# Velocity of gap soliton in nonlinear one dimensional photonic crystal

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Theoretical studies on the velocity of gap soliton in one-dimensional GaAs/AlGaAs nonlinear photonic crystal has been presented. Velocity of gap soliton through these multilayered structures has been calculated at different intensity of controlling wave by solving Maxwell's electromagnetic equations and using transfer matrix method. The study shows that gap solitons travel at high intensity of controlling wave and this property can be exploited to trap an optical pulse and in the design of an optical switch, active WDM coupler etc.

(Received September 3, 2011; accepted September 20, 2012)

Keywords: Nonlinear Photonic Crystal, Soliton, Multilayered

# 1. Introduction

Pulse spreading due to dispersion or diffraction in materials in the design of devices used in optical communication systems is a serious problem, particularly in modern day multi-channel, high band-width optical digital communication systems. However, it can be suppressed to the desired extent using nonlinear properties of some materials, and thus we can realize the propagation of solitary waves (or solitons) in such structured materials. Almost two and a half decade ago, it was predicted that the structures with periodically modulated parameters can support a new type of solitons known as gap solitons [1]. These solitons exist in band gaps of linear spectra in different structures like photonic crystals [2], Bragg gratings [3], and Bose-Einstein condensates loaded onto optical lattices [4]. Gap solitons are composed of left and right traveling waves, which are coupled non-linearly, and both experience Bragg scattering from the periodic structure. When the pulse amplitudes are balanced, the strongest coupling occurs, and formation of slow or immobile gap solitons takes place. After formation of solitons, stability of these solitons attracted much attention of researcher [5].

During the last decade a lot of interest has been given to the slowing down of the light [6-7]. Marangos observed slow light in atoms at nanokelvin temperatures [6] and Yong et al discussed the method of slow light generation in the optical fibers [7]. The fiber grating is a good candidate for a nonlinear medium where it may be possible to stop the light, as exact solutions of zerovelocity Bragg grating (BG) solitons, in which the forward and backward propagating waves are in permanent dynamical equilibrium [8,9], and some of them are stable [10,11]. In experimental observation of nonlinear propagation effects in fiber Bragg gratings, it was shown that the speed of these BG solitons were  $\approx 0.75$  times of the speed of light inside the fiber [12-13]. Local defect in Bragg gratings can be used to capture a zero velocity solitons [14-15].

Various aspects of gap solitons have been studied in last decades [16-20]. Very recently, existence of gap soliton in nonlinear one dimensional photonic crystal has been presented [21].

The main aim of the present communication is to calculate the band structure and velocity of gap soliton in GaAS/AlGaAs based one-dimensional nonlinear photonic crystal. Existence of this type of gap soliton has been already predicted by Arun et al [21]. Here, we have considered that the electromagnetic wave that carries the signal pulse incident making a small angle, say  $5^{0}$ , with respect to the layers. The controlling wave, which produces the nonlinear effect, is propagating perpendicular to the normal to the surface of the multi-layers. Also, we have considered the amplitude of the controlling wave much higher than the amplitude of the signal pulse thereby we can safely neglect the nonlinear effect due to the signal pulse on nonlinear layers.

### 2. Theoretical Analysis

To study the propagation of electromagnetic waves through such a periodic structure, we have assumed the materials of the multilayered structure to be nonlinear and select a particular axis as the z-axis which is along the direction normal to the layers. The refractive index profile of the structure has a form given by

$$n(z) = \begin{cases} n_1 = n_{01} + \Delta n_1 I, & 0 < z < d_1 \\ n_2 = n_{02} + \Delta n_2 I & d_1 < z < d_2 \end{cases}$$
(1)

with n(z+d) = n(z). Here, *d* is the lattice constant;  $d_1$  and  $d_2$  are the thicknesses of the alternate layers which have refractive indices  $n_{01}+\Delta n_1 I$  and  $n_{02}+\Delta n_2 I$ , where *I* is the intensity of the cross controlling wave. The schematic diagram of this structure is illustrated in Fig. 1.



Fig. 1. Schematic diagram of the structure.

Now, the wave equation for light propagation along the z-axis may be written as

$$\frac{d^{2}E}{dz^{2}} + \frac{n^{2}\omega^{2}}{c^{2}} = 0$$
 (2)

where n is given by equation (1). The solutions of equation (2) in any region, are given by the superposition of the left and right travelling waves.

