Third order material dispersion analysis in some types of glass optical fibres

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An estimation of material dispersion is presented in some types of glass optical fibres. Using broadly applicable principles for group velocity, a simple expression is derived for material dispersion that includes the third derivative of the index of refraction with respect to the wavelength, $d^3n/d\lambda^3$. It is shown that material dispersion persists around the so-called zero material dispersion wavelength when $d^2n/d\lambda^2=0$, meaning that a pulse of light centred on this zero material dispersion wavelength also suffers significant dispersion, necessitating that the third order term $d^3n/d\lambda^3$ be taken into account.

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1. Introduction

The broadening of light pulses caused by dispersion is a critical factor that limits the quality of signal transmission over optical links. Fundamentals about this topic, presented simply and illustrated by numerous examples, can be found in [1, 2]. Much research has been focused on theoretical investigations of the pulse broadening and the information-carrying capacity of multimode glass optical fibres [3-5]. The broadening and deformation of the light pulse in optical fibres has also been the subject of experimental measurements [6]. An important component of delay distortion in fibre-optic waveguides is produced by wavelength dispersion of the refractive index. Material dispersion is a delay-time dispersion caused by the fact that the refractive index of glass changes in accordance with the change of the signal frequency (or wavelength). The dependence of refractive indices of core and cladding on the frequency for optical fiber is nonlinear. This nonlinearity of the refractive index causes a non zero value for $d^2n/d\omega^2$. This material dispersion effect is characterized by the parameter M given by [7]

$$M=\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}, \tag{1}$$

where λ is the wavelength of light in vacuum, n is the refractive index, and c is the speed of light. In general, $M(\lambda)$ decreases rapidly in air and usually changes sign at some crossover wavelength λ_0. A review of literature reveals that much research effort (for example [8,9]) has focused on the computational simulation and modelling dispersion in optical fibres. However, there are only a few studies that have reported about material dispersion around the so-called zero material dispersion (ZMD) wavelengths λ_c when the second derivative of the index of refraction with respect to the wavelength is zero $d^2n/d\lambda^2=0$. The purpose of this paper is to model the material dispersion in context of the pulse distortion caused by effects of the term $d^3n/d\lambda^3$. Such modelling is based on the assumption that the refractive index of glass and plastic optical fibres follows a three-term Sellmeir's function of wavelength. For operation around ZMD wavelengths in the glass optical fibres, results show that even as $d^2n/d\lambda^2=0$, the material dispersion still occurs caused by effects of the third derivative of refractive index with respect to the wavelength.

2. Concept of zero material dispersion wavelengths

Over the wavelength region of greatest interest, the material dispersion can best be discussed by studying the propagation of plane waves in homogeneous dispersive medium. The group velocity of a wave is defined as

$$v_g=\frac{1}{(d\beta/d\omega)}, \tag{2}$$

where $\beta(\omega)=(\omega/c)n(\omega)$ represents the propagation constant, and $n(\omega)$ represents the frequency-dependent refractive index. In the case of a medium for which β is not a linear function of ω, the medium is said to be dispersive. Thus,

$$\frac{1}{v_g}=\frac{d\beta}{d\omega}=\frac{1}{c}\left[n(\omega)+\omega\frac{dn}{d\omega}\right]=\frac{1}{c}\left[n(\lambda)-\lambda\frac{dn}{d\lambda}\right], \tag{3}$$

where the free space wavelength λ is related to the frequency through the relation $\lambda=(2\pi c)/\omega$. The quantity $N_g=n(\lambda)-\lambda(dn/d\lambda)$ is referred to as the group refractive index since c/N_g determines the group velocity. Thus, the time it takes for a pulse to traverse some distance L through the fibre is given by

$$t(\lambda) = \frac{L}{v_g} = L\frac{d\beta}{d\omega} = L\frac{d\beta}{d\lambda}\frac{d\lambda}{d\omega}, \qquad (4)$$

which depends on the wavelength λ. With $\lambda = 2\pi c/\omega$ the following relation is obtained

$$t(\lambda) = \frac{L}{c}\left[n(\lambda) - \lambda\frac{dn}{d\lambda}\right]. \qquad (5)$$

