The temperature dependence of magnetic penetration depth in superconductors

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The magnetic penetration depth λ is one of the most fundamental parameters of superconductivity which is the order of several hundred angstroms. Thus in this work, temperature behavior of the penetration depth has been investigated. At first Gorter-Casimir two fluid model is viewed. Due to importance of the BCS energy gap, it appears that in high temperature superconductors (HTS) penetration depth is different from the two fluid model. At low temperatures, the condition of $T/T_c \leq 1/3$, unconventional superconductors variation of penetration depth with temperature was compared with available literature results.

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1. Introduction

One of the basic lengths of superconductivity is the magnetic penetration depth λ [l]. The concept of a characteristic length for magnetic field penetration into a superconductor was determined after the discovery of Meissner effect in 1933. C. J. Gorter and H. Casimir introduced a two fluid model of superconductivity in 1934 [2-5].

In 1935, F. and H. London brothers introduced a phenomenological model of superconductivity [6]. One of the most important characteristic of superconductivity is electrodynamics response. It has been generally accepted that the electrodynamics of superconductors in the London limit is described by the London equations [7]. Response to low-frequency electromagnetic fields is characterized by the two most important properties of superconductivity. These are perfect conductivity and perfect diamagnetism. London theory predicts that low magnetic fields are not completely excluded from a superconductor. But penetrates a small distance across the boundary. Hence, the penetration depth is of the order of several hundred angstroms [8].

The London theory could not predict correctly the absolute value of $\lambda(T=0)$ and could not explain the variation of the penetration depth with alloying [8]. Combined with the thermodynamic two-fluid model of the superconducting phase, the London theory makes definite predictions of the temperature variation of penetration depth. So, for the first time the BCS theory made the connection between the penetration phenomenon and the fundamental microscopic energy gap parameter of the superconducting phase [9].The temperature dependence of $\lambda(T)$ in the BCS theory reflects the temperature dependence of the energy gap [8]. Since it depends explicitly on the superconducting energy gap, is related fundamentally to the symmetry of the superconducting state and thus to the mechanism of pairing. Furthermore, the zero temperature value $\lambda(0)$ contains information on the effective mass and density of the superconducting carriers [1].

The temperature dependence of London penetration depth plays an important role in the ongoing debate about the mechanism of high-T_c superconductivity. It is indeed essential piece of information about this mechanism to know the symmetry of the order parameter [10]. Thus in HTS are important of the fact that the material is s-wave or d-wave pairing.

2. Theory

In a superconducting material, superelectrons experience no resistance to their motion. Therefore when a constant electric field $\vec{E}$ is retained in the material, the electrons accelerate steadily under the action of this field [11]. However, the first London equation is given by

$$\vec{E} = \frac{\partial}{\partial t}\left(\Lambda \vec{J_s}\right) \tag{1}$$

where $\Lambda \equiv m/n_s e^2$ [13]. Λ is a phenomenological parameter. Here, acceleration of the superfluid carriers with charge e and mass m give rise to the supercurrent $\vec{J}$ [7]. This equation describes perfect conductivity because of any electric field speeds up the superconducting electrons rather than simply maintaining their velocity against resistance as described in Ohm's law in a normal conductor [12]. The second London equation is given by [14];

$$\vec{\nabla} \times \vec{J_s} = -\frac{n_s e^2}{m}\vec{B} \tag{2}$$

To obtain the London penetration depth, we can write the following equations.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J_s} \tag{3}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \mu_0 \left(\vec{\nabla} \times \vec{J_s}\right) \tag{4}$$

$$= -\frac{\mu_0 n_s e^2}{m} \vec{B} \tag{5}$$

Recall and using that vector calculus identity $\vec{\nabla} \times \left(\vec{\nabla} \times B\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{B}\right) - \nabla^2 \vec{B}$ and on the ground that $\vec{\nabla}.\vec{B} = 0$, we can write as

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = -\frac{1}{\lambda_L^2} \vec{B} \tag{6}$$

And

$$\nabla^2 \vec{B} = \frac{1}{\lambda_L^2} \vec{B} \tag{7}$$

where λ_L is the London penetration depth which is defined as

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}} \tag{8}$$

[15, 16]. Eq. (7) should be pointed out that to describe the distribution of magnetic flux density inside a superconductor based on the known properties of superconductors [11].

