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Two photon multi mode laser model based on experimental observations

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We present experimental results and a model of Hamiltonian interactions which takes into account four photons process interaction of dressed field with ^{39}K atom. Using the method of elimination of virtual states, we have derived an effective interaction Hamiltonian which describes simultaneous generation of photon pairs. By taking into account good cavity limits in the process of two photon generation, a master equation for laser field is obtained. A steady state solution of the resulting equation for a threshold case, which takes into account the quantum fluctuations and photon statistics, is proposed.

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1. Introduction

Quantum Information Science (QIS) is an emerging field with the potential for revolutionary advances in fields of science and engineering involving computation, communication, precision measurement, and fundamental quantum science. Developments in the physical sciences produced trapped atomic ions, advanced optical cavities, quantum dots, and many other advances that made it possible to contemplate the construction of workable quantum logic devices and non-linear optical (NLO) applications. Quantum mechanics offers the potential for ultra-secure communications rendering eavesdropping, copying, and spying impossible. The property of entanglement between the emitted photons in the processes of light generation has a great impact towards applications dealing with quantum information, computing, and information security. The problem of quantum fluctuations and the generation of non-classical electromagnetic fields in multi-photon processes have been the subject of extensive theoretical and experimental studies in recent years, more specifically two-photon coherent generation of light has led to many experimental and theoretical studies recently.

An experimental realization of two-photon laser was firstly proposed by Gauthier et al. [1]. Two- photon laser consists of spin-polarized and laser-driven ^{39}K atoms placed in a high-finesse transverse-mode-degenerate optical resonator which produces a beam with a power of ~ 0.2 Watt at a wavelength of 770 nm. We observed complex dynamic instabilities of the state of polarization of the two-photon laser, which are made possible by the atomic Zeeman degeneracy. Following this experimental realization, we propose a model which takes into account one- and two-photon losses resulting from the system. The Fokker-Plank equation which describes the behavior of cavity field below the threshold of lasing process is obtained and solved in the stationary case.

2. Experimental observations

The two-photon amplification of light is possible in multi-level atoms and opens new perspective of application the coherence and entanglement between the photon in the quantum communication. As an example, laser amplification in a thermal vapor of potassium atoms ^{39}K when laser frequency is tuned in the vicinity of the $4S_{1/2} \to 4P_{1/2}$ transition. We demonstrate that two-photon amplification arises in the system by the process of four-quantum Hyper-Raman scattering effect as shown in Fig. 1.

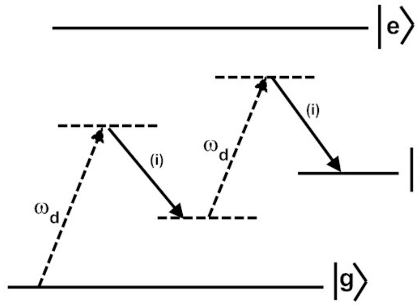

Fig. 1. Scattering diagram showing the Raman two-photon processes.

In this four photons process stimulated by dressed field with the frequency ω_d, the atom makes a transition from the ground state $|g\rangle$ to the new state quasi-stable state $|i\rangle$ by absorbing two photons from the dressing field with frequency ω_d, and generating two new photons to the probe lasing field with frequency ω_p, via virtual intermediate states. Taking into account the law of conservation of energy we find that the frequency of new generated light $\omega_p = \omega_d - \Delta_{gi}/2$, where Δ_{gi} is the energy difference between the states $|g\rangle$ and $|i\rangle$. In this experiment, the states $|g\rangle$ and $|i\rangle$ correspond to the $4S_{1/2}(F=1)$ and $4S_{1/2}(F=2)$ hyperfine states of ^{39}K, respectively and $\Delta_{gi}/2\pi = 462$ MHz. To obtain two-photon amplification based on this stimulated emission process, a steady-state imbalance must exist between the states $|g\rangle$ and $|i\rangle$ so that $N_g > N_i$, which is accomplished by optical pumping of the atom by the dressing field. Note that this process is similar to the multi-photon scattering. It is thus feasible to understand the origin of two-photon amplification process using the dressed-state bases for the three-level atom.

It was pointed out that for n - photon Raman scattering processes can occur in a system with probe beam frequencies:

$\omega = \omega_d - \Delta_{gi}/n$ for n= 1,2,3…[2]. Poelker at all [3] have extensively studied one-photon Raman process in a laser-driven sodium vapor, while Hemerich et al [4] and Cattaliotti et al. [5] observed multi-photon Raman scattering in cooled Rb atoms trapped in the potential wells of a 3D optical lattice.