If $dn/d\lambda \neq 0$, then the transit time for the pulse depends on the wavelength. Most light sources such as light emitting diodes (LEDs) or semiconductor lasers have a spectral width $\Delta\lambda$ that exceeds by far that of the pulse itself. Spectral widths $\Delta\lambda$ of light sources are comparatively large in the case of LEDs, approximately around 20 nm at half height, and small in the case of laser diodes (LDs), approximately between 1 and 2 nm at half height. If a source is characterized by a spectral width $\Delta\lambda$, then each wavelength component will traverse with a different group velocity resulting in temporal broadening of the pulse. A pulse produced by a light source with spectral width $\Delta\lambda$ will thus, after travelling distance L, spread out over a time interval determined by:

$$\Delta t = \frac{dt}{d\lambda}\Delta\lambda. \qquad (6)$$

The derivative $dt/d\lambda$ describes pulse broadening (or spreading) and may be of more interest than the time delay t itself. From equation (5), the pulse broadening can be derived as

$$\Delta t = \frac{L}{c}\left(\frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda\frac{d^2n}{d\lambda^2}\right) = -\frac{L}{c}\lambda\frac{d^2n}{d\lambda^2}\Delta\lambda \qquad (7)$$

or

$$\Delta t = -\frac{L}{c}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right)\left(\frac{\Delta\lambda}{\lambda}\right). \qquad (8)$$

The quantity $\lambda^2 d^2n/d\lambda^2$ is a dimensionless quantity. Since the material dispersion is proportional to the spectral width $\Delta\lambda$ and to the length L of the fibre traversed by the beam, dispersion is usually specified in units of picoseconds per kilometre (length of the fibre) per nanometre (spectral width of the source). Thus, the material dispersion coefficient D_m of the optical fibres is given by

$$D_m = \frac{\Delta t}{L\Delta\lambda} = \frac{1}{L}\frac{dt}{d\lambda} = -\frac{1}{\lambda c}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right)\cdot 10^9 \text{ ps/}(\text{km}\cdot\text{nm}) \qquad (9)$$

where λ is measured in micrometers and $c = 3\cdot 10^5$ km/s.

The wavelength λ_c at which $d^2n/d\lambda^2 = 0$, is referred to as the zero material dispersion wavelength. At this wavelength, pulses will suffer negligible dispersion as they propagate through an optical fibre. Below this wavelength, $d^2n/d\lambda^2$ is positive while above this wavelength $d^2n/d\lambda^2$ is negative. The former wavelength region is referred to as a normal group velocity dispersion region, while the latter region is known as an anomalous group velocity dispersion region. It should be noted that the pulse disperses in both regions: longer-wavelength components of the pulse travel faster than the shorter-wavelength ones in the normal dispersion region while the opposite is true for the anomalous dispersion region. It appears that, when an optical fibre is operated at ZMD wavelength, the pulses will not suffer any dispersion at all. However, for operation around ZMD wavelength, pulse distortion would be $d^3n/d\lambda^3$. This term has a significant impact in determining of the dispersion around ZMD wavelength. If we consider two closely spaced wavelengths λ and $\lambda + \Delta\lambda$, then $t(\lambda)$ and $t(\lambda+\Delta\lambda)$ will represent respective time delays. Then, we may write

$$t(\lambda+\Delta\lambda) = t(\lambda) + \Delta\lambda\frac{dt}{d\lambda} + \frac{(\Delta\lambda)^2}{2}\frac{d^2t}{d\lambda^2} \qquad (10)$$

The term $dt/d\lambda$ vanishes at the ZDM wavelength, so the pulse broadening due to material dispersion is given by

$$\Delta t = t(\lambda+\Delta\lambda) - t(\lambda) = \frac{(\Delta\lambda)^2}{2}\frac{d^2t}{d\lambda^2} = \frac{(\Delta\lambda)^2}{2}\frac{(-L\lambda)}{c}\frac{d^3n}{d\lambda^3}. \qquad (11)$$

Thus,

$$\Delta t = -L(\Delta\lambda)^2\left(\frac{\lambda}{2c}\frac{d^3n}{d\lambda^3}\right) \qquad (12)$$

or

$$\Delta t = -\frac{L}{2c}\left(\lambda^3\frac{d^3n}{d\lambda^3}\right)\left(\frac{\Delta\lambda}{\lambda}\right)^2 \qquad (13)$$

It should be noted that the broadening that occurs when a pulse propagates through an optical fibre at the wavelengths around ZMD wavelength, is proportional to $(\Delta\lambda)^2$ and distance L. Thus, the material dispersion coefficient D'_m of the optical fibres around zero material dispersion wavelengths is given by

$$D'_m = \frac{\Delta t}{L\Delta\lambda} = -\frac{\Delta\lambda}{\lambda^2 c}\left(\lambda^3\frac{d^3n}{d\lambda^3}\right)\cdot 10^9 \text{ ps/}(\text{km}\cdot\text{nm}) \qquad (14)$$

where λ is measured in micrometers and $c = 3\cdot 10^5$ km/s.

