 \quad \cos\alpha]\cdot[x \quad y \quad z+S]' = 0 \\ P_6:[0 \quad \sin\beta \quad \cos\beta]\cdot[x \quad y \quad z+S]' = 0 \end{cases} \tag{1}$$

3.2. Target space localization algorithm

Set the flying speed of target is V, the velocity component on the every axis are $\{V_x, V_y, V_z\}$. If the location coordinate of crossing light-screen P_1 is $M_1(x_1, y_1, z_1)$, the location coordinates that target cross the light-screen plane P_2, P_3, P_4, P_5, P_6 are respectively:

$$M_2 : ((x_1 + V_x \cdot t_2), (y_1 + V_y \cdot t_2), (z_1 + V_z \cdot t_2))$$
$$M_3 : ((x_1 + V_x \cdot t_3), (y_1 + V_y \cdot t_3), (z_1 + V_z \cdot t_3))$$
$$M_4 : ((x_1 + V_x \cdot t_4), (y_1 + V_y \cdot t_4), (z_1 + V_z \cdot t_4))$$
$$M_5 : ((x_1 + V_x \cdot t_5), (y_1 + V_y \cdot t_5), (z_1 + V_z \cdot t_5))$$
$$M_6 : ((x_1 + V_x \cdot t_6), (y_1 + V_y \cdot t_6), (z_1 + V_z \cdot t_6))$$

Set the time when the high-speed target crosses the first light-screen P_1 is t_1, so when the target crosses the P_2 to P_6, the time will be $t_2 \cdots t_6$. Introduce the whole coordinates into the plane equation (1), through simplification can write the matrix form:

$$\begin{bmatrix} 0 & -\sin\beta & \cos\beta & 0 & 0 & 0 \\ \sin\alpha & 0 & \cos\alpha & \sin\alpha t_2 & 0 & \cos\alpha t_2 \\ 0 & \sin\beta & \cos\beta & 0 & \sin\beta t_3 & \cos\beta t_3 \\ 0 & \sin\beta & -\cos\beta & 0 & \sin\beta t_4 & -\cos\beta t_4 \\ \sin\alpha & 0 & \cos\alpha & \sin\alpha t_5 & 0 & \cos\alpha t_5 \\ 0 & \sin\beta & 0 & 0 & \sin\beta t_6 & \cos\beta t_6 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ S \cdot \cos\beta \\ -S \cdot \cos\alpha \\ -S \cdot \cos\beta \end{bmatrix} \tag{2}$$

By the solving matrix equation of type (2) can get the space location coordinates when the target crosses the first light-screen plane P_1:

$$x_1 = \frac{S}{\tan\alpha}\left[\frac{t_2}{t_5 - t_2} - \frac{t_3}{2(t_6 - t_3)}\right] \tag{3}$$

$$y_1 = \frac{S \cdot t_3}{2\tan\beta \cdot (t_6 - t_3)} \tag{4}$$

$$z_1 = y \cdot \tan\beta = \frac{S \cdot t_3}{2(t_6 - t_3)} \tag{5}$$

At the same time, the flight vector $\{V_x, V_y, V_z\}$ of the target is:

$$V_x = \frac{S}{2\tan\alpha}\left[\frac{t_5 - 2t_4 - t_2}{t_4 \cdot (t_5 - t_2)} + \frac{1}{t_6 - t_3}\right] \tag{6}$$

$$V_y = \frac{(t_6 - t_4 - t_3) \cdot S}{2(t_6 - t_3) \cdot t_4 \cdot \tan\beta} \tag{7}$$

$$V_z = \frac{(t_6 + t_4 - t_3) \cdot S}{2t_4 \cdot (t_6 - t_3)} \tag{8}$$

If the plane equation of detection screen can be known, using the formula (3) to (8) can calculate the space crossover point coordinate between the target and light-screen.