Furthermore, Agarwal [6] investigated multi-photon parametric wave mixing process in laser driven sodium atoms. Their experiment uses emission and absorption features observed due to scattering from naturally abundant potassium ^{41}K, where: $\Delta_{gi} = 254$ MHz. The dressing laser beam was linearly polarized to a diameter of 150 μm as it passed through the cell, had a power of 850 mW at the entrance to the cell, and was tuned approximately 2.4 GHz to the low frequency side of the D_1 transition $4S_{1/2}(F=2)$ to $4P_{1/2}(F=1)$ occurring near λ=769.9 nm. The probe beam was collimated to a diameter of 65 μm and had a polarization orthogonal, which resulted in the maximum two-photon gain.

Pfister at al [7] demonstrated two photon amplification in laser-driven potassium atoms system using the orthogonal geometry, and is shown in Fig. 2. The interactions are somewhat more complex because the magnetic sublevels of the potassium hyperfine states have to be taken into account for this geometry (see Fig. 2) to the lowest order in the perturbation theory. Laser beam amplification occurred when two circularly polarized dressing field photons were annihilated and two linearly polarized probe photons were created as the atom underwent a transition from the $|g22\rangle$ to the $|g20\rangle$ Zeeman sublevels. The atomic states are denoted by $|\alpha F_\alpha M_\alpha\rangle$, where $\alpha = g$ for the potassium $4^2S_{1/2}$ and $a = g$ for the $4^2S_{1/2}$ levels. F and M are the quantum numbers for the total angular momentum and it's projection along the z.

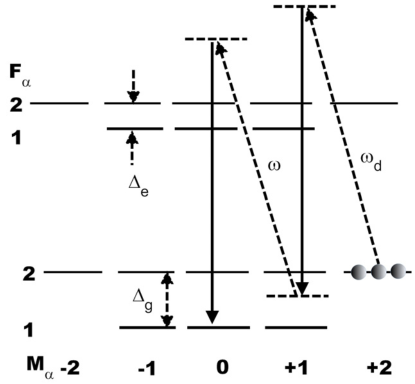

Fig. 2. Scattering diagram showing the Raman two-photon processes.

The necessary inversion between the states $|g22\rangle$ and $|g20\rangle$ was maintained using auxiliary optical pumping beams that continuously transferred population from all hyperfine states into $|g22\rangle$ (see Fig. 2). In their experiment, the atoms are produced by an atomic beam with a half-angle divergence of 30 mrad, giving rise to a residual Doppler width of 30 MHz with a diameter of 2.5 mm and an atomic number density of 2×10^{11} atoms/cm^3 in the interaction region. The atoms were dressed with a circularly polarized laser beam propagating along the quantization axis with an intensity of 25 W/cm^2 and its frequency tuned to the blue side of the $|g1M\rangle \leftrightarrow |e1M\rangle$ transition by 512 MHz. The generation with two-photon (continuous-wave) in laser-driven potassium atoms was first observed by [1], who combined the experimental apparatus from the measurements of amplification with a low-loss optical resonator. They used a linear cavity consisting of two high-reflectivity ultra-low loss mirror of radius of curvature of ~5 cm. Within the cavity mode volume, they estimated approximately ≈ 7×10^6 atoms. These experiments demonstrate that multi-level structure in a two-photon gain medium gives rise to a new behavior in the state of polarization of the generated beam. Further experiments are needed to fully explore the quantum statistical and nonlinear dynamics behavior of this new type of quantum oscillator.

In the next section we derive the model Hamiltonian which describes the two-photon lasing effect stimulated by dressed field. Using the method of adiabatic elimination is derived the master equation for lasing field in the similar way as this was made in the paper [8].