3. Results and discussion

3.1. Fused silica fibres

We have carried out a computer simulation for fused silica first. An empirical expression for the refractive index variation is given by [10]

$$n(\lambda) = C_0 + C_1\lambda^2 + C_2\lambda^4 + \frac{C_3}{(\lambda^2 - l)} + \frac{C_4}{(\lambda^2 - l)^2} + \frac{C_5}{(\lambda^2 - l)^3}, \quad (15)$$

where C_0= 1.4508554 , C_1= –0.0031268 , C_2= –0.0000381 , C_3= 0.0030270 , C_4= –0.0000779 , C_5= 0.0000018 , and l= 0.035 . Based on Eq. (15), the second derivative of refractive index for fused silica with respect to the wavelength is shown in Fig. 1. To gain numerical appreciation, we see from Eq. (8) that, at $\lambda_c = 1.2753\,\mu\text{m}$, the material dispersion for the fused silica is $\Delta\tau = 0$ (the value of λ_c is ZMD wavelength).

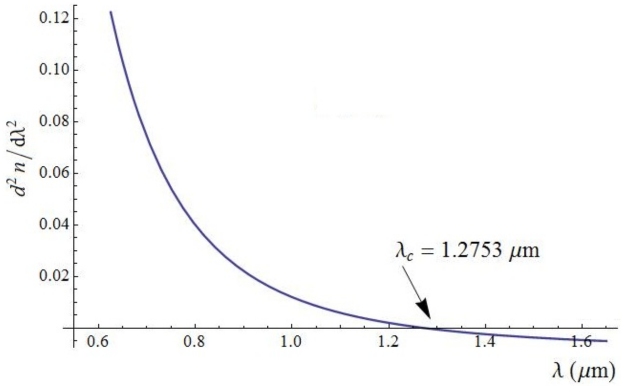

Fig. 1. Variation of $d^2n/d\lambda^2$ with λ of the fused silica.

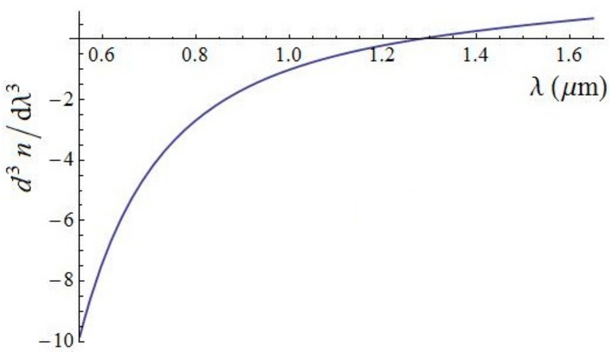

Fig. 2. Variation of $d^3n/d\lambda^3$ with λ of the fused silica.

Third derivative of refractive index for fused silica is shown in Figure 2. As mentioned already for operation around ZMD wavelength, the material dispersion would be determined by the next power term, namely by $d^3n/d\lambda^3$. It should be noted that the material dispersion still occurs at around $\lambda_c = 1.2753\,\mu\text{m}$ despite $d^2n/d\lambda^2 = 0$. Figure 3 shows the material dispersion for fused silica determined by $d^3n/d\lambda^3$ around ZMD wavelength. To gain some numerical appreciation, we see from Eq. (13) that for $\lambda_c = 1.2753\,\mu\text{m}$, the material dispersion for fused silica is $\Delta\tau \cong 31\,\text{ps/km}$.

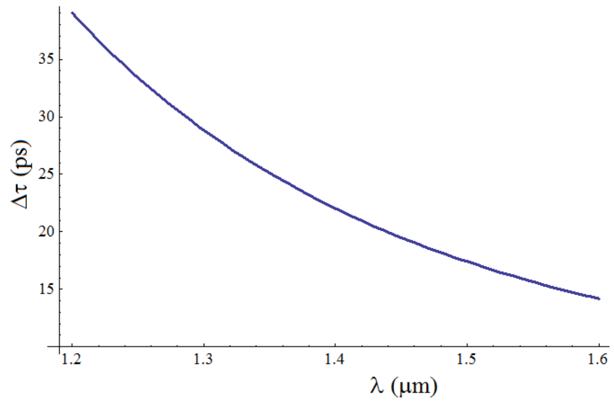

Fig. 3. Material dispersion of the fused silica-based optical fibres around ZMD wavelength.