4. The error analyses and simulation of target location

In the double-N light screen array, every measurement screen is declining in the double-N light screen array compared with coordinate planes, which will absolutely increase the complexity of coordinate measurement arithmetic in point of impact. But the screen form can be realized easily. If it still needs high measurement accuracy, for this high-speed target spatial orientation. double-N light screen array will be an ideal measurement model. The following are the measurement error's analyses on double-N light screen array.

During the evaluation of vertical target density, the results are acquired through measuring the flying target crosses vertical trajectory line of point of impact on certain plane. Given that the vertical range is S_2 on the reserved plane range is O_2 (along the direction of Z axis), the target coordinate on the reserved vertical plane is $M_6(x, y, z)$. So

$$x = x_1 - V_x(z_1 + S + S_2)/V_z \tag{9}$$

$$y = y_1 - V_y(z_1 + S + S_2)/V_z \tag{10}$$

$$z = -S - S_2 \tag{11}$$

Introduce the $x_1, y_1, z_1, V_x, V_y, V_z$ expression into (9)-(10), then

$$x = \frac{\cot\alpha}{(t_2 - t_5)\cdot(t_3 - t_4 - t_6)} \times \left\{S \cdot \left[t_3(t_4 - t_5) + t_4 t_5 - 2t_4 t_6 + t_5 t_6\right] + S_2 \cdot \left[2t_3 t_4 - t_3 t_5 + t_4 t_5 + t_2 \cdot (t_3 - t_4 - t_6) - 2t_4 t_6 + t_5 t_6\right]\right\} \tag{12}$$

$$y = \frac{\left[S \cdot (t_4 - t_6) + S_2 \cdot (t_3 + t_4 - t_6)\right] \cdot \cot\beta}{t_3 - t_4 - t_6} \tag{13}$$

Applying for the error spreading theory can get the space coordinate error spreading formula:

$$\Delta x = \left(\frac{\partial x}{\partial S} + \frac{\partial x}{\partial S_2}\right)\delta S + \frac{\partial x}{\partial \alpha}\delta\alpha + \left(\frac{\partial x}{\partial t_2} + \frac{\partial x}{\partial t_3} + \frac{\partial x}{\partial t_4} + \frac{\partial x}{\partial t_5} + \frac{\partial x}{\partial t_6}\right)\delta t \tag{14}$$

$$\Delta y = \left(\frac{\partial y}{\partial S} + \frac{\partial y}{\partial S_2}\right)\delta S + \frac{\partial y}{\partial \beta}\delta\beta + \left(\frac{\partial y}{\partial t_3} + \frac{\partial y}{\partial t_4} + \frac{\partial y}{\partial t_6}\right)\delta t \tag{15}$$

$$\Delta z = -2\delta S \tag{16}$$

The each error in the calculation formula (14) and

(16):

$$\frac{\partial x}{\partial S} = \frac{\left[t_3 \cdot (t_4 - t_5) + t_4 \cdot (t_5 - 2t_6) + t_5 t_6\right] \cot\alpha}{(t_2 - t_5) \cdot (t_3 - t_4 - t_6)} \quad (17)$$

$$\frac{\partial x}{\partial S_2} = \frac{\left[2t_3 \cdot (t_4 - t_5) + t_4 t_5 + t_2 \cdot (t_3 - t_4 - t_6) - 2t_6 \cdot (t_4 - t_5)\right] \cdot \cot\alpha}{(t_2 - t_5)(t_3 - t_4 - t_6)} \quad (18)$$

$$\frac{\partial x}{\partial \alpha} = -\frac{1}{(t_2 - t_5)(t_3 - t_4 - t_6)} \times \left\{S \cdot \left[t_3 \cdot (t_4 - t_5) + t_4 (t_5 - t_6) + t_5 t_6\right] + \right. \quad (19)$$

$$\left. S_2 \cdot \left[t_2 \cdot (t_3 - t_4 - t_6) + 2t_3 \cdot (t_4 - t_5) + t_4 \cdot (t_5 - 2t_6) + t_5 t_6\right]\right\} \cdot \csc^2\alpha$$