3. Model hamiltonian and master equations

Let us consider the Hamiltonian of the atomic system described above in interaction with dressed external field and new generated field

$$H = H_0 + H_I, \tag{1}$$

where H_0 and H_1 are the free part and interaction parts of the Hamiltonian, which can be represented in the following form

$$H_0 = \hbar\omega_e |e\rangle\langle e| + \hbar\omega_i |i\rangle\langle i| + \hbar\omega_g |g\rangle\langle g| + \hbar\omega_p b^+ b, \quad \text{(1a)}$$

and

$$\begin{aligned} H_{\text{int}} = &(d_{ei}, e_r)(E_d^+ e^{i(\omega_d t - k_d z)} |i\rangle\langle e| + h.c.) - (d_{eg}, e_r)(E_d^+ e^{i(\omega_d t - k_d z)} |g\rangle\langle e| + h.c.) \\ &- (d_{ei}, g)(b^+ e^{-ik_p x} |i\rangle\langle e| + h.c.) - (d_{eg}, g)(b^+ e^{-ik_p x} |g\rangle\langle e| + h.c.). \end{aligned} \quad \text{(1b)}$$

Here $\hbar\omega_e$, $\hbar\omega_i$ and $\hbar\omega_g$ are the energies of the excited, intermediary and ground levels respectively, E_d^+ and E_d are the amplitudes of positive and negative frequency parts of dressed field; b^+ and b are the creation and annihilation operators of generated field in the cavity, k_p and k_d are the wave vectors of generated and dressed fields direction of along the axis x and z respectively (see Fig. 3).

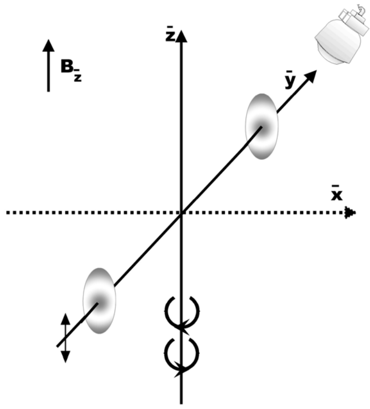

Fig. 3. The diagram of two photon laser.

In order to obtain the model Hamiltonian which describes the processes in which two-photon from dressed field are absorbed and two new photons are generated let us eliminate the operators of virtual level $|e\rangle$ from the Hamiltonian (1). Taking into account the solution of Heisenberg equation in the Born-Markov approximation [9] let us represent the state vector $|e(t)\rangle$ through the initial unpopulated and lowest $|g(t)\rangle$ and $|i(t)\rangle$ states respectively

$$|e(t)\rangle = |e(0)\rangle e^{i\omega_e t} - A_{e,i}^+(t,x,z)|i(t)\rangle - A_{e,g}^+(t,x,z)|g(t)\rangle,$$

where the coefficients $A_{e,i}(t,x,z)$ and $A_{e,g}(t,x,z)$ are determined by the expression

$$A_{e,\alpha}^+(t,x,z) = \frac{(d_{e\alpha}, e_r)E_d^+ e^{-izk_d + i\omega_d t}}{\hbar(\omega_{e\alpha} - \omega_d)} + \frac{(d_{e\alpha}, g)b^+(t)e^{-ixk_p}}{\hbar(\omega_{e\alpha} - \omega_p)}, \quad \alpha = g, i.$$

Introducing this vector state in the interaction part of Hamiltonian (1), we obtained the following expression

$$\begin{aligned} H_i^{eff} = &[(d_{ei}, e_r)\tilde{E}_d^+ + (d_{ei}, g)b^+ e^{-ik_p x}]A_{e,g}^-(t,x,z)|i\rangle\langle g| \\ &+ [(d_{eg}, e_r)\tilde{E}_d^- + (d_{eg}, g)b e^{ik_p x}]A_{e,i}^-(t,x,z)|i\rangle\langle g| \\ &+ [(d_{ei}, e_r)\tilde{E}_d^+ + (d_{ei}, g)b^+ e^{-ik_p x}]A_{e,i}^-(t,x,z)|i\rangle\langle i| \\ &+ [(d_{eg}, e_r)\tilde{E}_d^- + (d_{eg}, g)b e^{ik_p x}]A_{e,g}^-(t,x,z)|g\rangle\langle g|, \end{aligned} \quad (2)$$

represents the number of possibilities in which the bi-photons can be generated for the same mode K after L characteristics the all polarization for this mode $\tilde{E}_d^+ = E_d^+ \exp(-ik_d z + i\omega_d t)$.

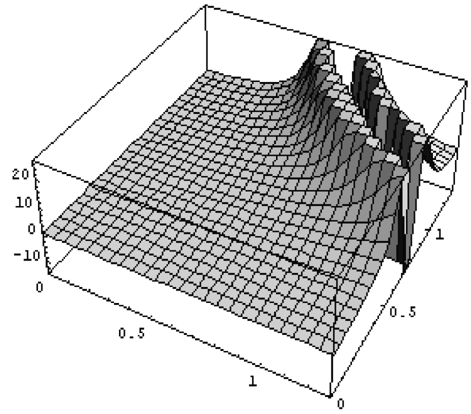

Fig. 4. The dependences of P-function on complex variables α and β for parameter $u_1/k=0.9$.