3. Two fluid model

This model is based on the presumption that all current carriers are normal electrons for temperatures greater than T_c. When a superconductor is cooled below T_c, normal electrons begin to transform to the superelectron state [17]. The conduction electron density is $n = N/V$, where N is the number of conduction electrons in the sample of volume V. n_n and n_s are the densities of normal state and superconducting electrons, where $n = n_n + n_s$ [18]. In the range of $0 < T < T_c$, the total current density $\vec{J}$ flowing through a material is the sum of the normal current density $\vec{J_n}$ and the superfluid density $\vec{J_s}$ [19],

$$\vec{J} = \vec{J_n} + \vec{J_s} \tag{9}$$

At absolute zero, all conduction electrons are coupled into "Cooper pairs" constituent of the superfluid.

The two-fluid model is a useful way to build temperature effects into the London relations. It is reasonable to derive the relation of temperature dependence in a logical way from the Gorter-Casimir two fluid model [20]. According to this model, the density of superconducting electrons at reduced temperature $\frac{T}{T_c}$ and the temperature dependence of n_s is that [17, 21]

$$n_s(T) = n_0 \left[1 - \left(\frac{T}{T_c}\right)^4\right] \tag{10}$$

(1.2)This equation is that this screeni

In Fig.1, we plotted that the number density of superelectrons $n_s(T)/n_0$ versus T/T_c for one of the conventional superconductor Nb.

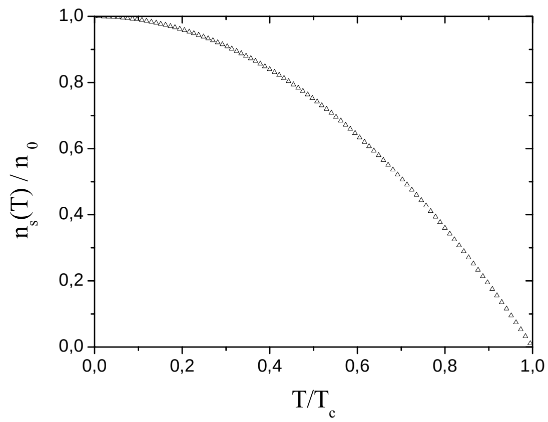

Fig. 1. Temperature dependence of the density of superconducting electrons n_s as given by Eq. (10) for Nb. Here $T_c = 9.25\ K$

4. The penetration depth

The penetration depth is a magnetic property of a superconductor. With a few angstroms deep, in a very thin surface layer can be any magnetic field. With a large superconductor, it is almost impossible to know it's overall size accurately enough for direct comparison with the penetration layer thickness [20].

The value of the penetration depth depends on the ratio of the Bardeen-Cooper-Schrieffer (BCS) coherence length ξ_0 and electron mean-free path l [22]. When the mean free path is much shorter than the coherence length the material is said to be in the dirty limit ($l \ll \xi_0$), while if the opposite is true, the material is in the clean limit ($\xi_0 \ll l$). That is, for the clean limit $\frac{l}{\xi_0}$ is equal to 100, for the dirty limit $\frac{l}{\xi_0}$ is equal to 0.01 [23].

Tinkham found that the BCS relationship between these two quantities at zero temperature is the best approximated by [24];

$$\lambda(0) = \lambda_L \left(1 + \frac{\xi_0}{l}\right)^{1/2} \tag{11}$$

The energy gap is characteristic of the material and dependent on temperature. In the BCS theory, the critical temperature T_c is related to Δ_0 by the following constraint, $\frac{2\Delta_0}{k_B T_c} = 3.52$ where k_B is Boltzmann's constant. This constraint is known as the weak-coupling limit and it is characteristic of the BCS theory. Most superconductors do not follow this relationship exactly, but instead have a higher value for $\frac{2\Delta_0}{k_B T_c}$. If this ratio is large, the material is said to be strongly-coupled, that is $2\Delta_0/k_B T_c > 3.52$. However, if this ratio is not too large, the superconductor is still considered to be weakly-coupled, and the BCS theory is still accurate. That is, for the weak coupling limit's value is $\frac{\Delta_0}{k_B T_c} = 1.75$ [23, 25]. In the literature, the weak coupling clean limit is usually represented as conventional BCS behavior. Even though energy gap vanishes at the transition temperature, the fundamental of BCS theory uses an energy gap. The BCS temperature dependence of the penetration depth can be approximated well by [27];

$$\lambda^2(T) = \lambda^2(0)/[1 - (T/T_c)^{3-(T/T_c)}] \quad (12)$$

In the two fluid model penetration depth is given by [26, 27]

$$\lambda^2(T) = \lambda^2(0)/[1 - (T/T_c)^4] \quad (13)$$

The energy gap is not considered in this formula; therefore Eq. (13) does not describe the penetration depth of a weak-coupling BCS superconductor. [28].