$$\frac{\partial x}{\partial t} = \frac{\partial x}{\partial t_2} + \frac{\partial x}{\partial t_3} + \frac{\partial x}{\partial t_4} + \frac{\partial x}{\partial t_5} + \frac{\partial x}{\partial t_6} = \quad (20)$$

$$\frac{\left[2S_2 \cdot (t_3 - t_6) + S \cdot (t_4 - t_6)\right] \cdot (t_3 - t_6) \cot\alpha}{(t_2 - t_5)(t_4 + t_6 - t_3)^2}$$

$$\frac{\partial y}{\partial S} = -\frac{(t_4 - t_6) \cdot \cot\beta}{-t_3 + t_4 + t_6} \quad (21)$$

$$\frac{\partial y}{\partial S_2} = -\frac{(t_3 + t_4 - t_6) \cot\beta}{-t_3 + t_4 + t_6} \quad (22)$$

$$\frac{\partial y}{\partial \beta} = \frac{\left[S \cdot (t_4 - t_6) + S_2 \cdot (t_3 + t_4 - t_6)\right] \cdot \csc^2\beta}{-t_3 + t_4 + t_6} \quad (23)$$

$$\frac{\partial y}{\partial t} = \frac{\left[2S_2 (t_3 - t_6) + S(t_4 - t_6)\right] cot\beta}{(t_4 + t_6 - t_3)^2} \quad (24)$$

Every error item formula above contains the moment of the bullet crossing the corresponding exploratory light curtains, and the distance of parameter summations S and S_2, but the measurement error of Z -coordination is double of the target-distance error, it's a constant. Therefore, if we are going to simulate that, we have to find out the time value when the bullet crosses the light curtain.

The following are $t_2 \sim t_6$ expression form of the screen plane equation:

$$\begin{cases} t_2 = (-\cos\alpha \cdot z_1 - \sin\alpha \cdot x_1) / (\sin\alpha \cdot V_x + \cos\alpha \cdot V_z) \\ t_3 = (-\cos\beta \cdot z_1 - \sin\beta \cdot y_1) / (\sin\beta \cdot V_y + \cos\beta \cdot V_z) \\ t_4 = (S \cdot \cos\beta + \cos\beta \cdot z_1 - \sin\beta \cdot y_1) / (\sin\beta \cdot V_y - \cos\beta \cdot V_z) \\ t_5 = (-S \cdot \cos\alpha - \cos\alpha \cdot z_1 - \sin\alpha \cdot x_1) / (\cos\alpha \cdot V_z + \sin\alpha \cdot V_x) \\ t_6 = (-S \cdot \cos\beta - \cos\beta \cdot z_1 - \sin\beta \cdot y_1) / (\cos\beta \cdot V_z + \sin\beta \cdot V_y) \end{cases} \quad (25)$$

During the actual test, bullet target must cross the light curtain array at a certain angle of inclination, at that time, the velocity vector of the bullet will respectively make three velocity component V_x, V_y, V_z at coordination of X, Y and Z . According to the previous test data, given the azimuth angle of the bullet enters the light curtain $\varphi = 3.2^\circ$, the pitching angle $\gamma = 1.78^\circ$, the speed of the bullet $V = 800 m/s$, so:

$$\begin{cases} V_x = V\cos\gamma\sin\varphi = 43.5 m/s \\ V_y = V\sin\gamma = 24.2 m/s \\ V_z = -V\cos\gamma\cos\varphi = 778.4 m/s \end{cases} \quad (26)$$

Synthesize every error item we've analyzed before according to formula (14) and (15), we can get the coordinates of the impact point (x, y) , the total measurement error. the simulation conditions are: $V_z = 800 m/s$, $\alpha = 30^\circ$, $\beta = 10^\circ$, $S = 3000mm$, $S_2 = 1500mm$, $\delta S = 2mm$, $\delta\alpha = \delta\beta = 0.15^\circ$, $\delta t = 5\mu s$, the simulation measurement of target plane is shown in Fig. 5.