Let us eliminate the rapidly oscillating terms from this equation. This procedure of elimination can be realized using eliminating procedure of oscillation parts from the Hamiltonian (2). Indeed using the initial (primitive) Hamiltonian (1), for the projectors $|i(t)\rangle\langle g(t)|$, $|i\rangle\langle i|$ and $|g\rangle\langle g|$ we obtain the following solutions in Born-Markov approximation.

$$\begin{aligned} |i(t)><g(t)| = &|i(0)><g(0)|e^{i\omega_{ig}t} + [A_{e,g}^-(t,x,z)][A_{e,g}^+(t,x,z)]|i><g| \\ &+ [A_{e,i}^-(t,x,z)]\left[A_{e,i}^+(t,x,z)\right]|i><g| \\ &+ \left[A_{e,i}^-(t,x,z)\right]\left[A_{e,g}^+(t,x,z)\right]|i><i| \\ &+ \left[A_{e,i}^-(t,x,z)\right]\left[A_{e,g}^+(t,x,z)\right]|g><g|, \end{aligned} \quad (3)$$

$$\begin{aligned} |i(t)\rangle\langle i(t)| = &-[A_{e,i}^-(t,x,z)]|i(t)\rangle\langle e(t)| - [A_{e,i}^+(t,x,z)]*|e(t)\rangle\langle i(t)| \\ &= [A_{e,i}^-(t,x,z)]*[A_{e,i}^+(t,x,z)]*|i(t)\rangle\langle i(t)| + [A_{e,i}^-(t,x,z)] \\ &*[A_{e,g}^+(t,x,z)]*|i(t)\rangle\langle g(t)| + h.c., \end{aligned} \quad (4)$$

$$\begin{aligned} |g(t)\rangle\langle g(t)| = &[A_{e,g}^-(t,x,z)]*[A_{e,i}^+(t,x,z)]|i(t)\rangle\langle g(t)| \\ &+ [A_{e,g}^-(t,x,z)]*[A_{e,g}^+(t,x,z)]|g(t)\rangle\langle g(t)| + h.c. \end{aligned} \quad (5)$$

After substitution for projectors $|i\rangle\langle g|$, $|i\rangle\langle i|$ and $|g\rangle\langle g|$ in expression in (2) we obtain following expression for the effective Hamiltonian:

$$H_I^{eff} = \sum_e \{G(z)b(t)b(t)e^{2ik_p x}|i(t)\rangle\langle g(t)| + G^+(z)b^+(t)b^+(t)e^{-2ik_p x}|g(t)\rangle\langle i(t)|\}, \quad (6)$$

where G(z) is presented in the following form:,

$$G(z) = \left(\frac{1}{\omega_{eg}-\omega_P} + \frac{1}{\omega_{ei}-\omega_D}\right) * \frac{(\overleftrightarrow{d}_{ei},\dot{e})(\overleftrightarrow{d}_{eg},\overleftrightarrow{g})^2(\overleftrightarrow{d}_{eg},\dot{e})}{\hbar^3(\omega_{eg}-\omega_D)(\omega_{eg}-\omega_P)} + \frac{(\overleftrightarrow{d}_{ei},\overleftrightarrow{g})^2(\overleftrightarrow{d}_{ei},\dot{e})(\overleftrightarrow{d}_{eg},\dot{e})}{\hbar^3(\omega_{ei}-\omega_P)(\omega_{eg}-\omega_D)} \times \left(\frac{1}{\omega_{ei}-\omega_P} + \frac{1}{\omega_{ei}-\omega_D}\right)\overleftrightarrow{E_D}^+(t,z)\overleftrightarrow{E_D}^+(t,z) \quad (7)$$

In the next section we will use the Hamiltonian (6) for study of the two photon emission in micro cavity. The Fokker-Plank equation which describes the emission processes below threshold is obtained.