The general transfer matrix method cannot deal with the non-linear propagation problem in the presence of other high intensity controlling wave. Hence, an approximate approach has been adopted to deal with nonlinearity. When the intensity of the controlling wave is high, the refractive index of nonlinear material could be calculated with the consideration of optical Kerr effect. Therefore, using the calculated refractive index of GaAs and AlGaAs, the transmittance of the proposed photonic crystal at different intensities of the controlling wave could be calculated. Johnson et al [22] and Xiaoyong et al [23] have confirmed the convergence and the correctness of this approximate approach. It has been pointed out that this approach could lead to the right results.

# 3. Result and discussion

In this section, numerical calculation on the group velocity of soliton has been presented. The band structure at three different intensities of the controlling wave has also been presented. We have considered a nonlinear medium i.e. GaAs, whose refractive index can be expressed as  $n_1 = 3.31 + \Delta n_{0l}I$ , where  $\Delta n_{0l}$  is the Kerr coefficient of GaAs, and  $\Delta n_{01} = 1.59 \times 10^{-13} \text{ cm}^2/\text{W}$  [24], and another nonlinear medium AlGaAs, having refractive index  $n_2 = 3.3 + \Delta n_{02}I$ , where  $\Delta n_{02}$  Kerr coefficient of AlGaAs, and  $\Delta n_{02} = 2 \times 10^{-13} \text{ cm}^2/\text{W}$  [25]. Here, *I* is the intensity of the controlling wave. In the proposed structure, we have taken the length of structure (L) to be equal to 0.7mm, having a lattice constant, d=235nm out of which the thickness of GaAs layer  $(d_1)$  is 0.2d and thickness of AlGaAs layer  $(d_2)$  is 0.8d. We have calculated the band structure and group velocity at three different intensities, namely 1 GW/cm<sup>2</sup> (low), 10 GW/cm<sup>2</sup> (moderate) and 100 GW/cm<sup>2</sup> (high) of the controlling wave.

Fig. (2.a), (2.b) and (2.c) show the photonic band structure for an oblique angle of incidence  $\theta_i$ , say  $5^0$ , for three different intensities of the controlling wave. From figures (2.a), (2.b) and (2.c), it is clear that the band gap of our interest shifts towards the shorter frequency region and becomes narrower as we increase the intensity of the controlling wave. This phenomenon confirms the results of our previous work [21] that reflection band shifts towards longer wavelength as we increase the intensity of the controlling wave for positive Kerr photonic crystal. It is noticeable here that the lattice constant d is arbitrary, thus the result obtained here is valid for arbitrary wavelengths and the existence of band-gap is possible as long as  $d\approx\lambda$ .

Fig. (3.a), (3.b) and (3.c) show the plot of the group velocity versus normalized frequency and a Gaussian signal pulse centered at 0.955 in the normalized frequency, group velocity decreases from a positive value to zero, and then zero to the negative minimum for a particular intensity of the controlling wave. It is clear from these figures that there are left-right and up-down symmetries in the plot of the group velocity and it support two different states. These states of symmetry may be used to make devices involving flip-flops, logic gates, optical switches (for optical computing), etc.

It is very interesting to note the dependence of the velocity of a pulse on the intensity of the controlling wave. If the intensity of the controlling wave is low  $(1 \text{ GW/cm}^2)$  and a Gaussian signal pulse centered at a normalized frequency of 0.955 is incident on the structure, then it is clear from figure (2.a) that it will fall inside the photonic band gap. And from figure (3.a), the velocity of this pulse is found to be zero; so theoretically, it would not be able to propagate through the crystal. If intensity of the controlling wave is moderate (10GW/cm<sup>2</sup>), it is clear from figure (2.b) that some spectral part of the pulse fall inside the photonic band gap and some spectral part of the pulse fall outside the photonic band gap. And it is clear from