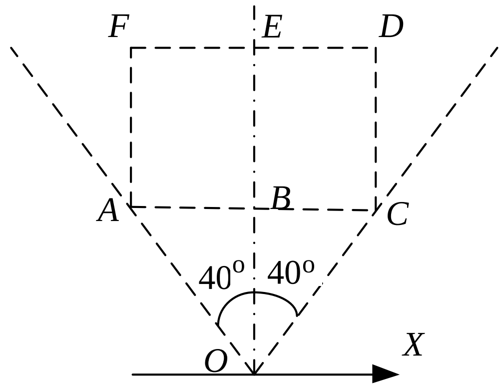
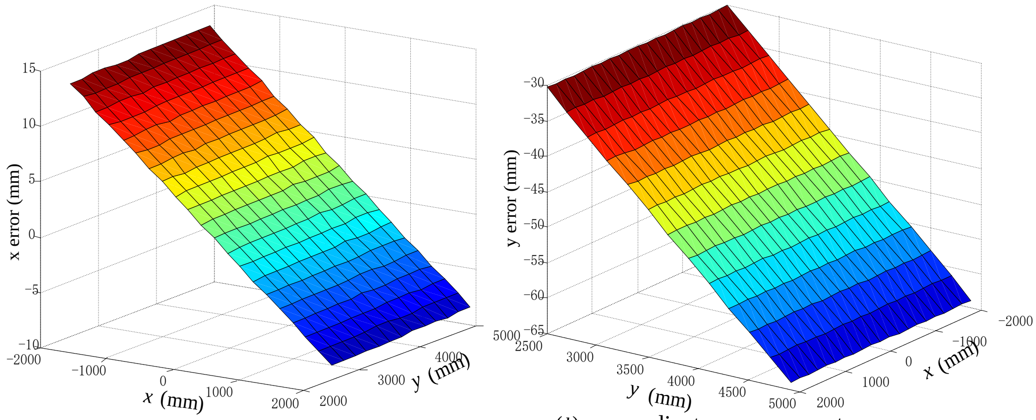


Fig. 5. The double-N array simulation measurement target plane

We generally use the form of sky-screen target to get double-N screen array. In order to give considerations to the measuring height and view field range apply the multi-shoots to realize the view field angle 80° , just like (Fig. 5). At that graph, rectangle $ACDF$ is the simulation target plane, given $\overline{AB} = \overline{BC} = 2000mm$, $\overline{OE} = 5000mm$, then $\overline{BE} = 2616.5mm$. So the range of the simulation target-screen coordination (x, y) is: $x(-2000, 2000)$, $y(2616.5, 5000)$. Introduce the simulation requirements and every error into the formula (14) and (15), we can get all the error distributions through simulation calculation on the simulation target screen, the whole measurement systematic errors when the bullets income tipsily is shown in Fig. 6.



(*a*) x coordinate measurement error (*b*) y coordinate measurement error

Fig. 6. The whole measurement systematic errors when the bullets income tipsily

In Fig. 6, there is a remarkable relationship between the x total measurement errors and the location of the x coordinates. They are merely symmetrical according to $x=0$ (the same y axis). The measurement errors are positive when $x<0$ while the measurement errors are negative when $x>0$. The total measurement errors of y coordinate have notable correlation with the position of y coordinate value (the same x axis), and it decreases obviously when the projectile incident height increases (negative direction). Overall, the measurement errors of x coordinate are clearly bigger than y. On the basis of these analytic results, we can choose the best shoot positions at the experiments and adjust the computer measurement results.