4. Master and fokker - plank equations for two photon lasing in micro cavities

Using the method of elimination of atomic variable proposed by Enaki et al [8] we obtained the following master equation for density matrix of bi-photon in the cavity,

$$\frac{\partial W(t)}{\partial t} = k[I^- W(t), I^+] + i\chi[I^+(1+pI^-I^+)^{-1}W(t), I^-] + u_1[I^+(1+pI^-I^+)^{-1}W(t), I^-] + u_2[I^+[I^-, I^+(1+pI^-I^+)^{-1}W(t)(1+\rho I^-I^+)^{-1}], I^-] + H.c. \quad (8)$$

Here $u_1 = 2|G|^2 N\sigma_0\gamma/[(\omega-2\Omega)^2+\gamma^2]$ represents the generation rate of photon pairs for full atomic inversion $N\sigma_0$, $\chi = 2|G|^2 N\sigma_0(\omega-2\Omega)/[(\omega-2\Omega)^2+\gamma^2]$ and $u_2 = \frac{T}{N\sigma_0}(u_1^2+\chi^2)$. k describes the losses of bi-photons (pairs of entangled photons) from the micro-resonator and $pu_1 = 2u_2$. We introduced here the bi-photon operators belonging to su(1,1) commutation algebra

$$[I^+, I^-] = -I_z, \quad [I_z, I^\pm] = \pm I^\pm,$$

where (a) for degenerate case the new operators are $I^+ = b^{+2}/2$; $I^- = b^2/2$; $I_z = (b^+b + bb^+)/4$ and for non-degenerate two photon process in which takes place two different photons we have $I^+ = b^+a^+$; $I^- = ba$; $I_z = (b^+b + aa^+)/2$. It is not difficult to observe from master equation (8), that in the process bi-photon exchanges the Kasimir vector is conserved

$$I^2 = I_z^2 - \frac{1}{2}\left(I^+I^- + I^-I^+\right)$$

here $I^2 = j(j-1)$, where j=1/4 and j=1/2 for degenerate and non-degenerate cases respectively. Number j represents the possibilities in which the bi-photons can be generated in the modes of cavity and depend on the mode structure of resonator.

Let us first discuss the emission below threshold $k > u_1$. In this situation one can neglect the terms proportional to the square value of amplification coefficient u_1 ($u_1 > u_2$).

The equation (8) can be represented in the following form,

$$\frac{\partial W(t)}{\partial t} = k[I^- W(t), I^+] + i\chi[I^+ W(t), I^-] + u_1[I^+ W(t), I^-] + H.c. \quad (9)$$

We introduce the complex P representation for su(1,1) algebra in the similar way as this representation was introduced by [10] for Bose quantum oscillators. Similar representation was introduced by [11] in parametrically amplification of photon pairs. Using this method, normalization of complex P representation can be found for many problems with similarities in diagonal P representation. In this case, many properties of P function and Q function remain specific and for su(1,1) algebra. Let us introduce the generalized P representations for bi-boson field in the cavity,

$$W = \int_D \Lambda(\alpha,\beta)P(\alpha,\beta)d\mu(\alpha,\beta); \quad \Lambda(\alpha,\beta) = \frac{|\alpha\rangle\langle\beta^*|}{\langle\beta^*|\alpha\rangle}, \quad (10)$$

where $d\mu(\alpha,\beta)$ is the integration measure which may be chosen to define different classes of possible representations and D is the domain of integration. The projection operator $\Lambda(\alpha,\beta)$, is analytic in (α,β). In this section we are interested in complex P representation in which $d\mu(\alpha,\beta) = d\alpha d\beta$. Here (α,β) are regarded as complex variables which are to be integrated an individual contours C and C′ . The coherent states for su(1,1) can be defined in the following form

$$|\alpha\rangle = (1-|\alpha|^2)^j \exp(\alpha I^+)|0\rangle, \quad \langle\beta^*| = (1-|\beta|^2)^j \langle 0|\exp(\beta I^-),$$

and linear product of such two states is

$$\left\langle \beta^* \middle| \alpha \right\rangle = \frac{(1-|\alpha|^2)^j (1-|\beta|^2)^j}{(1-\alpha\beta)^{2j}}$$

The inverse value of this product is considered as a normalization coefficient for the projector operator $|\alpha\rangle\langle\beta^*|$. Using the following action of operators I^+, I^-, I^z, of su(1,1) algebra on the coherent state

$$\begin{aligned} I^+|\alpha\rangle &= (1-|\alpha|^2)^j \frac{\partial}{\partial\alpha}\exp(\alpha I^+)|0\rangle, \\ I^z|\alpha\rangle &= (1-|\alpha|^2)^j (\alpha\frac{\partial}{\partial\alpha}+j)\exp(\alpha I^+)|0\rangle, \\ I^-|\alpha\rangle &= (1-|\alpha|^2)^j (\alpha^2\frac{\partial}{\partial\alpha}+2j\alpha)\exp(\alpha I^+)|0\rangle, \end{aligned} \tag{11}$$

we obtain the following expression for equation (9).