We continued our analysis of the material dispersion with pure and doped silica-based fibres whose refractive indices can be represented by the following three-term Sellmeier's function of wavelength:

$$n^2 - 1 = \sum_{i=1}^{3} \frac{A_i\lambda^2}{\lambda^2 - B_i}, \quad (16)$$

where n is the refractive index of the medium, A_i is the oscillator strength, B_i is the oscillator wavelength, and λ is the wavelength of light. Values of coefficients in Sellmeir's formula for pure and doped silica are shown in Table 1 [11]. The corresponding variation of refractive index with wavelength is shown in Fig. 4. It is apparent that the refractive index changes with the level of change depending on the dopant and the % of mole.

Table 1: The Sellmeir's coefficients for pure and doped silica [10].

Sample	Dopant (mole %)	A_1	A_2	A_3	B_1	B_2	B_3
1	Pure SiO_2	0.00469148	0.01351206	97.93400	0.6961663	0.4079426	0.8974794
2	GeO_2 (6.3)	0.007290464	0.01050294	97.93428	0.7083952	0.4203993	0.8663412
3	GeO_2 (19.3)	0.005847345	0.01552717	97.93484	0.7347008	0.4461191	0.8081698
4	B_2O_3 (5.2)	0.004981838	0.01375664	97.93353	0.6910021	0.4022430	0.9439644
5	P_2O_5 (10.5)	0.005202431	0.01287730	97.93401	0.7058489	0.4176021	0.8952753

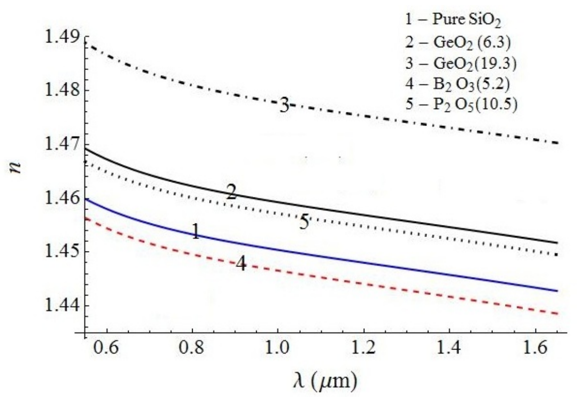

Fig. 4. Refractive index versus λ of the pure and doped silica

In Fig. 5, the variations of $d^2n/d\lambda^2$ are shown for pure and doped silica (the labels 1-5 correspond to various samples given in Table 1).

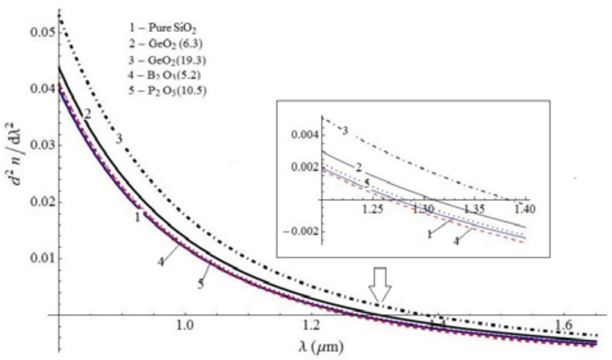

Fig. 5. Variations of $d^2n/d\lambda^2$ with λ of the pure and doped silica.

As an illustration, it follows from Eq. (16) that $\lambda_{c1} = 1.2728\mu m$, $\lambda_{c2} = 1.30947\,\mu m$, $\lambda_{c3} = 1.38325\,\mu m$, $\lambda_{c4} = 1.26453\,\mu m$ and $\lambda_{c5} = 1.28266\,\mu m$. It is apparent that the doping changes the ZMD wavelengths slightly.