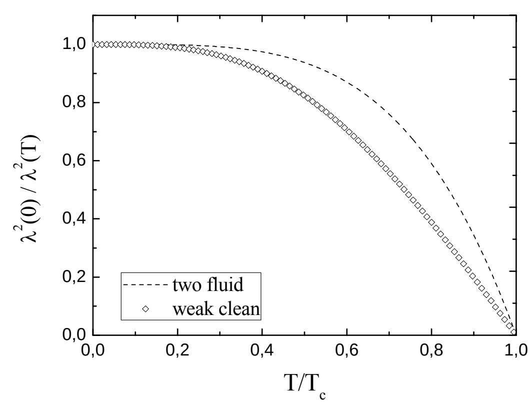

Fig.2. Various theoretical temperature dependencies of $\lambda^2(0)/\lambda^2(T)$. Weak clean line (BCS) is given by Eq. (12) and two-fluid line by Eq. (13) for Nb.

We plot the temperature dependence of $\lambda^2(0)/\lambda^2(T)$ by using Eq. (12) and Eq. (13) for Nb. This graphic is fitted to the literature [25]. In Fig. 2, we also compared with the two fluid approximations and the BCS theory. The change of the penetration depth in unconventional superconductors (for YBCO) with use of Eq. (12) and Eq. (13) is shown in Fig. 3.

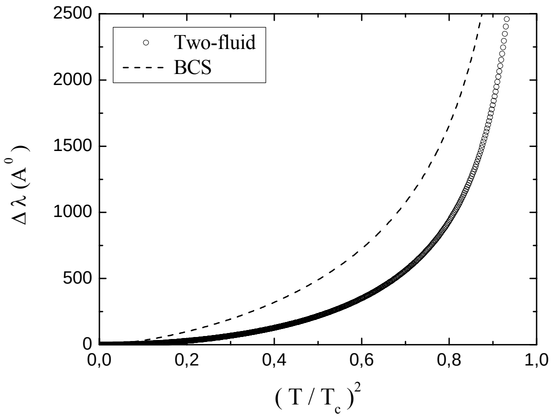

Fig. 3. The change in the penetration depth calculated with using two fluid model (Eq. 12) and equation (13).

In Fig. 3, open circle represents a two-fluid approximation with $\lambda(0) = 1400A^0$ and dotted line represents the clean weak-coupling penetration depth λ_L for $\lambda(0) = 1400A^0$ [1].

5. Penetration depth at low temperatures

At low temperatures, $\frac{T}{T_c} \leq \frac{1}{3}$ [30, 35], the temperature dependence of penetration depth is directly related to electrodynamic properties. Hence, its temperature dependence is calculated by using the electrodynamic theory of superconductors. This theory includes that two limiting behaviors, local and nonlocal [29].

In the local approximation, when $\xi_0 \ll \lambda(0)$, the low temperature behavior of $\Delta\lambda(T) = \lambda(T) - \lambda(0)$ are commonly used to determine the symmetry of the superconducting pairing state [30, 31]. Also, the analysis of temperature dependent superfluid density in superconductors is a powerful tool for examining pairing symmetry [32]. In order to determine the normalized superfluid density, $\rho = [\frac{\lambda(0)}{\lambda(T)}]^2$, it is necessary to know the absolute magnitude of the penetration depth $\lambda(0)$ [33]. The normalized superfluid density components are given by [34];

$$\rho_{ii}(T) = \frac{n_{ii}(T)}{n} = \left(\frac{\lambda_{ii}(0)}{\lambda_{ii}(T)}\right)^2 \quad (14)$$

Defining $\Delta\lambda = \lambda(T) - \lambda(0)$, Eq. (14) can be written,

$$\rho = \left(1 + \frac{\Delta\lambda(T)}{\lambda(0)}\right)^{-2} \approx 1 - 2\frac{\Delta\lambda(T)}{\lambda(0)} \quad (15)$$

For s-wave superconductors, in the low temperature limit ρ is given by;

$$\rho \approx 1-\sqrt{\frac{2\pi\Delta_0}{T}}\,exp\left(\frac{-\Delta_0}{T}\right) \quad (16)$$

Here Δ_0 is the zero temperature value of the energy gap in units of temperature [31, 35, 37]. The corresponding penetration depth from Eq. (15),

$$\frac{\Delta\lambda}{\lambda(0)} \approx \frac{1-\rho}{2} \quad (17)$$

when Eq. (16) is substituted in Eq. (17),

$$\frac{\Delta\lambda(T)}{\lambda(0)} = \sqrt{\frac{\pi\Delta_0}{2T}}\,e^{-\Delta_0/T} \quad (18)$$

is obtained [34,36, 38].