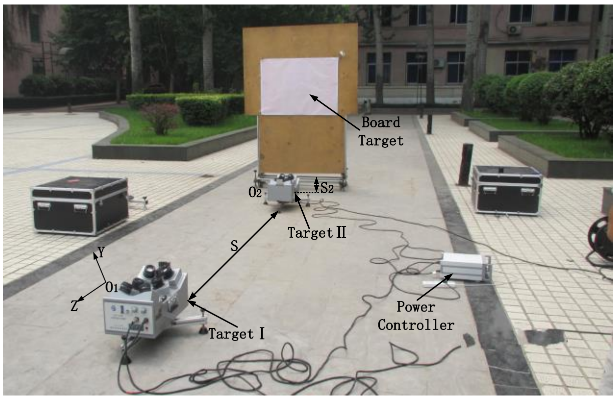


Fig. 7. The experiment of firing practice and designing target

5. Experiment verification

Fig. 7 is the schematic diagram of screen array target in the experiment. Making two same "N" form over the target I and II and the range between them is S. The origin of the whole measurement coordinate system establishes in the heart of target I .The coordinate system is accordant with Fig. 2. In order to satisfy the Right-Hand Rule and make the flight direction of bullets consistent with the negative direction of the Z axis.

When arranging the screen arrays, make the hearts of target I and II O_1, O_2 at the Z axis. If O_1, O_2 are not in the same height, we need to measure altitude difference between them.

In the certain experiment target distribution, we use the pistol bullets with 5.8mm caliber and rifle bullets with 7.62mm caliber and bullet with 30mm caliber to do the experiment. Now we get the data as follows:

The parameter of screen structure: $\alpha_1=\alpha_2=25^\circ$, $\beta_1=\beta_2=\beta_3=\beta_4=13.32^\circ$.

Table 1. The experimental data of 5.8mm pistol

No.	The coordinates of paper-target (mm)		The calculation coordinates of light-screen array (mm)		The distance between two target coordinates(mm)	
	X	Y	x	y	△x	△y
1	-647.00	1534.00	-645.50	1533.20	1.50	-0.80
2	-645.50	1707.00	-645.90	1703.50	0.40	-3.50
3	-270.00	1018.00	-265.00	1008.70	5.00	-9.30
4	-620.00	1232.00	-613.20	1227.90	6.80	-4.10

No.	The coordinates of paper-target (mm)		The calculation coordinates of light-screen array (mm)		The distance between two target coordinates(mm)	
	X	Y	x	y	△x	△y
5	147.50	1737.00	154.40	1730.80	6.90	-6.20
6	-114.00	1732.00	-114.40	1730.20	-0.40	-1.80
7	659.00	1492.00	660.50	1489.20	1.50	-2.80
8	649.00	1753.00	648.50	1752.60	-0.50	-0.40
9	662.00	1269.00	661.40	1264.60	-0.60	-4.40
10	460.00	1717.00	457.20	1716.10	-2.80	-0.90

Table 2. The experimental data of 7.62mm rifle

No.	The coordinates of paper-target (mm)		The calculation coordinates of light-screen array (mm)		The distance between two target coordinates(mm)	
	X	Y	x	y	△x	△y
1	-299.00	-76.00	-298.90	-75.46	-0.10	-0.54
2	-564.00	243.00	-567.90	240.74	3.90	2.26
3	-323.00	193.00	-323.00	195.24	0.00	-2.24
4	-474.00	153.00	-471.10	150.84	-2.90	2.16
5	-265.00	-436.00	-266.70	-434.36	1.70	-1.64
6	494.00	-71.00	496.5	-79.76	-2.50	8.76
7	476.00	201.00	475.6	193.64	0.40	7.36
8	334.00	-333.00	331.5	-336.66	2.50	3.66
9	477.00	-329.00	478.6	-331.06	-1.60	2.06
10	283.00	-357.00	282.1	-356.36	0.90	-0.64

Table 3. The experimental data of 30mm bullets

No.	The coordinates of paper-target (mm)		The calculation coordinates of light-screen array (mm)		The distance between two target coordinates(mm)	
	X	Y	x	y	△x	△y
1	-172.0	164.0	-168.6	163.61	-3.38	0.39
2	-219.0	362.0	-220.7	362.09	1.71	-0.09
3	-300.0	321.0	-300.0	320.43	-0.04	0.57
4	-406.0	266.0	-404.1	266.45	-1.87	-0.45
5	-52.0	92.0	-46.0	92.44	-5.95	-0.44
6	605.0	290.0	609.1	285.421	-4.15	4.58
7	600.0	117.0	600.2	113.427	-0.20	3.57
8	488.0	200.0	489.8	201.709	-1.83	-1.71
9	395.0	162.0	393.9	165.874	1.12	-3.87
10	423.0	340.0	424.3	342.567	-1.34	-2.57