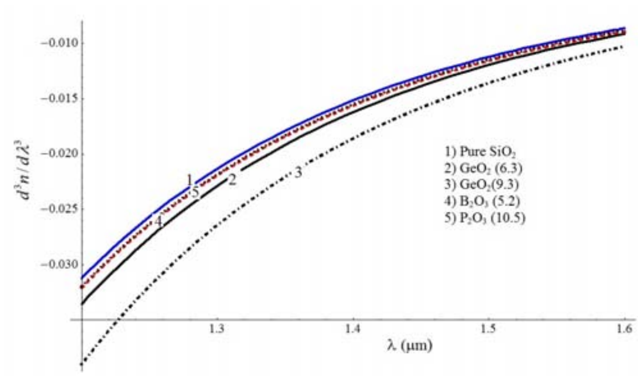

Fig. 6. Variation of $d^3n/d\lambda^3$ with λ of pure and doped silica.

In Fig. 6, the variation of $d^3n/d\lambda^3$ are shown for pure and doped silica. It is apparent that around ZMD wavelengths the third derivative of the refraction index is not zero and that it contributes to the existence of material dispersion in optical fibres (based on both, pure and doped silica). In Figure 7, the calculated material dispersion is shown for pure and doped silica optical fibres. We conclude that despite $d^2n/d\lambda^2 = 0$ around ZMD wavelengths, material dispersion occurs nevertheless; it is approximately 20-35 ps/km.

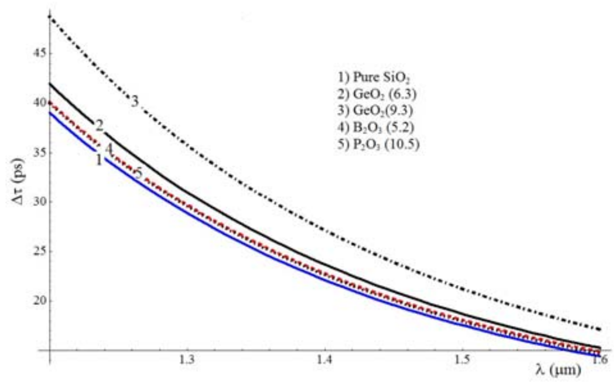

Fig. 7. Material dispersion of the pure and doped silica-based optical fibres around ZMD wavelength

3.2 Fluoride based fibres

In the last few years, fluoride-based fibres have been investigated in detail for operation in mid-infrared (2-5 µm) wavelength region because of their predicted ultra low loss of 10^{-3} dB/km. Such fibres may find applications in long-wave repeaterless telecommunication links and intercontinental submarine links. This has motivated us to continue with analysis of fluoride fibres in order to calculate a material dispersion around ZMD wavelength. Various fluoride glasses for applications in infrared fibre optic communication are shown in Table 2.

Table 2: Various fluoride glasses (adopted from [11]).

		Concentration (mole %)								
Sample	Material	ZrF_4	BaF_2	LaF_3	NaF	HfF_4	AlF_3	CaF_2	CdF_3	YF_3
1	ABCY		22				40	22		16
2	HBL		33	5		62				
3	ZBG	63	33						4	
4	ZBLAN	53	20	4	20		30			

For the fluoride glass, the refractive index versus wavelength can be represented by the following Sellmeier's expression [12]

$$n(\lambda) = A\lambda^{-4} + B\lambda^{-2} + C + D\lambda^{2} + E\lambda^{4} \tag{17}$$

where A to E are Sellmeier's coefficients listed in Table 3.

Table 3. Sellmeier's coefficients for the fluoride glasses [11].

Sample	Material	$A \cdot 10^{-6}$	$B \cdot 10^{-3}$	C	$D \cdot 10^{-3}$	$E \cdot 10^{-6}$
1	ABCY	7.67742	2.16195	1.42969	-1.28304	-5.35487
2	HBL	-28.61020	3.11470	1.50294	-1.17821	-2.64123
3	ZBG	93.67070	2.94329	1.51236	-1.25045	-4.01026
4	ZBLA	-300.80370	4.03214	1.51272	-1.21921	-6.77630
5	ZBLAN	93.67070	2.94329	1.49136	-1.25045	-4.01026

Average refractive indices of the fluoride glasses are summarized in Figure 8 for different wavelengths (the labels 1 to 5 denote the fluoride materials).