figure (3.b) that some spectral part of the pulse has zero velocity and some spectral part has the velocity ranging from zero to 0.3c. Thus a distorted pulse will be propagated through the structure. But on the other hand, if the intensity of the controlling wave is high (100  $GW/cm^2$ ), the situation will be quite different. Now, it is clear from figure (2.c) that the central frequency of the pulse will fall outside the band-gap; and from figure (3.c), the group velocity of the pulse comes out to be equal 0.3c. Thus, distortion-less propagation of the pulse is possible at high intensity of the controlling wave because of the positive Kerr effect. However at low intensity of the controlling wave, distortion-less propagation of an electromagnetic pulse is not possible. This also confirms the results of our previous work [21] that soliton exists at the high intensity of the controlling wave. Now, it is found that the velocity of this soliton is 0.3c. Thus, we can conclude that gap solitons can exist and it can propagate

through the crystal at high intensity of the controlling wave. Because the velocity of the pulse at low intensity of the contolling wave was zero inside the crystal and propagation of the pulse is prohibited. A very important application of the dependence of velocity on the intensity of the controlling wave may be the trapping of optical pulse. As shown in figure (3.c), if a Gaussian pulse centered at normalized frequency of 0.955 is incident on the crystal when the intensity of the controlling wave is 100GW/cm<sup>2</sup> (high), then the Gaussian pulse will propagate through the structure with a velocity of 0.3c. But if we suddenly change the intensity of the controlling wave from 100  $\text{GW/cm}^2$  to 1  $\text{GW/cm}^2$ , when the incident pulse is in middle of the crystal, then the pulse which was travelling with a velocity of 0.3c at 100GW/cm<sup>2</sup> intensity of the controlling wave, will get trapped. Thus, we can trap an optical pulse inside the structure by changing the intensity of the controlling wave.



Fig. 2. Dispersion vs normalized frequency for GaAS/AlGaAs at an incident angle  $\theta_i = 5^0$ , for three different intensities of the controlling wave.(a)  $1GW/cm^2$ , (b)  $10GW/cm^2$ , and (c)  $100GW/cm^2$ .



Fig. 3. Normalized Group velocity (v/c) vs normalized frequency ( $\omega d/2\pi c$ )(solid) for GaAS/AlGaAs at an incident angle  $\theta_i = 5^0$ , for three different intensities of the controlling wave. (a) 1GW/cm<sup>2</sup>, (b) 10GW/cm<sup>2</sup>, and (c) 100GW/cm<sup>2</sup>. The dashed curve indicates the shape of the Gaussian pulse.

# 4. Conclusion

This study shows that the intensity of the controlling wave controls the propagation characteristics of the crystal. A photonic band-gap of our interest may be shifted towards the short normalized frequency region in case of the positive Kerr crystals when the intensity of controlling wave increases. Thus, we can tune the band gap by changing the intensity of the controlling wave. So the proposed crystal may behave as an active WDM coupler. The group velocity becomes negative, positive, or zero for certain ranges of the frequency. Such structures may be considered as flip-flop as there is positive and negative symmetry of group velocity. Most interesting thing, for the crystal considered here, is the dependence of the group velocity on the intensity of the controlling wave. From the theoretical point of view, perhaps we can make the group velocity of a pulse equal to zero, negative, or positive by changing the intensity of the controlling wave. It is found

that the velocity of a Gaussian signal pulse is zero at low intensity of controlling wave for positive Kerr crystal. Thus, the propagation of a Gaussian pulse is prohibited at low intensity of controlling wave through the structure. At moderate intensity of the controlling wave, some spectral part of the pulse has zero velocity and some spectral part has the velocity ranging from zero to 0.3c. Thus a distorted pulse will be propagated through the structure. But situation is quite different at high intensity of the controlling wave. At high intensity of the controlling wave, the velocity of the Gaussian signal pulse becomes 0.3c. Thus, distortion less propagation of the Gaussian signal pulse is possible at high intensity controlling wave. This distortion less propagation of the pulse can be termed as soliton propagation. Hence, it is concluded that at high intensity of the controlling wave, soliton exists that can travel through Kerr crystal. Thus, the proposed structure may be used to propagate the soliton pulse and to trap an optical pulse by switching the intensity of the controlling

wave from low to high and high to low respectively. The proposed structure may also be used as active WDM coupler, optical switch and filter.

#### Acknowledgments

One of the authors Arun Kumar is highly thankful to Dr. Ashok K. Chauhan, Founder President, Amity University, Uttar Pradesh, India for his interest in research and constant encouragements. He is also grateful to Lt. Gen. P. D. Bhargava, Director General AITTM, Deputy Vice Chancellor, Amity University, Noida, Uttar Pradesh, India for his moral support.

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