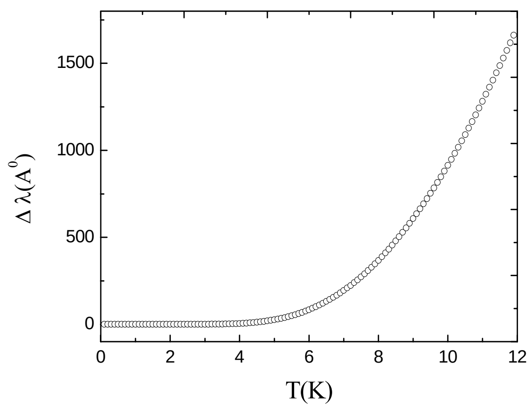

Fig. 4. Plot of $\Delta\lambda(T)$ versus temperature for MgB_2.

In Fig.4, the temperature dependence of the change in penetration depth $\Delta\lambda$ is plotted versus the temperature for MgB_2 using by Eq. (18). Fig. 4 represents a low temperature limit BCS behavior for an s-wave material which is fit to the literature [35, 37, 39].

In the nonlocal approximation, when $\xi_0 \gg \lambda(0)$, nonlocality may play an important role in the electromagnetic response of a d-wave superconductor [31]. For the $d_{x^2-y^2}$ symmetry now believed to describe high temperature cuprates. HTS are layered materials in which the superconducting pairing is accepted to occur in the CuO_2 planes [40]. The gap parameter is given by [33, 34]

$$\Delta = \Delta_0\cos(2\varphi) \quad (19)$$

At low temperatures this leads to a superfluid density varying as [34],

$$\rho = 1-\frac{2ln2}{\Delta_0}T \quad (20)$$

That is, in the case of d-wave pairing is given by [41, 42];

$$\left[\frac{\lambda(0)}{\lambda(T)}\right]^2 = 1-2\left(\frac{T}{\Delta_0}\right)ln2 \quad (21)$$

or

$$\frac{\Delta\lambda(T)}{\lambda(0)} = \frac{Tln2}{\Delta_0} \quad (22)$$

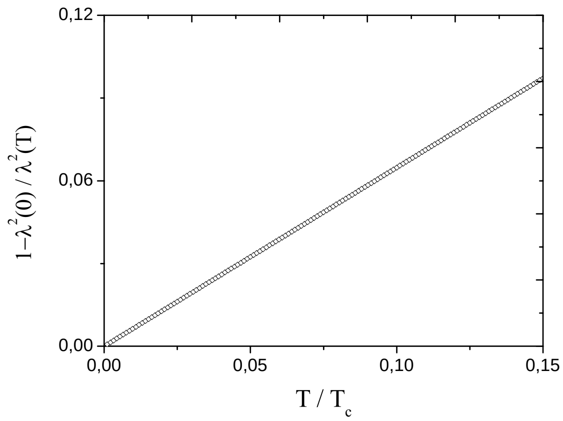

Fig. 5. Plots of $1-[\lambda(0)/\lambda(T)]^2$ versus T/T_c for $YBa_2Cu_3O_7$ [34, 40]

The materials such as YBCO possess d-wave symmetry. We plot that $1-[\lambda(0)/\lambda(T)]^2$ versus T/T_c for $YBa_2Cu_3O_7$ by using Eq. (21). It is clear that the linear term in $1-[\lambda_{ab}(0)/\lambda_{ab}(T)]^2$ increases as T_c decreases [42].

6. Discussion and conclusions

We have studied the temperature dependence of magnetic penetration depth in superconductors. First of all, this dependence is examined by two fluid model. However, since the energy gap is not considered in this model, it is modified in the literature. The penetration depth is impressed by the energy gap. Taking account to this property, two fluid approximation and BCS like behaviors are compared. It is seen that BCS behavior is better agreement with experiments in the literature than empirical two fluid model.

In HTS, the temperature dependence of λ contains important information on the involved pairing mechanism that is s or d wave, weak or strong coupling. For s-wave materials that have not contained nodes, penetration depth was examined. We describe the penetration depth in the context of the BCS theory and the low temperature effects. Also, we have described that $d_{x^2-y^2}$ wave superconductors which have nodes exhibits different behaviors than with s-wave.

In addition, in the temperature dependence of the penetration depth, the energy gap has remarkable role. At low temperature limit, as the temperature dependence of penetration depth is exponential in s-wave superconductors, is linear in d-wave superconductors.

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