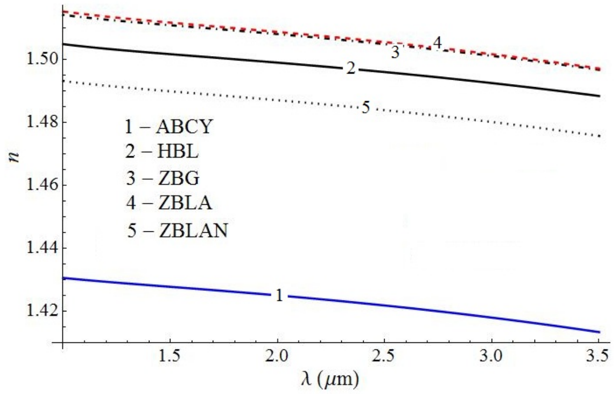

Fig. 8. Average refractive indices versus λ of the fluoride glasses.

The corresponding variations of the $d^2n/d\lambda^2$ and $d^3n/d\lambda^3$ with wavelength are shown in Figures 9 and 10. The following values of the ZMD wavelengths were obtained: $\lambda_{c1} = 1.48152\,\mu m$, $\lambda_{c2} = 1.65832\,\mu m$, $\lambda_{c3} = 1.62595\,\mu m$, $\lambda_{c4} = 1.69593\,\mu m$ and $\lambda_{c5} = 1.62595\,\mu m$. It is apparent that around ZMD wavelengths the third derivatives of index refraction are different from zero (Figure 10), which should be taken into account when calculating the material dispersion.

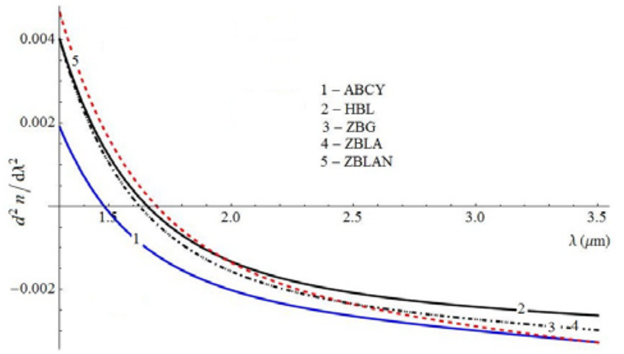

Fig. 9. Variation of $d^2n/d\lambda^2$ with λ of the fluoride glasses.

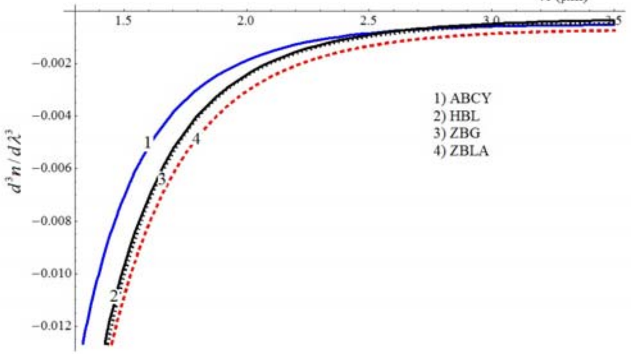

Fig. 10. Variation of $d^3n/d\lambda^3$ with λ of the fluoride glasses.

In Fig. 11, the material delay distortion of pulse in the fluoride-based optical fibres was obtained from Eq. (13). The material dispersion is approximately 10-25 ps/km around the ZMD wavelengths.

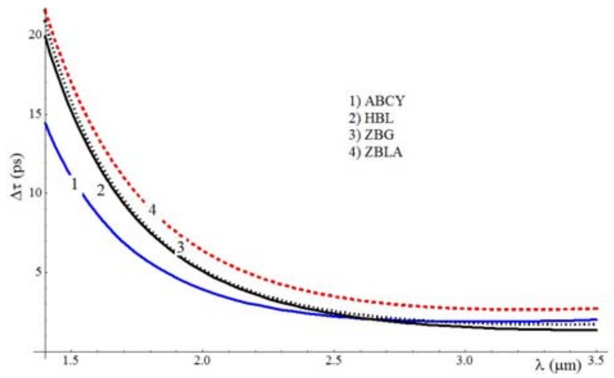

Fig. 11. Material dispersion of the fluoride glasses-based optical fibres around ZMD wavelength

4. Conclusion

Details of material dispersion are investigated in the context of pulse distortion in multimode glass optical fibres. Significance of the $d^3n/d\lambda^3$ term in an earlier reported concept is estimated. This is done for the operating wavelength around the so-called zero material dispersion wavelength when $d^2n/d\lambda^2 = 0$. The strong material dispersion persisting under this condition (a pulse of light centred on this zero material dispersion wavelength also suffers significant dispersion) attests to strong influence of the next higher-order term, $d^3n/d\lambda^3$.

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