$$\begin{aligned} &\int_{C'} d\alpha \int_{C} d\beta \frac{\partial}{\partial t} P(\alpha,\beta,t)\exp(\alpha I^+)(1-\alpha\beta)^{2j}\,|0\rangle\langle 0|\exp(\beta I^-) \\ &= \int_{C} d\alpha \int_{C} d\beta P(\alpha,\beta,t)(1-\alpha\beta)^{2j} k\left(\alpha^2\frac{\partial}{\partial\alpha}+2j\alpha\right)\left[\left(\beta^2\frac{\partial}{\partial\beta}+2j\beta\right)-\frac{\partial}{\partial\alpha}\right] \\ &+(i\chi+u_1)\frac{\partial}{\partial\alpha}\left[\frac{\partial}{\partial\beta}-\left(\alpha^2\frac{\partial}{\partial\alpha}+2j\alpha\right)\right]\exp(\alpha I^+)|0><0|\exp(\beta I^-)+H.c. \end{aligned} \tag{12}$$

Following the method of partial integration of right hand site of this equation taking into account that notations, one can obtain the following Fokker-Plank equation

$$\begin{aligned} \frac{\partial}{\partial t}P(\alpha,\beta) &= 2[jk-(j-1)u_1][\frac{\partial}{\partial\alpha}\alpha P(\alpha,\beta)+\frac{\partial}{\partial\beta}\beta P(\alpha,\beta)] \\ &+2\frac{\partial^2}{\partial\alpha\partial\beta}[k\alpha^2\beta^2+u_1]P(\alpha,\beta)-(k+u_1) \\ &\times[\frac{\partial^2}{\partial\beta^2}\beta^2 P(\alpha,\beta)+\frac{\partial^2}{\partial\alpha^2}\alpha^2 P(\alpha,\beta)], \end{aligned} \tag{13}$$

where P is complex representation for su(1,1) algebra, $u = 2|G|^2 N\sigma_0\gamma/[(\omega-2\Omega)^2+\gamma^2]$ represent the generation rate of photon pairs for full atomic inversion $N\sigma_0$, k is constant describing the losses of bi-photons (pairs of entangled photons) from the micro-resonator.

The Fokker-Plank equation is obtained in the first order approximation on the interaction constant $|G|^2$. The solution of this equation can be easily found.

$$P(\alpha,\beta) = D\frac{|\alpha\beta-u_1/k|^{2(j-1)}}{|1-\alpha\beta|^{2j}}, \tag{14}$$

here D is the integration constant. In figure (4) we present the dependence of P as function of the complex variables α and β for j=1/2. Number j represents the possibilities in which the bi-photons can be generated in the modes of cavity and depend on the mode structure of resonator.

This solution describes the amplification of two photon lasing effect in the cavity stimulated by pump field. The density matrix one can be determined by the following expression,

$$\begin{aligned} W &= D\int_{C'} d\alpha \int_{C} d\beta \frac{|\alpha\beta-u_1/k|^{2(j-1)}}{|1-\alpha\beta|^{2j}}\Lambda(\alpha,\beta) \\ &= D\int_{C'} d\alpha \int_{C} d\beta |\alpha\beta-u_1/k|^{2(j-1)}\exp(\alpha I^+)|0\rangle\langle 0|\exp(\beta I^-). \end{aligned} \tag{15}$$

In the good cavity limits when the losses from cavity k are less then the generation processes of bi-photons, the numbers of bi-photons in the cavity drastically increase. In this case the last terms in equation (8) increases more rapidly then terms proportionally with u_1. The numerical and analytical behavior of the field in the cavity will be studied as part of future and continuing investigations.

5. Conclusion

Using the methods of elimination operator for virtual states, we obtained the interaction Hamiltonian of atoms with pump laser field and generation in the cavity two photon field. This generation and annihilation processes is described by bi-photon operators I^+ and I^- which correspond to su(1,1) symmetry. Using the P representation for su(1,1) bi-boson field, the Fokker-Plank equation, which describes the behavior of cavity field below the threshold for two-photon laser field, is obtained. The solution for this equation provides the rate of generation photons in the cavity. The numerical stimulation of $P(\alpha,\beta)$ strongly depends on the ratio between the coupling constant u_1 with external field and cavity field and losses from cavity, k.

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