From the experimental data of the bullets shooting in the Table 1 and Table 2, we can see that the errors of the x coordinate of a shoot between the simulation coordinate are basically consistent. But the y coordinate is a little bigger than the simulation coordinate. Following are the probable reasons: (1) The artificial measurement errors; (2) The position of target is not vertical axis Z, there may be a certain mistakes; (3) The changes of sky brightness lead to the decreases of device's sensibility so that we could not get the correct timetable when the bullets cross the light-screen; (4) The test target is too large. Table 3 shows that the experimental effects of the 30mm artillery firing bullets are better than of the 5.8mm and 7.62mm bullets. The main reason is that after the projectile diameter increases, the single resolution of the system is strong, thus improving the measurement accuracy of time when the bullets cross the screen. How to solve the big error of small projectile? Under the condition of minimizing the artificial measurement error and the external environment influence, the test target will be resized. From the test results of three different kinds of caliber projectiles, the measurement algorithm proposed in this paper is correct and has a comparable high accuracy.

6. Conclusions

In this paper, using the theory of analytic geometry establishes the space surface equation of each detection screen of N-shaped multi-screen array. And using the space line with unknown parameters takes place of high speed target trajectory theory. Solving the linear equations obtain the coordinates of the intersection when the high-speed target crosses the first detection light-screen. Then we can convert to the predetermined vertical target plane to get the target x coordinate and y coordinate in the reserved target plane. The algorithm has the following advantages: (1) Not only the coordinates of the impact point but also the flight vector of the projectile passing through the interval of the light curtain array can be calculated according to the relationship between the six detection planes; (2) The conclusion can be obtained through the analyses of the live test data of three different types of projectiles: there is almost no error between the projectile coordinates calculated by the light curtain array target and the actual bullet hole coordinates left on the paper target; (3) Through numerical simulation can get the distribution rules of x and y coordinates in order to provide the principles of error correction for the actual engineering measurement.

According to the error spreading theory, the distribution law of the measurement error of x and y coordinates is analyzed. By numerical simulation, the measurement error of x coordinate is symmetrical distribution along $x = 0$ and the maximum value in the simulation target is 12mm. The measurement error of y coordinates increases with the increasing of y value, and the maximum value is 65mm. Finally, by firing experimental data to verify the coordinate calculation formula given in this paper is correct, and the error distribution trend and the simulation results are consistent.

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References

[1] B. Feng, J. P. Ni, L. Yang, J. Ballistics **20**, 59 (2008).
[2] H. S. Li, H. Y. Gao, M. Jiang, J. Ballistics **19**, 33 (2007).
[3] H. Li, Optelectron. Adv. Mat. **8**(7–8), 653 (2014).
[4] Y. X. Liu, Y. Ghanu, R. S. Xionu, Acta Photon. Sinica **25**(6), 550 (1996).
[5] H. Li, L. Lei, H. M. H. Chang, Acta Photonica Sin. **37**, 1238 (2008).
[6] Journal of Hand Surgery **31**(9), 1535 (2015).
[7] N. I. Jin-Ping, L. U. Hong-Wei, D. Tao, Journal of Ballistics **24**, 1(2012).
[8] H. Tian, J. P. Ni, X. W. Lu, IEEE International Conference on Computer Science and Automation Engineering, (605) 2011.
[9] B. Feng, J. P. Ni, Z. C. Wu, Optoelectron. Adv. Mat. **10**(1-2), 40 (2016).
[10] B. Feng, Xiangzhong Meng, International Journal of Applied Mathematics and Statistics **51**, 366 (2013).
[11] B. Wenzhuo, C. Mingyu, Z. Wei, et al. Acta Opt. Sin. **30**, 2249 (2012).

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